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THE

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CHAP. VII.

OF ALGEBRA.

SECT. I.

OF NOTATION.

Definitions.

1. ALGEBRA is the art of reſolving difficult queſtions more readily than by the rules of common arithmetic.

In algebra, the value of quantities is expreſſed by ſome letters of the alphabet, which have ſometimes figures, and certain characters added to them, whereby their value is increaſed, or diminiſhed; and each letter may repreſent any quantity at pleaſure. But, generally, the firſt letters in the alphabet, *a*, *b*, *c*, *d*, &c. are uſed to ſignify quantities, the value whereof is known; and the latter letters, as *w*, *x*, *y*, *z*, &c. are uſed for quantities which are unknown: the letters are then managed according to the rules of art.

2. The ſign + ſignifies *addition*, and in algebra it is called *plus*; it denotes that the characters or letters placed on each ſide of it are to be added together, thus, $a+b$ ſignifies that

the quantity expreſſed by a is to be added to that repreſented by b. Thus, if a ſtand for 3, and b for 6; then $a+b$ will be equal to 9.

3. The ſign — ſignifies *ſubtraction*; and ſhows that the quantity following it is to be ſubtracted from the quantity preceding it: thus, $a-b$ ſignifies that the quantity repreſented by b is to be ſubtracted from that repreſented by a; as, if a was 8, and b was 3, then $a-b$ would be equal to 5: this ſign is called *minus*.

4. The ſign + repreſenting addition, is called a *poſitive*, or an *affirmative* ſign. The ſign —, ſignifying ſubtraction, is called a *negative* ſign.

5. Like ſigns are when ſeveral quantities have all the ſign + or —; and unlike ſigns are quantities where ſome have the ſign +, and others the ſign —.

6. The ſign = denotes *equality*, and is placed between two quantities, to ſhow they are equal: thus, $a=b$ ſignifies that a and b are equal to each other.

7. The ſign × ſtands for *multiplication*, and ſignifies that the quantities placed on each ſide are to be multiplied together: thus, $a\times b$ ſignifies the quantity a is to be multiplied by the quantity b; as, if a be equal to 5, and b equal to 6, they will, with the product, ſtand thus $a\times b=30$, which ſignifies that a multiplied by b is equal to 30. But the product of two or more ſimple quantities is generally ſignified by merely joining the letters. Thus, the product of the above quantity is expreſſed $a\,b$, and if there be three or more quantities to be multiplied together, as $a\times b\times c$, they will be expreſſed thus, $a\,b\,c$.

8. The ſign ÷ expreſſes *diviſion*: thus, $a\div b$ ſignifies that a is to be divided by b; but this ſign is not much uſed, for diviſion is generally expreſſed in the manner of a fraction: thus, $\frac{a}{b}$ and $\frac{a-b}{c+d}$ ſignifies that a is to be divided by b, and $a-b$ divided by $c+d$.

9. The ſign ∽ ſignifies the *difference* between two quan-

ties: thus, $a \backsim b$ ſtands for the difference between a and b. Thus if a ſtand for 9, and b for 4, $a \backsim b$ repreſents 5.

10. The ſign $\sqsubset$ or 7 are ſigns of *majority*, and ſhow that the quantity placed before the ſign is greater than that which follows it: thus, $a \sqsubset b$, or $a \, 7 \, b$, ſhows that a is greater than b.

11. The ſign $\sqsupset$ or $\angle$ ſignifies *minority*, and ſhows that the quantity placed before the ſign is leſs than that which follows it; thus, $a \angle b$, or $a \sqsubset b$, ſignifies that a is leſs than b.

12. The ſign $\sqrt{\ }$ is the ſign of the *ſquare root*. It alſo expreſſes the cube root, biquadrate root, &c. by placing 3 or 4, &c. over it; thus, $\sqrt{a}$, or $\sqrt[3]{a}$, or $\sqrt[4]{a}$, denote the ſquare root, cube root, and biquadrate root of a reſpectively.

13. *Involution* is the raiſing of a quantity to any power, according as it is joined to the figures 2, 3, 4, &c. reſpectively.

14. The ſign *lu* ſignifies *evolution*, and denotes that the quantity to which it is joined is the ſquare or cube root, &c. as it is joined to the numbers 2, 3, &c. reſpectively.

The *power* of a quantity is often expreſſed in algebra by placing a figure over the quantity; thus, a^2, a^3, and a^4, denote the ſquare, cube, and biquadrate, of a reſpectively; or the ſecond, third, and fourth power; and the figures 2, 3, and 4, placed over a, are called the indices or exponents of a.

15. *Like quantities* are thoſe that conſiſt of the ſame letters, as a, $4a+2a$, or $b-2b+3bb$, &c.

16. *Unlike quantities* conſiſt of different letters; as a, $2b$, $3c$; or $2a$, $cd-d$.

17. *Simple quantities* conſiſt of one term only; as $4b$, or $3a^2$, or $12d$, &c.

18. *Compound quantities* conſiſt of ſeveral terms; as $a+c$, $2b-d$, &c.

19. A vinculum is a line drawn over ſeveral quantities, and ſhows that they are to be taken as a compound quantity; as $\overline{a+b-c}$.

20. The *coefficient* of a quantity is the number prefixed

to it; as $6d$; here 6 is the coefficient, and ſignifies that the quantity d is multiplied thereby.

21. A *binomial quantity* conſiſts of two terms; as $b+c$. A *trinomial quantity*, of three terms; as $a+b+c$. A *quadrinomial quantity*, of four terms; as $a+b+c+d$.

22. A *reſidual quantity* is a binomial, where one of the terms is a negative one; as $a-b$.

23. A *rational quantity* has no radical ſign.

24. A *ſurd quantity* is that which has not a proper root; as the ſquare root of b ($\sqrt{b}$), the biquadrate root of bb ($\sqrt[4]{bb}$).

25. The ſign : :: : ſignifies *proportion*; as 5 : 10 :: 40 : 80, that is, as 5 to 10, ſo is 40 to 80.

26. An *equation* is the compariſon of two quantities which are equal to one another, having the ſign of equality between them, as 5, 9=6, 8, which ſignifies that 5 and 9 are equal to 6 and 8, or 14. Of equations there are ſeveral ſorts:

1. A *dependant equation* is that which may be deduced from ſome others.—2. An *independent equation*, that which cannot be deducible from another.—3. A *pure equation*, that which contains but one power of the unknown quantity.—4. An *affected equation*, that which has ſeveral powers of the unknown quantity.

Axioms.

1. If equal quantities be added to equal quantities, the ſums will be equal. And if equal quantities be taken from equal quantities, the remainders will be equal.

2. If equal quantities be multiplied by equal quantities, the products will be equal. And if equal quantities be divided by equal quantities, the quotients will be equal.

3. If equal quantities be raiſed to equal powers, the products will be equal.

4. Quantities equal to any other quantity are equal to one another.

5. The whole is equal to all its parts taken together.

SECT.

SECT. II.

OF THE FOUR SINGLE RULES OF ALGEBRA.

OF ADDITION.

RULE 1. When like quantities having like ſigns are to be added together, add together the coefficients (if there be any), and to the ſum prefix the ſign, and ſubjoin the common quantity.

2. When the quantities are like, but have unlike ſigns, take the difference between the ſum of the affirmative coefficients, and the ſum of the negative ones; to which difference prefix the ſign of the greater ſum, and annex the common quantity.

3. But when the quantities are all unlike, they cannot be brought into one ſum, but muſt be written down one after another, prefixing to each its proper ſign; as in the following examples:

$7ab$	$12a^2c+2ab$	$-10x^2y^2-2a^2$
$12ab$	$9a^2c+5ab$	$-7x^2y^2-7a^2$
$9ab$	$10a^2c+2ab$	$-4x^2y^2-2a^2$
$2ab$	$4a^2c+3ab$	$-31x^2y^2-a^2$
$30ab$ Sum.	$35a^2c+12ab$ Sum.	$-52x^2y^2-12a^2$ Sum.

Where there is no coefficient prefixed to a quantity, the coefficient is 1. And when there is no ſign prefixed, the quantity is affirmative; as in the quantities of the firſt and ſecond of the foregoing examples.

Examples of like Quantities, and unlike Signs.

$$\begin{array}{l}
-3ac-7cd+de \\
+7ac+4cd-de \\
-6ac-2cd+de \\
\hline
-2ac-5cd+de
\end{array}
\qquad
\begin{array}{l}
+12a^2b^2-de-12x^2 \\
+\ 4a^2b^2+de+\ 6x^2 \\
+\ 5a^2b^2-de-10x^2 \\
\hline
+21a^2b^2-de-16x^2
\end{array}$$

Unlike Quantities, and unlike Signs.

$$\begin{array}{l}
+15a-6a^2-2b^2+5cd \\
-2de+4xy-x^2+xy \\
+3b^2-2x+y^2 \\
\hline
+15a-6a^2+b^2+5cd-2de+5xy-x^2-2x+y^2
\end{array}$$

SUBTRACTION.

Rule. Place the quantities one under the other, and change all the ſigns of the ſubtrahend; that is, where there is an affirmative ſign, place a negative one; and *vice verſa.* Then add the quantities together, as in addition.

Examples.

$$\begin{array}{ll}
\text{From} & +2a^2b^2-cd \\
\text{Take} & +\ a^2b^2+cd \\
\hline
 & +\ a^2b^2-2cd
\end{array}
\qquad
\begin{array}{ll}
\text{From} & -\ ab+6a^2-9cd \\
\text{Take} & +4ab-2cd+x^2 \\
\hline
 & 5-ab+6a^2-7cd-x^2
\end{array}$$

Subtraction, as well as each of the other four rules in algebra, is proved in the ſame manner as in common arithmetic *.

---

* The reaſon of this rule is evident from hence, that if a decrement or a negative quantity be taken away from an affirmative quantity, the remainder will be the ſame as if an increment or an affirmative quantity of equal quantity be added to the original quantity. For every negative quantity always decreaſes the value of any quantity with which it is joined. Thus, if $-b$ be taken from $a-b$, there will remain a, and if $+b$ be added to $a-b$, the ſum is likewiſe a.

MULTI-

MULTIPLICATION.

Rule. Multiply each term of the multiplier into every term of the multiplicand: that is, the coefficients into the coefficients, and the letters into the letters; and to each prefix its ſign; viz. + to like ſigns, and — to unlike ſigns.

Examples.

Multiply	$7a$	Multiply	$3d^2$	Multiply	$-5c^2d^2$
By	$3b$	By	$5c^2$	By	$+7c^2d^2$
Product	$21ab$	Product	$15d^2c^2$	Product	$-35c^4d^4$

Mult.	$7a-3c$	Multiply	a^2-ab+c^2
By	$12a+4c$	By	$-a+2b^2$
	$84aa-36ac$		$-a^3+a^2b-ac^2$
	$+28ac-12cc$		$+2a^2b^2-2ab^3+2c^2b^2$
Pro.	$+84aa-8ac-12cc$	P.	$-a^3+a^2b+2a^2b^2-ac^2-2ab^3+2c^2b^2$

The foregoing examples may be proved by diviſion, as in common arithmetic *.

---

* This rule depends upon the ſame principle as multiplication in common arithmetic. And that two quantities having like ſigns, ſhould give a product with the ſign +, and two quantities of unlike ſigns the ſign —, may be proved from hence: viz. 1. If an affirmative quantity be multiplied by an affirmative quantity, the product muſt of courſe be an affirmative quantity.—2. If a negative quantity be multiplied by an affirmative one, the negative quantity muſt be taken as often as there are units in the affirmative one, and the ſum of any number of negative quantities will be negative. And if an affirmative quantity be multiplied by a negative one, the affirmative quantity muſt be ſubtracted as often as there are units in the negative one; and the ſum of any number of negatives will be negative.—3. Again, if a negative quantity be multiplied by a negative quantity, the multiplicand is to be ſubtracted, as often as there are units in the multiplier: but, to ſubtract a negative quantity is the ſame thing as to add an equal affirmative one: therefore, the product will be affirmative. From hence the general rule, that like ſigns produce +, and unlike ſigns —.

DIVISION.

Rule. If the quantities be ſimple, divide the coefficient of the dividend, by the coefficient of the diviſor, and place the anſwer in the quotient; annexing thereto thoſe letters in the dividend, which are not found in the diviſor: obſerving that like ſigns produce +, and unlike ſigns —.

2. But if the quantities be compound, divide the firſt term of the dividend by the firſt term of the diviſor, and place the reſult in the quotient. Then multiply the whole diviſor thereby, and ſubtract the product from the dividend, and to the remainder bring down the next term in the dividend. And repeat the operation as in common arithmetic. But the terms in the dividend ſhould be ranged in a proper order, that is, according to the dimenſions of ſome letter; the quantities repreſented by *a* being generally placed firſt; thoſe by *b* or *a b* next, as follows;

Examples.

$$\begin{array}{rl} 9ac) & 36abcd(4bd \\ & 36abcd \\ \hline & \qquad\quad 0 \\ \hline\hline \end{array} \qquad\qquad \begin{array}{rl} 4a^2b^2) & -24a^2b^3d(-6bd \\ & \;\;24a^2b^3d \\ \hline & \qquad\qquad 0 \\ \hline\hline \end{array}$$

$$\begin{array}{rl} 4a-6c) & 16abb-24cbb(4bb \\ & 16abb-24cbb \\ \hline & \qquad\qquad\qquad 0 \\ \hline\hline \end{array}$$

$$\begin{array}{rllll} 3a-b) & 3a^3-12a^2 & -ba^2 & +10ab-2b^2 & (a^2-4a+2b \\ & 3a^3 & \;\;ba^2 & & \\ \hline & \;\;-12a^2 & & +10ab & \\ & \;\;-12a^2 & & +\;4ab & \\ \hline & & & +\;6ab-2b^2 & \\ & & & +\;6ab-2b^2 & \\ \hline & & & \qquad\quad 0 & \\ \hline\hline \end{array}$$

When the diviſor will not exactly divide the dividend, as is often the caſe, the dividend is to be placed over the diviſor, and a line drawn between them, like a fraction, throwing out ſuch letters as are found in both the diviſor and dividend. Thus, If $ab+c^3$ was to be divided by $ab-c^2$, it would ſtand thus, $\frac{ab+c^3}{ab-c^2}=\frac{+c^3}{-c^2}$. When the power of a quantity is to be divided by any other power of the ſame quantity, it is done by ſubtracting the exponent of the diviſor, from that of the dividend:

Thus, $\frac{b^8}{b^4}=b^4$*.

SECT. III.

OF FRACTIONAL QUANTITIES.

BEFORE the ſtudent proceed to equations, it is neceſſary that he know how to manage fractional quantities; and to raiſe a quantity to any given power; and, on the contrary, to extract the root of any quantity; to manage ſurd quantities, &c.

The rules for managing Algebraic Fractions are exactly the ſame as thoſe for Vulgar Fractions in arithmetic, and therefore need not be repeated; as few perſons would attempt Algebra, till they were ſufficiently ſkilled in common arithmetic. An example or two may, however, be of ſervice.

---

* To prove the reaſon of this rule, that like ſigns give +, and unlike ſigns −, it is only neceſſary that the diviſor be multiplied by the quotient, and the product will be equal to the dividend.

Example 1. Reduce the mixed quantity $a-\frac{c}{d}$, to a fraction. Here $\frac{da-c}{d}$, is the fraction required.

Example 2. Let the fraction $\frac{bd-b^2}{d}$, be reduced to a mixed quantity. Here dividing $bd-b^2$ by d, the quantity will be b, and the remainder $-b^2$, and therefore the mixed quantity will be $b-\frac{-b^2}{d}$.

Example 3. Reduce the fractions $\frac{a}{b}\,\frac{c}{d}$ and $\frac{e}{f}$ to fractions of the ſame value, having a common denominator. Here $\frac{adf}{bdf}\,\frac{cbf}{bdf}\,\frac{ebd}{bdf}$ are the fractions required.

Involution; or, to find any given Power of any given Quantity.

Rule. Multiply the quantity into itſelf as often as the index contains units, except one, and the laſt product will be the required power; or, which is more convenient, multiply the index of the quantity by the index of the power.

Thus, let it be required to raiſe the quantities $b+c$ and $b-c$ to the third power, or the cube,

$$\begin{array}{ll}
b+c & \text{Root} \\
\underline{b+c} & \\
b^2+bc & \\
\underline{\quad +bc+c^2} & \\
b^2+2bc+c^2= & \text{Square} \\
\underline{b+c} & \\
b^3+2b^2c+bc^2 & \\
\underline{\quad b^2c+2bc^2+c^3} & \\
\underline{\underline{b^3+3b^2c+3bc^2+c^3=}} & \text{Cube.}
\end{array}$$

$$\begin{array}{l}
b-c \text{ Root}\\
\underline{b-c\qquad}\\
b^2-bc\\
\underline{\quad -bc+c^2}\\
b^2-2bc+c^2= \text{ Square}\\
\underline{b-c\qquad\qquad}\\
b^3-2b^2c+bc^2\\
\underline{\quad -b^2c+2bc^2-c^3}\\
b^3-3b^2c+3bc^2-c^3 \text{ Cube.}
\end{array}$$

Theſe quantities are raiſed to the third power only; but by the ſame method of proceeding, quantities may be raiſed to any higher power.

If a trinomial, or quadrinomial, &c. be required to be raiſed to any power, it will be beſt done by taking the firſt or the laſt term of the quantity, and for all the other terms ſubſtitute any ſingle term. Theſe two terms being raiſed to the required power, the anſwer will be obtained by replacing, inſtead of the ſubſtituted term, the proper value.

Thus, if it be required to raiſe $b-c+d-ef+g$ to the fourth power, for the terms $b-c+d-ef$, ſubſtitute the term a, then the quantity will be $a+g$, which being raiſed to the fourth power, the quantity a may be taken away, and the proper value placed inſtead thereof in the product, which the learner may prove at his leiſure.

In involving a fractional quantity, both the numerator and denominator muſt be raiſed to the required power, by which a new fraction will be obtained, being the anſwer of the queſtion. Thus, the third power $\frac{a^2}{b} = \frac{a^6}{b^3}$.

*The *Binomial Theorem*, invented by *Sir Iſaac Newton*, is the moſt elegant and conciſe method of raiſing a quantity to any power, and is as follows: let n denote any number at pleaſure, and let $a+b$ be a binomial, then the *nth* power thereof will be as follows:

$$\text{by } a^{n}+na^{n-1}b+\frac{n.\overline{n-1}}{1.\ 2}.a^{n-2}b^{2}+\frac{n.\overline{n-1}.\overline{n-2}}{1.\ 2.\ 3}a^{n-3}b^{3}$$

$$+\frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}}{1.\ 2.\ 3.\ 4}.a^{n-4}b^{4}+\frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}}{1.\ 2.\ 3.\ 4.\ 5}a^{n-5}.\ b^{5},\ \&c.$$

And the *nth* power of $a-b$, is expreſſed in the ſame manner, except that the ſigns of every other term will be negative.

To illuſtrate this theorem, let $a+b$ be involved to the third power. In this caſe, the index is 3, which muſt be placed in the theorem inſtead of n, then the firſt term will be a^{3}, the ſecond term $3a^{3-1}b=3a^{2}b$; the third term, $\frac{3\times 2}{2}a^{3-2}b^{2}=3ab^{2}$, the fourth term $\frac{3\times 2\times 1}{2\times 3}a^{3-3}b^{3}=b^{3}$: the fifth and following terms are equal to nothing. Therefore theſe four terms together, or the third power of $a+b$, is $a^{3}+3ab^{2}+3ab^{2}+b^{3}$.

It may here be obſerved that the coefficients increaſe till the indices of the two letters a and b become equal or change values; then they return or decreaſe again in the ſame order: thus, having the coefficients of half the terms, the reſt are known.

Evolution; or, to extract the Root of a given Power.

RULE 1. If the quantities be ſimple, extract the root of the coefficient for the new coefficient, and divide the index of the letters by the index of the power, and the quotient will be the root required.

Thus, the ſquare root of $25b^{2}$ will be $5b^{\frac{2}{2}}=5b$, and the cube root of $27x^{3}=3x^{\frac{3}{3}}=3x$.

RULE 2. If the quantities be compound, after ranging the terms according to the dimenſions of ſome letter, ſo that the higheſt power of that letter may ſtand firſt in order, and the lower powers of the ſame letter follow, according to the dimenſions of their power: take the root of the firſt term, and place it in the quotient, and if it be the ſquare, or cube root, ſubtract the ſquare, or cube thereof, from the firſt term, bring down two, or three of the next terms for a dividend, according as the caſe ſhall be the ſquare, or cube root; then proceed to find the diviſor as in extracting the ſquare, or cube root, in common arithmetic. And if the root of a higher power be to be extracted, it is performed in the ſame manner as in common arithmetic.

Examples.

EXAMPLE 1. What is the ſquare root of

$$36x^4+108x^2+81\,(6x^2+9$$
$$\underline{36x^4\qquad\qquad\qquad}$$
$$12x^2+9)+108x^2+81$$
$$\underline{+108x^2+81}$$

EXAMPLE 2. What is the ſquare root of $9x^4+16a^4+4b^4-24a^2x^2+12b^2x^2-16a^2b^2$. Anſwer $3x^2-4a^2+2b^2$.

EXAMPLE 3. What is the cube root of

$$x^3-6x^2y+12xy^2-8y^3\ (x-2y \quad \text{Root}$$
$$\underline{x^3\qquad\qquad\qquad\qquad}$$
$$3x^2 6xy+4y^2)\ -6x^2y+12xy^2-8y^3$$
$$\underline{-6x^2y+12xy^2-8y^3}$$

Surd Quantities.

When a quantity has not a perfect root, it is called a ſurd quantity; and the root cannot be expreſſed any other way, than by either inſerting the quantity with its proper radical ſign, or throwing it into an infinite ſeries. Thus, the ſquare root of a, can be expreſſed in no other way than by $\sqrt{a}$, or $a^{\frac{1}{2}}$; the cube root of b^2 by $\sqrt[3]{b^2}$ or $b^{\frac{2}{3}}$; the cube root of $\frac{a^2b}{c^2}$ by $\sqrt[3]{\frac{a^2b}{c^2}}$.

To reduce a rational Quantity to the Form of a Surd.

Rule. Multiply the index of the quantity by the index of the ſurd, and over the product place the radical ſign, and it will be the form required. Thus, let 3 be reduced to the form of $\sqrt{5}$. Here $3^1\times{}^2$, or $3^2=9$, therefore, $\sqrt{9}$ is the ſurd required.

Again, let a^2 be reduced to the form of a cube ſurd of the form of $\sqrt[3]{b}$ Here $a^2\times{}^3=a^6$ and $\sqrt[3]{a^6}$ is the ſurd quantity.

To reduce Quantities of different Indices to other Quantities equal in Value, and having one given Index.

Rule. Divide the indices of the quantities by the given index, and the quotients will be the new indices of thoſe quantities. Then over the ſaid quantities with their new indices place the given index, and they will be the equivalent values required.

Example. Let $12^{\frac{1}{2}}$, and $9^{\frac{1}{4}}$, be reduced to equal quantities, having the common index $\frac{1}{3}$. Here $\frac{1}{2}\div\frac{1}{3}.=\frac{3}{2}=$, the index of the firſt quantity, and $\frac{1}{4}\div\frac{1}{3}=\frac{3}{4}=$, the index of the ſecond quantity, therefore, $\overline{12^{\frac{3}{2}}}\big|^{\frac{1}{3}}$ and $\overline{9^{\frac{3}{4}}}\big|^{\frac{1}{3}}$, are the quantities required.

To reduce a Surd Quantity to its moſt ſimple Terms.

Rule. Divide the ſurd by the greateſt power which it contains, and place the root of ſuch power before the quotient with the radical ſign between them.

Thus, let $\sqrt{32}$ be reduced to its moſt ſimple terms. Here 16 is the greateſt ſquare number that will divide 32, which being divided by 16, the quotient is 2, therefore, $4\sqrt{2}$ is the ſurd required.

Again the moſt ſimple term of the ſurd $\sqrt{81a^2b}$ is $9a\sqrt{b}$.

To find whether Surds are commenſurable, or not.

Rule. Reduce the ſurds to the leaſt common index; and the quantities, if fractions to a common denominator, except when like terms are commenſurable; then divide them by the greateſt common diviſor, or by ſuch a diviſor as will give one rational quotient, and if both the quotients are rational, the ſurds are commenſurable; otherwiſe, not.

Example. Let $\sqrt{27}$ and $\sqrt{12}$ be given to find whether they are commenſurable; theſe two ſurds have already one common index, and are equal to $\sqrt{3 \times 9}$, and $\sqrt{3 \times 4}$, reſpectively. Therefore, divide 27 and 12 by 3, and the quotients are $\sqrt{9}$ and $\sqrt{4}$, that is, 3 and 2; therefore, they are commenſurable.

To add, or ſubtract Surd Quantities.

Rule. If the quantities have unlike indices, reduce them to quantities with like indices; and fractional quantities muſt be reduced to a common denominator, or to other fractions that have rational denominators or numerators; then reduce the quantities to their ſimpleſt terms, and if the ſurd part be the ſame in all, annex it to the ſum or difference with the ſign $\times$; but if the ſurd part is not the ſame in all, the quantities muſt be added or ſubtracted by joining them together with the ſign + or −.

Example 1. Let $\sqrt{32}$ $\sqrt{72}$ be added together. Here $\sqrt{32}$ $=\sqrt{\overline{16\times 2}}=4\sqrt{2}$; and $\sqrt{72}=\sqrt{36\times 2}=6\sqrt{2}$, and the ſum $=4+6\times\sqrt{2}=10\sqrt{2}$.

Example 2. Let $\sqrt{4a}$, and $\sqrt[4]{a^6}$ be added together. Here $\sqrt{4a}=\sqrt[4]{\overline{16a^2}}=2\sqrt[4]{a^2}$. And $\sqrt[4]{a^6}=\sqrt[4]{\overline{a^4\times a^2}}=a\sqrt[4]{a^2}$. Therefore, their ſum $=\overline{a+2}\times\sqrt[4]{a^2}=\overline{a+2}\times\sqrt{a}$. If it were required to have ſubtracted $\sqrt{4a}$ from $\sqrt[4]{a^6}$, the remainder would have been $\overline{a-2}\times\sqrt{a}$. Alſo if $\sqrt[3]{a^2}-\sqrt{a^3}+\sqrt{7}$ be added to $2\sqrt[3]{a^2}+\sqrt{a}-\sqrt{3}$, the ſum will be $3\sqrt[3]{a^2}-\sqrt{a^3}+\sqrt{7}+\sqrt{a}-\sqrt{3}$.

To multiply and divide Surds.

Rule. Reduce the ſurds to the ſame index; and the product or quotient of the rational quantities being annexed to the product or quotient of the ſurds, will give the product or quotient required.

Example 1. Multiply $2\sqrt{2}$ by $3\sqrt{3}$, theſe ſurds have the ſame index already, therefore, $7\times 3\times\sqrt{\overline{2\times 3}}=6\sqrt{6}$, thus, $6\sqrt{6}$ is the product required.

Example 2. Multiply $a^{\frac{1}{3}}$ by $b^{\frac{1}{4}}$. Here $a^{\frac{1}{3}}=\overline{a^4}\rceil^{\frac{1}{12}}$, and $b^{\frac{1}{4}}=\overline{b^5}\rceil^{\frac{1}{12}}$, therefore, their product is $\overline{a^4b^3}\rceil^{\frac{1}{12}}$.

Example 3. Let $x^{\frac{1}{2}}$ be divided by $x^{\frac{1}{3}}+y^{\frac{1}{2}}$, this is the ſame as if the dividend $x^{\frac{1}{2}}$ was multiplied by $\frac{1}{x^{\frac{1}{3}}+y^{\frac{1}{2}}}$; therefore, the quotient $\overline{\frac{x^3}{x^2+y3}}.\Big|^{\frac{1}{6}}$

Involution or Evolution of Surd Quantities.

Rule. If the ſurd be a ſimple quantity, multiply the index of the quantity by the index of the power, to which the ſurd is to be involved; or by the fraction, expreſſing the root to which it is to be evolved; and if there is a rational part, its proper power or root is to be prefixed thereto.

Compound ſurds are involved and extracted as integers or rational quantities, having regard to the operation of ſimple ſurds.

Example 1. What is the ſquare of $a\sqrt{x}$? Here the ſquare of $a=aa=a^2$ and $\sqrt{x}=x$; therefore the ſquare of $a\sqrt{x}=a^2x$.

Example. What is the cube of $a\sqrt{x}\frac{1}{2}$? Here the cube of $a=a^3$, and the cube of $\sqrt{x}\frac{1}{2}=\sqrt{x}^{1\,5}$, therefore the cube of $a\sqrt{x}\frac{1}{2}=a^3\sqrt{x}^{1\,5}$.

Example 3. What is the cube root of $\overline{a+x}|\frac{1}{2}$? Here $\overline{a+x}|\frac{1}{2}\times\frac{1}{3}=\overline{a+x}|\frac{1}{6}$.

SECT. IV.

OF EQUATIONS.

An Equation is the mutual comparing of two equal quantities, having the ſign = between them. Thus, if a be equal to 3, and b to 6, and c to 4, and d to 13, then, a added to b, will be equal to d made leſs by c; and is thus expreſſed in algebra $a+b=d-c$.

To reduce an Equation.

When a queſtion is brought to an Equation, in order to underſtand the value thereof, the quantity or quantities ſought muſt be placed on one ſide of the equation, and the known quantities on the other ſide. For this purpoſe, the following rules muſt be attended to:—

Firſt. When any quantity is expreſſed on both ſides of the equation, it may be entirely rejected, or thrown out of both. Thus, if $3d+7x=4c-2b+7x$. Here $7x$ ſhould be rejected from both ſides of the equation, then it will ſtand thus, $3d=4c-2b$.

Second. When known and unknown quantities are both on the ſame ſide of the equation, the known quantities muſt be brought to one ſide of the equation, and the unknown quantities to the other ſide, and thoſe quantities ſo tranſpoſed muſt have their ſigns changed. That is, thoſe which have the ſign +, muſt, after they are tranſpoſed to the other ſide, have the ſign —. And thoſe which have the —, muſt, after tranſpoſition, have the +. Thus, if $10+5=x-5$: here if x be the quantity ſought, -5 muſt be tranſpoſed to the other ſide of the equation with the ſign +; it will then ſtand thus, $+5+10+5=x$: therefore $x=20$. Again, what is the value of x in this equation? $24-4x+10=60-12x$. Here $+24$ and $+10$ on the firſt ſide of the equation muſt be transferred to the other ſide, and $+12x$ on the ſecond ſide of the equation being transferred to the firſt ſide, it will ſtand thus, $-4x+12x=60-24-10$. And by ſubtracting $4x$ from $12x$, and 24 and 10 from 60, the equation will be $8x=26$: therefore $x=3\frac{1}{4}$.

Third. If there be fractions in the equation, multiply both ſides of the equation by the denominators of the fractions; and the product will be the true integral quantities.

Example 1. Reduce the fractional equation $a+\frac{b^2}{x}=c$ to integral quantities. Here, by multiplying the whole by x, we ſhall have $ax+b^2=cx$.

Again, If there be given $\frac{a^2}{a+x}+\frac{b^2}{x}=c$, then will $a^2x+b^2\overline{a+x}=\overline{ca+x}\times x$.

Fourth. If in the unknown quantity there be a ſurd, all the other terms muſt be tranſpoſed to the contrary ſide, and each ſide of the equation involved according to the index of the ſurd; and if there be more ſurds than one, the operation muſt be as often repeated as there are ſurd quantities. Thus, if $\sqrt{x^2+ax}+d=c$, by tranſpoſing d, the equation will be $\sqrt{x^2+ax}=c-d$, and by ſquaring both ſides, the equation is $x^2+ax=c^2-2cd+d^2$, thus the equator is freed from the ſurd.

Fifth. When any quantity is multiplied into both ſides of the equation, or into the higheſt term of the unknown quantity, divide the whole equation thereby. Thus, the equation $5bx^2=3bc$ is divided by b, and it becomes $5x^2=3c$; again if it were divided by 5 it would be $x^2=\frac{3c}{5}$.

Sixth. When the ſide of the equation containing the unknown quantity is a pure power, or when being affected it has a rational root, extract the root from both ſides of the equation. Thus in the equation $a^2=b^2+ax$, the equation will be $a=\sqrt{b^2+ax}$. Again, if $x^2+4x+9=25c$ be given by taking the ſquare root we have $x+2x+3=5c$.

Each or all of the foregoing rules are to be uſed as may be neceſſary, till the equation be brought to a proper form.

Examples, wherein the foregoing Rules appear.

EXAMPLE 1. What is the value of x, in the equation $10+\frac{36}{12-x}=16$.

By ſubtracting 10 from each ſide of the equation, we have $\frac{36}{12-x}=6$, both ſides of which divided by 6, the quotient is $\frac{6}{12-x}=1$, this multiplied by $12-x$, gives $6=12-x$, whence by tranſpoſing x and 6, we have $x=12-6$, or $x=6$.

EXAMPLE 2. What is the value of x in the equation $\frac{ax^2+ac^2}{a+x}=ax+b^2$. Here multiplying by $a+x$, there comes out $ax^2+ac^2=\overline{ax+b^2}\times\overline{a+x}$, or $ax^2+ac^2=a^2x+ab^2+ax^2+b^2x$, which tranſpoſed and ordered according to the foregoing rules, is $a^2x+b^2x=-ab^2+ac^2$, wherefore, $x=\frac{-ab^2+ac^2}{a^2+b^2}$, ſo that if $a=1$, $b=2$, $c=3$, then will $x=\frac{-4+9}{1+4}=1$.

EXAMPLE 3. What is the value of x in the equation $\sqrt{a^2+x^2}=\sqrt[4]{b^4+x^4}$. Both ſides of this equation being raiſed to the fourth power, we have $a^4+2a^2x^2+x^4=b^4+x^4$, which by tranſpoſition, &c. becomes $2a^2x^2=b^4-a^4$, which divided by $2a^2$, becomes $x^2=\frac{b^4-a^4}{2a^2}$, therefore $x=\left.\overline{\frac{b^4-a^4}{2a^2}}\right|^{\frac{1}{2}}$.

EXAMPLE 4. What is the value of x in the following equation: $x=\sqrt{c^2+x\sqrt{b^2+x^2}}-c$. In this equation $x+c=\sqrt{c^2+x\sqrt{b^2+x^2}}$, which ſquared gives $x^2+2cx+c^2=c^2+$

$x\surd$

$x\sqrt{b^2+x^2}$, or $x^2+2cx=x\sqrt{b^2+x^2}$; dividing this by x, it quotes $x+2c=\sqrt{b^2+x^2}$; this ſquared gives $x^2+4cx+4c^2=b^2+x^2$, and by tranſpoſition it becomes $4cx=b^2-4c^2$; and dividing by $4c$ we have $x=\frac{b^2-4c^2}{4c}$.

To exterminate an unknown Quantity out of ſeveral Equations; or, to reduce two or more Equations to a ſingle one.

Rule. If the quantity to be exterminated has but one dimenſion in the equation, find the value of it in two equations, and put thoſe values equal to each other; or having found the value in one equation, ſubſtitute it in the room of the quantity in the other equations. Proceed in the ſame manner with every unknown quantity. But if the quantity to be exterminated be of ſeveral dimenſions, find the value of its higheſt power in two equations. Then if the coefficients are not the ſame, multiply the leſs quantity, ſo that it may become equal to the greater. Put theſe values equal to each other, and there will ariſe a new equation, with a leſs power of the unknown quantities: and the operation muſt be repeated till the quantity be exterminated.

Examples:

Example I. What is the value of x and y in theſe two equations, $7x-5y=28$ and $3x+4y=55$? By tranſpoſing 28 in the firſt equation, and $5y$, we have $7x-28=5y$, therefore the value of y is $\frac{7x-28}{5}$.

In the ſecond equation by proceeding in the ſame manner, viz.:—By tranſpoſing $3x$, the value of y is found to be $\frac{55-3x}{4}$; therefore, theſe two quantities being put equal to each other, we have the equation $\frac{7x-28}{5}=\frac{55-3x}{4}$. In this equation only x is concerned. Multiply this equation by 20, which is the production of 4 and 5; or, which is the ſame thing, multiply the numerators and the denominators croſsways, and there will ariſe the equation $28x-112=275-15x$, which by tranſpoſition becomes $43x=387$, or $x=\frac{387}{43}$; therefore, $x=9$; therefore, 9 being ſubſtituted in either of the given equations inſtead of x, the value of y will be found. Thus, in the firſt equation if 9 be ſubſtituted for x, it will be $63-5y=28$, which tranſpoſed is $\frac{63-28}{5}=y$, or $7=y$.

Example 2. Required the value of x, y, and z, in the three following queſtions:

$x+100=y+z$; $y+100=2x+2z$; $z+100=3x+3y$, by tranſpoſing 100 in the firſt equation, $x=y+z-100$ ariſes, which value ſubſtituted in the other two equations, inſtead of x, we have the two following:—

$y+100\ (=2y+2z-200+2z)=2y+4z-200$

$z+100\ (=3y+3z-300+3y)=6y+3z-300$, then by tranſpoſing y and $4z-200$, in the firſt of theſe two equations we have $300-4z=y$, which ſubſtituted for the y in the laſt equation, is $z+100=1800-24z+3z-300$, that is $z+100=1500-21z$; wherefore $22z=1400$, or $z=\frac{1400}{22}=63\frac{7}{11}$: therefore, $y=300-4z=45\frac{5}{11}$, and $x=y+z-100=9\frac{1}{11}$.

Of

Of the Nature and Compoſition of Equations, containing different Dimenſions of the ſame unknown Quantity.

It often happens that the unknown quantity will be of ſeveral different dimenſions; then ſuch equation is called a quadratic, a cubic, a biquadratic equation, &c. according as the dimenſion of the higheſt power is a ſquare, cube, or biquadrate: in ſuch equations we muſt diſcover the root or value of the unknown quantities.

All equations are derived (or may be conſidered ſo) from thoſe of a more ſimple form. Thus, if $x-b=0$, which is a ſimple equation, be raiſed to the ſecond power, there ariſes $x^2-2bx+b^2=0$, which is called a quadratic equation; if the former equation be raiſed to the third power, we have $x^3-3bx^2+3b^2x-b^3=0$, which is called a cubic equation, and ſo on. It is but ſeldom that equations occur in this regular form, for the coefficients of the terms will generally be more or leſs than thoſe produced by the involution of one quantity, as $x-b$, and therefore a quadratic equation is a compound one, generally derived from $\overline{x-b\times x-c}$. A cubic equation is derived from $\overline{x-b}\times\overline{x-c}\times\overline{x-d}$. A biquadratic equation from $\overline{x-b}\times\overline{x-c}\times\overline{x-d}\times\overline{x-e}$, or from a quadratic ſquared, &c. But, the letters b, c, d, &c. may have either affirmative or negative ſigns.

In equations of this nature, as the whole is equal to nothing, it is obvious that ſome or other of the factors muſt be equal to nothing. It is alſo evident that any ſuch equation may be divided by its factors, till there remain only one factor; and as each of the inferior equations obtained by ſuch diviſion muſt ſtill be equal to nothing, it muſt follow that each of theſe factors themſelves are equal to nothing; therefore, b, c, d, e, &c. exhibit ſo many different values of x with contrary ſigns; therefore every equation has as many roots as there are dimenſions of the unknown quantity in its higheſt power. And where b, c, d, e, &c. are found negative, x is affirmative; and where any of theſe are affirmative, x is negative. By multiplying the factors or roots together, under different ſigns, it is obſervable than when b, c, d, &c. are all negative, or, which is the ſame thing, when all the values of x are affirmative, the ſigns in the equation are +, and — alternately. But when there is a negative root, one affirmative quantity will follow another; therefore, there will be as many affirmative roots in the equation as there are changes of the ſigns from + to —, and from — to +, and all the reſt will be negative.

What is here delivered, concerns only poſſible roots. An impoſſible root is when b, c, d, &c. denote the ſquare or any other even root of a negative quantity: an equation derived from ſuch roots is an impoſſible or imaginary one: if there be one poſſible root, the equation will admit of one poſſible anſwer.

In the multiplication of the roots of ſuch equations, the coefficient of the ſecond term is the ſum of all the roots with contrary ſines; the coefficient in the third term is equal to the ſum of the rectangles of thoſe roots; or, of all the products that can poſſibly ariſe by combining them two and two: the coefficient of the fourth term is equal to the ſum of all the products that can poſſibly ariſe by the combination of them three and three, &c.; and the laſt term is always equal to the product of all the roots with contrary ſines.

The Reſolution of Quadratic Equations.

If it be a pure quadratic, as $x^2 = b^2$, or $x^2 - b^2 = 0$, it is produced from the rectangle of $x - b$ and $x + b$, and therefore has one affirmative, and one negative root, and the affirmative root is equal in number to the negative. The root in this caſe is found by extracting the ſquare root of the number dented by b^2. Thus, if $x^2 + = 576$, then $x = + \sqrt{576} = + 24$.

All other quadraties are comprehended under ſome of the following forms: viz. $x^2+2bx=\pm d=0$; or, $x^2-2bx\pm d=0$; or, $2bx-x^2\pm d=0$; and this laſt form, by tranſpoſition, becomes the ſame as the ſecond form, only the negative roots are changed into affirmative roots, and the affirmative into negative; therefore we may conſider the two other forms as applicable to all caſes, and in the ſolution of them it will be more commodious to tranſpoſe d, and then they will ſtand thus: $x^2+2bx=\pm d$ and $x^2-2bx\pm d$. Now if to each of theſe equations be added b^2 (the ſquare of half the coefficient of the ſecond term) we ſhall have in the former caſe $x^2+2bx+b^2=\pm d+b^2$, and in the latter caſe $x^2-2bx+b^2=\pm d+b^2$, and by extracting the ſquare roots, the equations become $x+b=\sqrt{\pm d+b^2}$ and $x-b=\sqrt{\pm d+b^2}$ reſpectively; and x in the former caſe $=\sqrt{\pm d+b^2}-b$, and in the latter $=\sqrt{\pm d+b^2}+b$ which expreſſions give the affirmative values of x: but the ſquare roots of the above equations may alſo be $-x-b=-\sqrt{\pm b+b^2}$ and $-x+b=-\sqrt{\pm d+b^2}$ reſpectively; and therefore in the former caſe $x=-\sqrt{\pm d+b^2}+b$, and in the latter $=-\sqrt{\pm d+b^2}-b$. That is, if $B=\sqrt{\pm d+b^2}$, where d is + or —, according as it is + or — in the ſecond ſide of the given equation; then in the firſt caſe, where $x^2+2bx=\pm d$ the values of $\pm x$ are $\pm\overline{B+b}$, where b is — or + according as x is affirmative or negative; and in the ſecond caſe, where $x^2-2bx=\pm d$, the values of $\pm x$ are $\overline{\pm B\pm b}$, where b is + or —, as x is + or —.

Examples.

Example 1. What are the two values of x in this equation, $x^2+6x=2295$? Here $b=3$, and $2295=+d$, and $\sqrt{+d+b^2}=\sqrt{2295+9}=48=B$, and $\pm B\mp b=\pm 48\mp 3=+45$, and -51 for the two values of x.

Example 2. What are the two values of x in $x^2-11x=-28$? Here $b=\frac{11}{2}$, and $-28=-d$, alſo $b^2=30\frac{1}{4}$, and

$\sqrt{-d+b^2}=\sqrt{-28+30.25}=\sqrt{2.25}=1.5=B$; and $\pm\overline{B\pm b}=\pm\overline{1.5\pm 5.5}=+7$, and $+4$ the two values of x, which are here both affirmative.

To increaſe or diminiſh the Roots of Equations.

Rule. Subſtitute a new letter for the unknown quantity; the given increment, or + the given decrement; and ſubſtitute the powers thereof in the equation, inſtead of the unknown letter.

Example. Increaſe the roots of the following equation by 2. $x^3+x^2-10x+8=0$. Let $x+2=z$, or $z-2=x$, then $x^3=\overline{z-2}|^3$ $x^2=\overline{z-2}|^2$ and $-10x=-10\times z-2=-10z+20$ then the powers of x being actually involved, and the ſeveral terms collected, we have $x^3+x^2-10x+8=z^3-5z^2-2z+24=0$, where the roots of z are greater than thoſe of x by 2.

Thus all the negative roots of an equation may be made affirmative, by increaſing them with a proper quantity.

To complete a deficient Equation.

Rule. Increaſe or diminiſh the roots of the equation, by ſome given quantity, as ſhown in the laſt example.

To multiply or divide the Roots of any Equation, by a given Quantity.

Rule. Multiply or divide any new letter by the given number, and ſubſtitute its powers in the equation for the unknown quantity.

Example. Divide the roots of the equation $x^3-2x+\sqrt{3}=0$ by $\sqrt{3}$. Here by putting $x=y\sqrt{3}$, and ſubſtituting it for x, we have $3y^3\sqrt{3}-2y\sqrt{3}+\sqrt{3}=0$, which, by dividing by $\sqrt{3}$, is $3y^3-2y+1=0$ for the equation required.

By this rule, fractions and ſurds may be taken out of an equation, viz. by dividing the new letter by the common denominator; or by multiplying the new letter by the ſurd quantity.

To take away any Term out of an Equation.

Rule. Add an unknown quantity to a new letter, and ſubſtitute this ſum and the powers thereof, for the root in the given equation; then any term, or any of thoſe quantities wherein the new letter is of the ſame power, being put into an equation, and made equal to nothing, will give the value of the unknown quantity, which being put into the equation with the new letter, the power of the new letter, which was equated, will vaniſh.

Example.

Suppoſe $x^4-3x^3+3x^2-5x-2=0$
Put $y+e=x$ then $x^4=y^4+4y^3e+6y^2e^2+4ye^3+e4$
$-3x^3=-3y^3-9y^2e-9ye^2-3e^3$
$+3x^2=+3y^2+6ye+3e^2$
$-5x=-5y-5e$
$+2=+2$

If the ſecond term is to be taken away, we have $4y^3e-3y^3=0$; and dividing by y^3, $4e-3=0$, or, $e=\frac{3}{4}$, which ſubſtituted for e, the ſecond term will vaniſh. If the third term is to be taken away, we have $6y^2e^2-9y^2e+3y^2=0$, and dividing by the y^2, we have $6e^2-9e+3=0$, from which quadratic equation e may be determined. In like manner the fourth term may be taken away by ſolving the cubic equation; and the fifth term by ſolving a biquadratic equation, &c.

To reſolve or extract the Root of a cubic Equation.

Rule. Take the ſecond term of the equation away, as taught in the laſt example; then the equation will be in this form, $x^3+ax=b$, and the following general expreſſion will give the value of x.

$$\overline{\frac{b}{2}+\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}}\Big|^{\frac{1}{3}} \qquad \frac{\frac{1}{3}a}{\overline{\frac{-b}{2}+\sqrt{\frac{b^2}{4}+\frac{a}{27}^3}}\Big|^{\frac{1}{3}}}$$

There are ſeveral other particular rules for the ſolution of biquadratic and other higher equations; but the method of approximating the roots of equations has ſuperſeded all the other methods, on account of its diſpatch.

To approximate the Roots of Equations, in general.

Rule. By ſeveral trials, chooſe ſome number to repreſent the unknown quantity, and ſuch a number that approaches pretty near to the true value. Then aſſume ſome letter, as v, to denote the effect or exceſs of the number ſo found, and put that number + or —v, inſtead of the unknown quantity in the equation; by which a new equation will ariſe affected with v only, and known quantities; wherein all the terms that contain two or more dimenſions of v may be rejected as inconſiderable in reſpect to the reſt. This being done, the value of v will be found by a ſimple equation, which, added to or ſubtracted from the ſaid number, according as it was taken, too little or too big, will give a number ſtill nearer the truth. Then with this number and the letter v, proceed as before to find another value of v, which muſt be applied as above: repeat the operation till the unknown quantity be diſcovered to a ſufficient degree of exactneſs.

Example. Let it be required to find the value of x, in this equation, $x^3+24x=587814$. Here by a few trials it will be found that x is ſomething above 80; wherefore, let $80+v=x$; then, $x^3=512000+192000v+1680v+v^3$, and $24x=1920+24v$. Therefore (rejecting thoſe terms affected with v^2 and v^3) we have $513920+19224v=587914$, and $v=\frac{587914-513920}{19224}=\frac{73994}{19224}=3\,.\,8$: this, added to 80, gives 83 . 8 for the approximate value of x; this being ſubſtituted in the equation, it will be found too great; therefore, for another operation take $83.8-v=x$, and $x^3=588480.472-21067.32v$, and $24x=2011.2-24v$, and conſequently $590491.672-21091.32v=587914$, and $v=\frac{2577.672}{21091.32}=$ 0.1222 nearly. Hence $x=83.8-0.1222.=83.6778$. true to five figures, and the ſixth being too much by only two.

If the operation be repeated, it will give the anſwer true to eleven figures. But when five or ſix figures of the root have been obtained, and more exactneſs is ſtill required, it will ſhorten the work to ſeek a correction for v, inſtead of one for the whole root, which may be had by ſubſtituting the laſt value of $v+$ or $-$ (new) v, inſtead of the laſt v in the equation, including all the powers of the laſt v, but rejecting thoſe of the new v (in the equation, thence ariſing), as before.

Thus, in the laſt example, the whole equation, including the terms affected with v^2 and v^3, is $513920+19224v+1680v^2+v^3=587914$, or $19224v+1680v^2+v^3=73994$, and in this equation putting $3\,.\,8-v$, inſtead of v, and rejecting the terms of v^2 and v^3 as before; a correction will be obtained for the laſt found v, which will give the anſwer as above. In like manner, a ſecond or third correction may be found, and the operation carried on to any degree of exactneſs. This rule doubles the number of figures true in the root at each operation. If the term wherein v^2 is found be retained, v will be had by ſolving a quadratic, and then treble the number of figures will be had each time; therefore, if the firſt figure only be taken true, nine or ten figures will be had at two operations. This rule affords various theorems for ſolving particular equations as well as general formulas adapted to all, of which I ſhall give two examples:

Example 1. $x^2+6x=61495=c$. Here, by trial, it will be found that x is above 200; therefore, let $200=r$, then $-b=r-1200$ and $c-r^2-br=20295$. The diviſion is as follows:

The Operation.

$$\begin{array}{rll}
2r+b=406 & & (-45=245=x)\\
v=40 & \Big)\,20895(45=v,\ \text{and}\ r+v-=200 & \\
\hline
2r+b+v=446 & \Big) & \\
40 & \;\underline{1784} & \\
2r\,(\text{new})+b=486 & \Big)\;2455 & \\
v^2=\quad 5 & \Big)\;\underline{2155} & \\
\hline
2r+b+v^2=491 & & \\
\hline\hline
\end{array}$$

Example 2. Again, for a cubic equation, let $x^3+bx^2+dx=c$. Put $r\pm v=x$, ſo will $x^3=r^3\pm 3r^2v+\pm 3rv^2\pm v^3$; $bx^2=br^2\pm 2brv+bv^2$, and $dx=dr\pm dv$. Hence $r^3+br^2\pm\overline{3r^2+2br+d}\times v+\overline{3r+b}\times v^2+dr=c$, ($v^3$ being rejected, as ſmall in compariſon of the reſt,) and by tranſpoſition $\pm\overline{3r^2+2br+d}\times v+\overline{3r+b}\times v^2=c-r^3-br^2-dr$; and $v=\pm\dfrac{c-r^3-br^2-dr}{\overline{3r+2br+d}+\overline{3r+b}v}$, which is wrought after the ſame manner as the laſt example.

To find the Limits of Equations.

When an equation contains ſeveral unknown quantities, it will admit of an infinite number of ſolutions, when both fractional and negative numbers are admitted; for all of them but one may be taken at pleaſure, and their value ſubſtituted in the equation, which quantity will de determined. But ſometimes both fractional and negative quantities will be excluded from an equation, and ſuch equation will be confined to a determinate number of ſolutions. I ſhall, therefore,

fore, aſſign the limits of ſuch equations in the following caſes:

CASE I.

When ſeveral unknown Quantities are in one Equation; to find the Limits.

Rule. Tranſpoſe all the negative quantities to the contrary ſide, that all the terms may be affirmative; then to find the limits of any one quantity, ſuppoſe all the reſt to vaniſh in the equation, then the value of that one will become determinate, and will be one limit thereof. And to know which limit it is, ſuppoſe the other quantities to increaſe, and become of ſome certain value: then if the value of the unknown quantity under conſideration, increaſe, it is the leaſt limit; but if it decreaſe, it is the greateſt.

When fractional quantities are to be excluded, inſtead of ſuppoſing the other quantities to vaniſh, put each of them $=1$, and an equation will ariſe, from which the limits of the remaining quantity will be found as before. Proceed in the ſame manner to find the limits of the other unknown quantities.

Example. What are the limits of x and y, in the equation $4x+5y=67$? Let $y=o$ or be ſuppoſed to vaniſh, and then $4x=67$, and $x=16\frac{3}{4}$. Now let y be ſuppoſed to be equal to ſome quantity; then it is evident, that as y increaſes, x decreaſes, therefore $16\frac{3}{4}$ is the greater limit; wherefore x is leſs than $16\frac{3}{4}$.

If $x=o$ then $5y=67$ and $y+13\frac{2}{5}$. Now if x be ſuppoſed to increaſe, y will decreaſe; and therefore $13\frac{2}{5}$ is the greater limit of y; whence y is leſs than $13\frac{2}{5}$: and the leſs limit of both x and y is o.

CASE II.

To determine the Limits of three or more unknown Quantities, when they are in two Equations.

Rule. Fix upon a quantity to be limited, and expunge one of the other quantities; then there will be had one limiting equation. Do the ſame with another unknown quantity, and there will be had another limiting equation; from each of which equations find a limit for the quantity fixed on.

Example. What is the limit of x in the two following equations? $x+y+z=56$, and $32x+20y+16z=1232$. To exterminate y, multiply the firſt equation by 20, and there ariſes $20x+20y+20z=1120$; ſubtract this from the ſecond equation, and we have $12x-4z=112$; then excluding the fractions, the leſs limit of x in this equation is $9\frac{2}{3}$.

Again, to exterminate z, multiply the firſt equation by 16, and ſubtract the product from the ſecond equation, and there remains $16x+4y=336$. And the greater limit of x in this equation is $20\frac{3}{4}$. Hence x is greater than $9\frac{3}{4}$, and leſs than $20\frac{3}{4}$. In the ſame manner may y and z be limited; and ſo alſo in any other equation.

Of indeterminate Problems.

Example. What is the leaſt integer for the value of x, that will alſo cauſe the value of the following fraction to be an integer? $\frac{ax+b}{c}$

Rule. Divide the denominator (c) by the coefficient (a) of the indeterminate quantity; then divide the diviſor by the remainder, and the laſt diviſor again by the laſt remainder, and continue this operation till an unit only remains. Write down all the quotients in a line, as they riſe, under the firſt quotient,

quotient write an unit, and under the ſecond quotient write the firſt quotient; then multiply theſe two together: to the product add the firſt term of the lower line, or an unit, and place the ſum under the third term of the upper line; multiply in like manner the next two correſponding terms of the two lines together, and add the ſecond term of the lower line to the product; put down the ſum under the fourth term of the upper line; proceed in the ſame manner till you have multiplied by every number in the upper line. Then multiply the laſt number, thus found by the abſolute number (b), in the numerator of the fraction, and divide the product by the denominator; then the remainder will be the true value of x required, provided the number of terms in the upper line be even, and the ſign of b be negative; or that the number be odd, and the ſign of b affirmative: but if the number of terms be even, and the ſign of b affirmative, or *vice verſa*, then the difference between the ſaid remainder and the denominator of the fraction will be the true anſwer.

Operation.

The given fraction $\frac{ax+b}{c} = \frac{71x+10}{89}$

$$\begin{array}{lll}
71)89(1 & 1\,.\,3\,.\,1 & \text{Total quotients} \\
\quad 18)71(3 & 1\,.\,1\,.\,4\,.\,5 & \\
\qquad 17)\underline{18}(1 & \underline{10 = b} & \\
\qquad\quad \underline{\underline{1}} & \underline{\underline{50}} & \text{Product.}
\end{array}$$

Here, if the product 50 be divided by 89, the remainder is 50 = the leaſt value of x.

In this rule it is always ſuppoſed, that a is leſs than c; and that they are prime to each other; for if they were to admit of a common meaſure, whereby b is not diviſible, no integer could be aſſigned for x, ſo as to give the value of the fraction $\frac{ax+b}{c}$ an integer.

Example 2. What is the leaſt value of x and y in whole numbers, in the equation $24x-13y=16$? Here by tranſpoſing $13y$, and dividing by 24, we have $x=\frac{13y+16}{24}$; and by tranſpoſing $-13y+16$, we have $y=\frac{24x-16}{13}$; therefore the leaſt value of $y=8$, and the leaſt value of $x=5$.

To find the Value of a Fraction in an infinite Series.

Rule. Divide the numerator by the denominator, and continue the operation as far as is neceſſary. For in many caſes, after the quotient is continued to a few terms, it may be ſeen how the terms converge, and thus any number of terms may be aſſigned at pleaſure.

Example 1. What is the value of $\frac{1}{1+x^2}$

$$
\begin{array}{llllll}
1+x^2)\,1\,(1-x^2+x^4-x^6+x^8-\&c. & & & & & \\
\quad\underline{1+x^2} & & & & & \\
\quad -x^2 & & & & & \\
\quad \underline{-x^2-x^4} & & & & & \\
 & x^4 & & & & \\
 & \underline{x^4+x^6} & & & & \\
 & & -x^6 & & & \\
 & & \underline{-x^6-x^8} & & & \\
 & & & x^8 & & \\
 & & & \underline{x^8+x^{10}} & & \\
 & & & \underline{\underline{\quad -x^{10}}} & &
\end{array}
$$

If a quantity, which is not a fraction, is to be thrown into an infinite ſeries, it muſt be brought into a fraction, by placing one underneath it, as the denominator.

After a few terms are found in the ſeries, the law by which it converges will ſoon be diſcovered, and the terms may be continued to any number.

Sometimes the ſeries cannot eaſily be diſcovered by reaſon of the coefficients; then it will be neceſſary to aſſume a ſeries with unknown coefficients to repreſent it; which being multiplied or involved as the queſtion requires, and the quantities of the ſame dimenſion being put equal to each other, new equations will be had, wherein the coefficients may be diſcovered.

Example 2. Suppoſe $\frac{1}{a-x}$ be the given quantity, and the aſſumed ſeries be $A+Bx+Cx^2+Dx^3+Ex^4$, &c. $=\frac{1}{a-x}$ Multiply both by $a-x$, and there ariſes $1=aA+aBx+aCx^2+aDx^3+aEx^4$, &c. And $-Ax-Bx^2-Cx^3-Dx^4$, &c. and by equating the coefficients of the ſame powers of x, $aA=1$, $aB-A=0$, $aC-B=0$, $aD-c=0$, $aE-D=0$, &c. Thus, in the firſt equation $A=\frac{1}{a}$; in the ſecond equation $B=\frac{A}{a}=\frac{1}{a^2}$; in the third $C=\frac{B}{a}=\frac{1}{a^3}$; in the fourth $D=\frac{C}{a}=\frac{1}{a^4}$, and in the like manner $E=\frac{1}{a^5}$; therefore, $\frac{1}{a-x}$ brought to a ſeries, is $\frac{1}{a}+\frac{x}{a^2}+\frac{x^2}{a^3}+\frac{x^3}{a^4}+\frac{x^4}{a^5}$, &c.

Some of the Properties of ſquare Numbers.

1. All even ſquare numbers are diviſible by 4; therefore, if a number conſiſts of two even ſquare numbers, it will be diviſible by 4.

2. Any odd ſquare number divided by 4 leaves a remainder of 1; therefore, if a number conſiſting of two odd ſquare numbers be divided by 4, there will be a remainder of 2.

3. Therefore, if a number conſiſting of an odd and an even ſquare number, be divided by 4, there will be a remainder of 1.

4. From hence it follows, that if any number compoſed of two ſquare numbers be divided by 4, it cannot leave a remainder of 3; therefore, a number compoſed of two ſquare numbers, cannot fall within this progreſſion 3, 7, 11, 15, 19, 23, &c.

5. Any number ending in 2, 3, 7, or 8, is not a ſquare number.

6. The ſum of any number of terms of the ſeries 1, 3, 5, 7, 9, 11, &c. beginning with the firſt, is a ſquare number, whoſe root is equal to the number of terms.

7. The difference between any two ſquare numbers is equal to the product of the ſum and difference of their roots. Thus, if a and b be the roots, then $a+b\times a-b=a^2-b^2$, and the ſame is alſo equal to the ſum of the two roots, together with twice the ſum of the roots of all the intermediate ſquare numbers. Thus, the difference between 36 and $9=\overline{6+3}+2\times\sqrt{16+25}=9+18=27$.

To reſolve queſtions of this nature, the chief point is, to make ſuch aſſumptions for the root of the required ſquare, or cube, as ſhall, when involved, cauſe either the given number, or the higheſt power of the unknown quantity, to vaniſh from the equation; whereby at length there will be only one dimenſion of the unknown quantity, and ſo the queſtion will be ſolved by reducing the equation.

SECT. V.

THE RESOLUTION OF SEVERAL ALGEBRAIC PROBLEMS.

Qu. 1. What are thoſe two numbers, the ſum whereof is 108, and the proportion of the leſs to the greater is as 5 to 7?

Let x repreſent the greater number, then $108-x$ is equal to the leſs number; and the proportion of the numbers will be as follows: $108-x : x :: 5 : 7$, theſe four quantities being in a direct proportion, the product of the two means $5x$ is equal to the product of the two extremes $756-7x$, therefore, we have this equation $5x=756-7x$; and by tranſpoſing $7x$, we have $12x=756$. Hence, by dividing 756 by 12, x is found equal to 63, which is the greater number, therefore, $108-63=45$, the leſs number.

Qu. 2. Bought apples at ſix for a penny, and pears at five for twopence. The number of apples and pears together was 100; the money given for the whole was 2*s.* 2*d*; how many were there of each ſort?

Let a repreſent the number of apples, then, $100-a$ will be the number of pears: and as $6 : 1d. :: a : \frac{a}{6}=$, price of the apples. Alſo, as $5 : 2d. :: 100-a : \frac{200-2a}{5}=$, price of the pears; then $\frac{a}{6}+\frac{200-2a}{5}=26$. This equation multiplied by 30, gives $5a+1200-12a=780$. By tranſpoſing and dividing $7a=1200-780=420$, and by diviſion $a=60$, the number of apples; and $100-60=40$, the number of pears.

Qu. 3. It is required to divide the number 128 into four ſuch parts, that if the firſt part be added to 7, the ſecond made leſs by 7, the third multiplied by 7, and the fourth divided by 7, the reſults may be equal among themſelves.

Let the four parts into which the number is divided be repreſented by the letters v, x, y, z. Then $v+7=x-7=7y=\frac{z}{7}$ will repreſent the four quantities; and from the equality of the two firſt equations we have $x=v+7+7$; from the equality of the firſt and third equations $y=\frac{v+7}{7}$, and from the equality of the firſt and fourth $z=\overline{v+7}\times 7=7v+49$; therefore, by collecting theſe together, there ariſes $\overline{v+14}+\frac{v+7}{7}+\overline{7v+49}=128$. By collecting the terms and tranſpoſition $9v+\frac{v+7}{7}=65$; and multiplying this by 7, collecting the terms, tranſpoſing and dividing, we have $v=7$. And hence $x=7+14=21$; $y=\frac{7+7}{7}=2$, and $z=\overline{7+7}\times 7=98$, the ſeveral parts required.

Qu. 4. There are two cubical pieces of marble, the ſide of one exceeding the ſide of the other by three inches, the ſolid inches of both are 2457 inches; what is the length of the ſide of each piece?

Let the ſide of each piece be repreſented by x, then the ſide of the greater will be $x+3$, and $x^3+\overline{x+3}^3=2457$ inches; therefore, $2x^3+9x^2+27x=2430$; this equation ſolved, gives $x=9$, and conſequently $y=12$.

Qu. 5. A gentleman left a ſum of money to be divided among three ſervants, in ſuch proportion, that one half of the ſhare of the firſt, one third of the ſecond ſhare, and one quarter of the ſhare of the third, ſhould be equal to 62*l.*; and one third of the firſt, one fourth of the ſecond, and one fifth of the third, equal to 47*l.*; and one fourth of the firſt, one fifth of the ſecond, and one ſixth of the third, equal to 38*l.*;—what is each ſervant's ſhare?

Put $a=62$, $b=47$, and $c=38$, and let the three ſhares required, be denoted by x, y, and z; then the conditions of the queſtion will ſtand thus: $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=a$. $\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=b$. $\frac{x}{4}+\frac{y}{5}+\frac{z}{6}=c$. Theſe equations brought out of fractions, give $6x+4y+3z=12a$. $20x+15y+12z=60b$. $15x+12y+10z=60c$. Here by ſubtracting the ſecond equation from four times the firſt, in order to exterminate z, there ariſes $4x+y=48a-60b$; then, by taking three times the third equation from ten times the firſt, we have $15x+4y=120a-180c$, which, ſubtracted from four times the laſt equation, leaves $x=72a-240b+180c=24$; wherefore $y=48a-60b-4x=60$ and $z=\frac{12a-6x-4y}{3}=120$. demonſtrated thus:

$$\frac{24}{2}+\frac{60}{3}+\frac{120}{4}=12+20+30=62$$

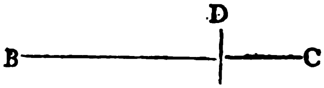
$$\frac{24}{3}+\frac{60}{4}+\frac{120}{5}=8+15+24=47$$

$$\frac{24}{4}+\frac{60}{5}+\frac{120}{6}=6+12+20=38$$

Qu. 6. A grocer bought 120 pounds of tea, and as many pounds of coffee; he had one pound of coffee more for 20 ſhillings than of tea, and the whole price of the tea exceeded that of the coffee by 6*l.*; I demand how many pounds of tea he had for 20 ſhillings, and how many pounds of coffee?

Let the number of pounds of tea bought for 20 ſhillings be repreſented by x, then the number of pounds of coffee, for 20 ſhillings, will be $x+1$, and the whole price of the tea will be $\frac{120}{x}$ pounds, and that of the coffee $\frac{120}{x+1}$ pounds; therefore, $\frac{120}{x}-\frac{120}{x+1}=6$; wherefore, $120x+120-120x=6x^2+6x$, therefore $x^2+x=20$; which reſolved gives $x=4$ the pounds of tea for 20 ſhillings, and $x+1=5$ the pounds of coffee for 20 ſhillings.

Qu. 7. Two travellers ſet out on a journey at the ſame time; the one ſets out from C to go to B, the other from B to go to C; they both travel uniformly, and in ſuch proportion that he that ſet out from B, four hours after meeting the other, arrives at C, and the other arrives at B, nine hours after meeting. How many hours did each perſon take to perform his journey?



In this figure, D is the place of their meeting; put $a=4$, $b=9$, and x for the number of hours which they travel before they meet. Then the diſtances they have travelled, with the ſame uniform pace, will be to each other as the times in which they are deſcribed; therefore, B D : D C :: x, (the time in which the traveller who ſet out from B goes the diſtance B D,) : $a=$ the time in which he travels from D to C; and by the ſame manner as B D : D C :: b, (the time the other traveller goes from D to B,) : $x=$ the time he goes from C to D; now as x is to a in the ratio of B D to D C, and b to x in the ſame ratio, it will follow, that as $x : a :: b : x$, whence $x^2=ab$ and $x=\sqrt{ab}=6$, therefore, $a+\sqrt{ab}=10$, and $b+\sqrt{ab}=15$, are the two numbers required.

Qu. 8. The ſum of three numbers in geometrical proportion, and the ſum of the ſquares of three numbers being given; to find the numbers themſelves.

Put a for the ſum of the three numbers, and b for their ſquares, and x, y, and z for the numbers themſelves, then we ſhall have $x+y+z=a$, and $x^2+y^2+z^2=b$ and $xz=y^2$, whence, by tranſpoſing y in the firſt equation, and involving both ſides to the ſecond power, there ariſes $x^2+2xz+z^2=a^2-2ay+y^2$, from which ſubtracting the ſecond equation, we have $2xz-y^2=a^2-2ay+y^2-b$; but $2xz$ by the third equation is $=2y^2$, therefore, $2y^2-y^2=a^2-2ay+y^2-b$, or $a^2-2ay-b=0$, whence, $y=$

$\frac{a}{2}-\frac{b}{2a}$. Now to find x and z, we muſt look upon y as a known quantity, and then by the ſecond equation we ſhall have $x^2+z^2=b-y^2$, from which ſubtracting $2xz=2y^2$, we have $x^2-2xz+z^2=b-3y^2$, and by taking the root we have $x-z=\sqrt{b-3y^2}$, but by the firſt equation $x+z=a-y$, therefore, $x=\frac{a-y+\sqrt{b-3y^2}}{2}$ and $z=\frac{a-y-\sqrt{b-3y^2}}{2}$.

Qu. 9. A farmer ſold as many ſheep and oxen as brought him 100*l.*; for the ſheep he received 17 ſhillings each, and for the oxen 7*l.* each. It is required to know how many he ſold of each?

Let the number of ſheep be x, and that of the oxen y; then we have this equation $17x+140y=2000$, and conſequently $x=\frac{2000-140y}{17}=117-8y+\frac{71-4y}{17}$ which being a whole number $\frac{11-4y}{17}$ or $\frac{4y-11}{17}$ muſt therefore be a whole number likewiſe; whence by proceeding as above, we have $y=7$, and $x=60$, and this is the only anſwer the queſtion will admit of.

Qu. 10. What are the dimenſions of a cubical block of marble, whoſe ſide in inches is expreſſed by two digits; the ſuperficies of the block is equal to 864 times the ſum of the ſaid digits; and its ſolidity is equal to 576 times the ſquare of the ſum of the ſaid digits?

Put x for the digit in the place of tens, and y for the digit in the place of units, then $10x+y$ is equal to the ſide of the cube and $\overline{10x+y}|^2\times 6=864\times\overline{x+y}$; or $\overline{10x+y}|^2=144\times\overline{x+y}$. Alſo, $\overline{10x+y}|^3=576\times\overline{x+y}|^2$ per queſtion; and multiplying theſe equations croſs ways, we have $\overline{10x+y}|^2\times 576\times x+y^2=144\times\overline{x+y}\times\overline{10x+y}|^3$. Then dividing both ſides by $144\times$

$\overline{x+y}\times\overline{10x+y}\rvert^2$, we have $x+y\times 4=10x+y$, or $4x+4y=10x+y$, and by tranſpoſition $3y=6x$, therefore, $y=2x$. This being ſubſtituted for y in the former equation, we have $\overline{10x+2x}\rvert^2=144\times\overline{x+2x}$ or $144x^2=144\times 3x$, and dividing by $144x$ we have $x=3$ and $y=2x=6$; therefore, the ſide of the cube is 36.

CHAP. X.

OF THE VALUE OF LIVES;

OR,

DOCTRINE OF ANNUITIES.

SECT. I.

THE VALUE OF AN ANNUITY FOR A SINGLE LIFE.

AN Annuity is a ſum of money payable yearly, half-yearly, or quarterly; to continue either for life, for a certain number of years, or for ever.

When an annuity remains unpaid after it is due, it is ſaid to be in *arrear*. When the purchaſer of an annuity does not immediately enter upon poſſeſſion, the annuity is ſaid to be in *reverſion*.

The intereſt upon annuities in arrear may be computed either in the way of ſimple or compound intereſt. But compound intereſt being found moſt equitable, both for buyer and ſeller, is in moſt general uſe.

Annuities may be divided into certain and uncertain.

A certain annuity is that which continues for a certain time, or for ever. An uncertain annuity depends upon one or more lives.

Before I proceed to give the doctrine of contingent annuities, it will be neceſſary to deliver the rules for calculating of annuities certain.

PROBLEM I.

To find the Amount of an Annuity for a given Term of Years, at a given Rate of Intereſt.

EXAMPLE. What will an annuity of 50*l.* amount to, at the end of 8 years, at the rate of 5 *per cent. per annum*, ſimple intereſt?

In this example, the intereſt being at 5 per cent. multiply the rate of intereſt of 1*l.* for 1 year, or .05 by 50 the annuity, and the product by 8, the number of years, and the product hence ariſing is 20: the half whereof (10) multiplied by the number of years, made leſs by one, (7,) produces 70, the ſimple intereſt; which added to the product of 50, and 8, (400,) give 470, the amount required.

PROBLEM II.

To find the Amount of an Annuity, at compound Intereſt.

RULE. Multiply the amount of 1*l.* for 1 year, as often into itſelf as there are years, except one; or, which is the ſame, raiſe it to the power whoſe index is equal to the number of years, and from the reſult ſubtract 1; then divide the remainder by the intereſt of 1*l.* for 1 year, and multiply the quotient by the annuity, and the product will be the amount required.

Example. What is the amount of an annuity of 50*l.* for 3 years, at 5 *per cent. per annum, compound intereſt?* Here the amount of 1*l.* for 1 year is 1.05, which multiplied twice into itſelf, produces 1.157625, and 1 ſubtracted from this, the remainder is .157625, which divided by .05, the quotient is 3.1525; this multiplied by 50, produces 157.625, or 157*l.* 12*s.* 6*d.* the anſwer required.

Note. If the payments are half-yearly or quarterly, the amount, and intereſt of 1*l.* muſt be taken for a half, or a quarter of a year. And then the double or quadruple of the time muſt be taken. And the amount of 1*l.* for half a year at compound intereſt is equal to the ſquare root of the amount for a year; and the amount for a quarter of a year is equal to the ſquare root of that for half a year.

PROBLEM III.

To find the preſent Value of an Annuity, having the Time and Rate.

Rule. Multiply the amount of one year as often into itſelf as there are years, leſs 1; or involve it to the power denoted by the time: by this reſult, divide 1, and ſubtract the quotient from 1, divide the remainder by the intereſt of 1*l.* for a year; then multiply this laſt quotient by the annuity, and the product will be the preſent value.

Example. What is the preſent value of an annuity of 40*l.* for 5 years, diſcounting 5 *per cent. per annum, compound intereſt?* Here 1.05 involved to the fifth power is 1.27628. By which dividing 1, the quotient is .78353, which ſubtracted from 1, leaves .21647; this divided by .05 gives 4.3294, which multiplied by 40 is 173.176, or 173*l.* 3*s.* 6½*d.* the preſent worth.

PROBLEM IV.

Having the preſent Worth, Rate, and Time, to find the Annuity.

Rule. Find the preſent value of 1*l.* annuity at the given rate and time; and then by the rule of three, ſay, as the preſent worth, thus found, is to 1*l.* annuity, ſo is the preſent worth given to its annuity; that is, divide the given preſent worth by that of 1*l.* annuity.

Example. What annuity will 173*l.* 3*s.* 7*d.* purchaſe to continue 5 years, allowing *compound intereſt at 5 per cent. per annum?*

.05:1::1:20*l.*
1.05 × 1.05 × 1.05 × 1.05 × 1.05 = 1.2762815625
1.2762815625)20.00000000(15.6705
20
15.6705
4.3295 preſent worth of 1*l.* annuity.
4.329)173.179(40*l.* annuity, Anſwer.

Annuities for ever, or Freehold Eſtates.

In calculating the value of an annuity for ever, commonly called an *Annuity in fee ſimple*, three things are to be conſidered: 1. The annuity, or yearly rent. 2. The price, or preſent worth. 3. The rate of intereſt.

PROBLEM I.

Having the Rent and Rate of Intereſt, to find the Price or Value.

Rule. As the intereſt of 1*l.* is to 1*l.* ſo is the rent to the price or value.

Example. What is the preſent worth of an annuity of 40*l.* per annum in fee ſimple, *compound intereſt* 3½ *per cent. per ann.?* As .035 the intereſt of 1*l.* for a year is to 1*l.* ſo is 40*l.* the rent of the annuity to 1142.857142 or 1142*l.* 17*s.* 1½*d.*

PROBLEM II.

Having the Price and Rate of Intereſt, to find the Annuity.

Rule. As 1*l.* is to its intereſt, ſo is the price to the annuity.

Example. What annuity will 4000*l.* purchaſe, at 4½ *per cent. per ann. compound intereſt?*

As 1*l.* is to .045, ſo is 4000*l.* to 180*l.* the annuity.

PROBLEM III.

Having the Price and Rent of the Annuity, to find the Rate of Intereſt.

Rule. As the price is to the rent, ſo is 1*l.* to the rate of intereſt.

Example. If an annuity of 180*l.* coſt 4000*l.* what is the rate of intereſt compound *per ann.?*

As 4000:180::1: .045 or 4½ per cent. rate of intereſt.

PROBLEM IV.

Having the Rate of Intereſt, to find how many Years Purchaſe an Eſtate is worth.

Rule. Divide 1 by the rate of intereſt, and the quotient is the anſwer.

Example. How many years purchaſe is an annuity, when the purchaſer has 2½ *per cent.* for his money?

.025)1.000(40 years purchaſe.

PROBLEM V.

Having the Number of Years Purchaſe, to find the Rate of Intereſt.

Rule. Divide 1 by the number of years purchaſe, and the quotient is the rate of intereſt.

Example. What intereſt has a purchaſer, who gives 40 years purchaſe for an annuity ?

40)1.000(.025 intereſt required.

Though the foregoing examples are moſtly performed by a ſingle diviſion or multiplication, yet they give the anſwers at compound intereſt; but in caſes where there is a reverſion, recourſe muſt be had to the tables of annuities, on compound intereſt, as in the following Problems:

PROBLEM VI.

Having the Rate of Intereſt, and the Annuity, to find the preſent Value of the Reverſion.

Rule. Find the preſent value of the annuity by Problem I. then, by the tables, find the preſent value of the annuity for the years before the reverſion takes place. Subtract this value from the former value, and the remainder is the preſent value of the reverſion.

Example. What is the value of an eſtate, or an annuity of 130*l. per ann.* to continue 20 years? What is the value of the ſame, after the expiration of 20 years, to continue for ever? and what is the value of the whole, at 6 *per cent. compound intereſt per ann.*?

.06)130.000(2166.6666 Value of the whole.
1491.0896 Value of the poſſeſſion.
675.5770 Value of the reverſion.

PROBLEM

PROBLEM VII.

Having the Value of an Annuity in Reverſion, the Time prior to the Commencement of the Reverſion, and Rate of Intereſt, to find the Annuity.

Rule. Find the amount of the price of the reverſion, for the years prior to the commencement, by the tables: then find the annuity, which that amount will purchaſe.

Example. If the reverſion of an annuity, to commence 20 years hence, be bought for 675.577*l.* what is the annuity, *compound intereſt, at* 6 *per cent. per ann.?*

Here by the tables, the amount of 675,577*l.* for 20 years, at 6 per cent. is 2166.6*l.* which will purchaſe an annuity of 130*l.*

Life Annuities.

Annuities for lives are calculated from obſervations made on the bills of mortality, which, however imperfect they may ſeem to be in themſelves, when individually applied, are nevertheleſs become in very general uſe, and are found to anſwer pretty accurately, for ſuch perſons, or public offices, as buy and ſell annuities.

It muſt, however, be obſerved, that calculations of this nature are at beſt but bare probabilities, or mere chance-work; yet every one muſt be ſenſible, that a perſon of 60 years of age has not ſo great a chance of living as a perſon of 30, provided both be in good health; therefore, an annuity to continue for the life of the former, cannot be worth near ſo much as an equal annuity for the life of the latter.

The principal writers on calculations of this kind, are Dr. Halley, Mr. Simpſon, Monſieur De Moivre, and Dr. Price, each of whom has formed tables on the probability of human life, deduced from the bills of mortality.

Dr. Halley, from the bills of mortality at Breſlaw, conſtructed a table, in which he ſhows how many perſons died in every year out of 1000 promiſcuouſly choſen, and all born in the ſame year, till the death of the laſt. He choſe the city of Breſlaw, the capital of Sileſia, for this purpoſe, which he thought might ſerve as a ſtandard to the reſt of Europe, being a central town, at a diſtance from the ſea, and not much crowded with foreigners. His table is, however, found not to be ſo well adapted to the probability of human life in England, as thoſe of ſome others ſince his time. I ſhall, however, give his table, as well as that of Mr. Simpſon, and Dr. Price, that the reader may ſee the comparative merit of each.

DR. HALLEY's TABLE,

CALCULATED

From the Bills of Mortality at Breſlaw.

TABLE I.

Age.	Perſons living.	Decrement of Life.	Age.	Perſons living.	Decrement of Life.	Age.	Perſons living.	Decrement of Life.
1	1000	145	32	515	8	63	212	10
2	855	57	33	507	8	64	202	10
3	798	38	34	499	9	65	192	10
4	760	28	35	490	9	66	182	10
5	732	22	36	481	9	67	172	10
6	710	18	37	472	9	68	162	10
7	692	12	38	463	9	69	152	10
8	680	10	39	454	9	70	142	11
9	670	9	40	445	9	71	131	11
10	661	8	41	436	9	72	120	11
11	653	7	42	427	10	73	109	11
12	646	6	43	417	10	74	98	10
13	640	6	44	407	10	75	88	10
14	634	6	45	397	10	76	78	10
15	628	6	46	387	10	77	68	10
16	622	6	47	377	10	78	58	9
17	616	6	48	367	10	79	49	8
18	610	6	49	357	11	80	41	7
19	604	6	50	346	11	81	34	6
20	598	6	51	335	11	82	28	5
21	592	6	52	324	11	83	23	3
22	586	7	53	313	11	84	20	5
23	579	6	54	302	10	85	15	4
24	573	6	55	292	10	86	11	3
25	567	7	56	282	10	87	8	3
26	560	7	57	272	10	88	5	2
27	553	7	58	262	10	89	3	2
28	546	7	59	252	10	90	1	1
29	539	8	60	242	10	91	0	0
30	531	8	61	232	10			
31	523	8	62	222	10			

TABLE II.

SHOWING

The Probabilities from Ten Years Obſervation on the Bills of Mortality in London.

By Mr. SIMPSON.

Age.	Perſons living.	Decrement of Life.	Age.	Perſons living.	Decrement of Life.	Age.	Perſons living.	Decrement of Life.
0	1000	320	27	321	6	54	135	6
1	680	133	28	315	7	55	129	6
2	547	51	29	308	7	56	123	6
3	496	27	30	301	7	57	117	5
4	469	17	31	294	7	58	112	5
5	452	12	32	287	7	59	107	5
6	440	10	33	280	7	60	102	5
7	430	8	34	273	7	61	97	5
8	422	7	35	266	7	62	92	5
9	415	5	36	259	7	63	87	5
10	410	5	37	252	7	64	82	5
11	405	5	38	245	8	65	77	5
12	400	5	39	237	8	66	72	5
13	395	5	40	229	7	67	67	5
14	390	5	41	222	8	68	62	4
15	385	5	42	214	8	69	58	4
16	380	5	43	206	7	70	54	4
17	375	5	44	199	7	71	50	4
18	370	5	45	192	7	72	46	4
19	365	5	46	185	7	73	42	3
20	360	5	47	178	7	74	39	3
21	355	5	48	171	6	75	36	3
22	350	5	49	165	6	76	33	3
23	345	6	50	159	6	77	30	3
24	339	6	51	153	6	78	27	2
25	333	6	52	147	6	79	25	2
26	327	6	53	141	6	80	23	2

TABLE III.

The Probabilities of Life, calculated from Forty-ſix Years Obſervation on the Bills of Mortality at Northampton: viz. from 1735 *to* 1780.

Age.	Perſons living.	Decrement of Life.	Age.	Perſons living.	Decrement of Life.	Age.	Perſons living.	Decrement of Life.
0	11650	1894	34	4085	75	69	1312	80
½	9756	1106	35	4010	75	70	1232	80
1	8650	1367	36	3935	75	71	1152	80
2	7283	502	37	3860	75	72	1072	80
3	6781	335	38	3785	75	73	992	80
4	6446	197	39	3710	75	74	912	80
5	6249	184	40	3635	76	75	832	80
6	6065	140	41	3559	77	76	752	77
7	5925	110	42	3482	78	77	675	73
8	5815	80	43	3404	78	78	602	68
9	5735	60	44	3326	78	79	534	65
10	5675	52	45	3248	78	80	469	63
11	5623	50	46	3170	78	81	406	60
12	5573	50	47	3092	78	82	346	57
13	5523	50	48	3014	78	83	289	55
14	5473	50	49	2936	79	84	234	48
15	5423	50	50	2857	81	85	186	41
16	5373	53	51	2776	82	86	145	34
17	5320	58	52	2694	82	87	111	28
18	5262	63	53	2612	82	88	83	21
19	5199	67	54	2530	82	89	62	16
20	5132	72	55	2448	82	90	46	12
21	5060	75	56	2366	82	91	34	10
22	4985	75	57	2284	82	92	24	8
23	4910	75	58	2202	82	93	16	7
24	4835	75	59	2120	82	94	9	5
25	4760	75	60	2038	82	95	4	3
26	4685	75	61	1956	82	96	1	1
27	4610	75	62	1874	81		299198	11650
28	4535	75	63	1793	81			
29	4460	75	64	1712	80			
30	4385	75	75	1632	80			
31	4310	75	66	1552	80			
32	4235	75	67	1472	80			
33	4160	75	68	1392	80			

TABLE IV.

The Expectation of Life at every Age, according to the Bills of Mortality for both London and Northampton.

Age.	Expectation.		Age.	Expectation.		Age.	Expectation.	
	London.	North.		London.	North.		London.	North.
0	19.5	25.18	34	22.4	26.20	68	9.9	9.50
1	27.5	32.74	35	22.0	25.68	69	9.6	9.05
2	32.5	37.79	36	21.6	25.16	70	9.3	8.60
3	34.5	39.55	37	21.2	24.64	71	8.9	8.17
4	36.1	40.58	38	20.8	24.12	72	8.6	7.74
5	36.5	40.84	39	20.4	23.60	73	8.3	7.33
6	36.5	41.07	40	20.1	23.08	74	8.0	6.92
7	36.3	41.03	41	19.7	22.56	75	7.7	6.54
8	36.1	40.79	42	19.3	22.04	76	7.3	6.18
9	35.7	40.36	43	19.0	21.54	77	6.9	5.83
10	35.3	39.78	44	18.6	21.03	78	6.5	5.48
11	34.8	39.14	45	18.3	20.52	79	6.0	5.11
12	34.2	38.49	46	17.9	20.02	80	5.5	4.75
13	33.6	37.83	47	17.5	19.51	81		4.41
14	33.0	37.17	48	17.2	19.00	82		4.09
15	32.4	36.51	49	16.8	18.49	83		3.80
16	31.8	35.85	50	16.5	17.99	84		3.58
17	31.2	35.20	51	16.1	17.50	85		3.37
18	30.6	34.58	52	15.7	17.02	86		3.19
19	30.0	33.99	53	15.4	16.54	87		3.01
20	29.4	33.43	54	15.0	16.06	88		2.86
21	28.8	32.90	55	14.7	15.58	89		2.66
22	28.2	32.39	56	14.3	15.10	90		2.41
23	27.7	31.88	57	13.9	14.63	91		2.09
24	27.1	31.36	58	13.6	14.15	92		1.75
25	26.6	30.85	59	13.2	13.68	93		1.37
26	26.1	30.33	60	12.9	13.21	94		1.05
27	25.6	29.82	61	12.5	12.75	95		0.75
28	25.1	29.30	62	12.1	12.28	96		0.50
29	24.6	28.79	63	11.7	11.81			
30	24.1	28.27	64	11.3	11.35			
31	23.6	27.76	65	11.0	10.80			
32	23.2	27.24	66	10.6	10.42			
33	22.8	26.72	67	10.3	9.96			

TABLE V.

Showing the Number of Years Purchaſe an Annuity on a ſingle Life is worth according to the Probabilities of Life, at London and Northampton, from the Age of 6 to 75 Years; at 3, 4, and 5 per Cent.

Age.	3 *per cent.*		4 *per cent.*		5 *per cent.*		Age.	3 *per cent.*		4 *per cent.*		5 *per cent.*	
	Lon.	*North.*	*Lon.*	*North*	*Lon.*	*North*		*Lon.*	*North*	*Lon*	*North.*	*Lon.*	*North*
6	18.8	20.73	16.2	17.48	14.1	15.04	41	13.0	14.62	11.4	13.02	10.2	11.70
7	18.9	20.85	16.3	17.61	14.2	15.17	42	12.8	14.39	11.2	12.84	10.1	11.55
8	19.0	20.89	16.4	17.66	14.3	15.23	43	12.6	14.16	11.1	12.66	10.0	11.41
9	19.0	20.81	16.4	17.63	14.3	15.21	44	12.5	13.93	11.0	12.47	9.9	11.26
10	19.0	20.66	16.4	17.52	14.3	15.14	45	12.3	13.69	10.8	12.28	9.8	11.11
11	19.0	20.48	16.4	17.39	14.3	15.04	46	12.1	13.45	10.7	12.09	9.7	10.95
12	18.9	20.28	16.3	17.25	14.2	14.94	47	11.9	13.20	10.5	11.89	9.5	10.78
13	18.7	20.08	16.2	17.10	14.1	14.83	48	11.8	12.95	10.4	11.68	9.4	10.62
14	18.5	19.87	16.0	16.95	14.0	14.71	49	11.6	12.69	10.2	11.47	9.3	10.44
15	18.3	19.66	15.8	16.79	13.9	14.59	50	11.4	12.44	10.1	11.26	9.2	10.27
16	18.1	19.44	15.6	16.62	13.7	14.46	51	11.2	12.18	9.9	11.06	9.0	10.10
17	17.9	19.22	15.4	16.46	13.5	14.33	52	11.0	11.93	9.8	10.85	8.9	9.93
18	17.6	19.01	15.2	16.31	13.4	14.22	53	10.7	11.67	9.6	10.64	8.8	9.75
19	17.4	18.82	15.0	16.17	13.2	14.11	54	10.5	11.41	9.4	10.42	8.6	9.57
20	17.2	18.64	14.8	16.03	13.0	14.01	55	10.3	11.15	9.3	10.20	8.5	9.38
21	17.0	18.47	14.7	15.91	12.9	13.92	56	10.1	10.83	9.1	9.98	8.4	9.19
22	16.8	18.31	14.5	15.80	12.7	13.83	57	9.9	10.61	8.9	9.75	8.2	9.00
23	16.5	18.15	14.3	15.68	12.6	13.75	58	9.6	10.34	8.7	9.52	8.1	8.80
24	16.3	17.98	14.1	15.56	12.4	13.66	59	9.4	10.06	8.6	9.28	8.0	8.60
25	16.1	17.81	14.0	15.44	12.3	13.57	60	9.2	9.78	8.4	9.04	7.9	8.39
26	15.9	17.64	13.8	15.31	12.1	13.47	61	8.9	9.49	8.2	8.80	7.7	8.18
27	15.6	17.47	13.6	15.18	12.0	13.38	62	8.7	9.21	8.1	8.55	7.6	7.97
28	15.4	17.29	13.4	15.05	11.8	13.28	63	8.5	8.91	7.9	8.29	7.4	7.74
29	15.2	17.11	13.2	14.92	11.7	13.18	64	8.3	8.61	7.7	8.03	7.3	7.51
30	15.0	16.92	13.1	14.78	11.6	13.07	65	8.0	8.30	7.5	7.76	7.1	7.28
31	14.8	16.73	12.9	14.64	11.4	12.97	66	7.8	7.99	7.3	7.49	6.9	7.03
32	14.6	16.54	12.7	14.50	11.3	12.85	67	7.6	7.68	7.1	7.21	6.7	6.79
33	14.4	16.34	12.6	14.35	11.2	12.74	68	7.4	7.37	6.9	6.93	6.6	6.54
34	14.2	16.14	12.4	14.20	11.0	12.62	69	7.1	7.05	6.7	6.65	6.4	6.28
35	14.1	15.94	12.3	14.04	10.9	12.50	70	6.9	6.73	6.5	6.36	6.2	6.02
36	13.9	15.73	12.1	13.88	10.8	12.38	71	6.7	6.42	6.3	6.08	6.0	5.76
37	13.7	15.52	11.9	13.72	10.6	12.25	72	6.5	6.10	6.1	5.79	5.8	5.50
38	13.5	15.30	11.8	13.55	10.5	12.12	73	6.2	5.79	5.9	5.51	5.6	5.24
39	13.3	15.08	11.6	13.38	10.4	11.98	74	5.9	5.49	5.6	5.23	5.4	4.99
40	13.2	14.85	11.5	13.20	10.3	11.84	75	5.6	5.20	5.4	4.96	5.2	4.74

THE USE OF THE TABLES.

Table I.

Shows the probability of life, according to Dr. Halley's computation: the firſt column ſhows the ages; the ſecond column, the number of perſons living at thoſe ages; and the third column, the decrement of life, or the number of perſons that died each year: thus, oppoſite age 1, is 1000 in the ſecond column, and 145 in the third column; which ſhows, that of 1000 perſons born in the ſame year, 145 died before the expiration of the year; and of 855, the remainder of the perſons living, 57 died the ſecond year, and ſo on.

But the calculations according to this table differ from thoſe made from the tables of the probabilities of life in London, partly owing to the different ſituations of theſe two places. Breſlaw being an inland town not much frequented by ſtrangers or foreigners, and London being a mercantile port, and crowded with traffickers and travellers from all parts of the world; and partly owing to the difference of climate, difference of food, and different manners of life, between the inhabitants of theſe two places, which is always found to occaſion a different proportion in the deaths at the ſame ages. Theſe conſiderations induced Mr. Simpſon to compoſe a table of the probability of life, calculated from the bills of mortality of London, and which table will conſequently much better anſwer the purpoſe of calculating the value of an annuity for a life at London, than the other table of Dr. Halley. This table may be ſeen page 51.

It muſt, however, be obſerved, that Dr. Halley's table is better adapted for the uſe of all Europe in general than any other particular table.

Table III.

Shows the probability of life at all ages, from 46 years obſervation on the bills of mortality at Northampton: viz.

from the year 1735 to 1780 incluſive. This table is juſtly reckoned to be the moſt correct of any extant, as it is taken from a large manufacturing town, and which conſiſts generally of the ſame perſons: and alſo that the table is conſtructed from a larger number of perſons born than any other table, being 11650, which affords an opportunity of obſerving the decrement of life to a greater exactneſs.

Table IV.

Shows the expectation of life, or the number of years which any perſon may be ſuppoſed to have a fair chance of living, according to an equality of chance at every age, according to the bills of mortality for both London and Northampton. Thus, againſt age 2 ſtands 32.5 in the ſecond column, under London; and 37.79 in the third column, under Northampton; which ſhows, that a child of the age of two years has an equal chance of living 32.5 years, according to the London tables, or 32 years 6 months; and according to the Northampton tables 37.79 years, or 37 years and upwards of 9 months.

Table V.

Shows the value of an annuity for a ſingle life, according to the probabilities of life at London and Northampton, from the age of 6 to 75 years incluſive, at 3, 4, and 5 *per cent.* Thus, ſuppoſe, it were required to find the number of years purchaſe which an annuity is worth to a perſon of the age of 30: here in the ſecond column, and oppoſite the age 30, ſtands 15.0, which ſhows that an annuity for a perſon of 30 years of age in London, and at 3 *per cent.* is worth 15 years purchaſe; and an annuity for a Northampton life, at the ſame rate of intereſt, is worth 16.92 years purchaſe; and for a London life, at 4 *per cent.* an annuity is worth 13.1 years purchaſe; and for a Northampton life, at the ſame rate and

intereſt,

is worth 14.78 years purchaſe: and an annuity at 5 *per cent.* for a London life, is worth 11.6 years purchaſe; and for a Northampton life, at the ſame rate of intereſt, 13.07 years purchaſe. This table alſo ſhows the value of an annuity of 1*l.* for a ſingle life, at all the above-mentioned rates of intereſt: thus, an annuity of 1*l.* *per ann.* for a ſingle life, at 30 years of age, according to the London tables, at 3 *per cent.* is worth 15*l.*; and the ſame, according to the Northampton tables, is worth 16*l.* and upwards of 18*s.* &c.

This table is eſteemed the beſt of any extant, and preferable to any other of a different form. But thoſe who ſell annuities have generally a table of 2 years more value than the lives in this table, for purchaſers who are upwards of twenty years of age.

Definitions.

1. The *probability of life* is the chance that any perſon or perſons have of living to any certain time, and is denoted by a fraction, whoſe numerator is the chance of living, and denominator that of living and dying. Thus, ſuppoſe it were required to find the probability of a perſon of the age of 20 attaining to the age of 37, according to Mr. Simpſon's table. Here it muſt be obſerved, that of 360 perſons living at the age of 20, only 252 ſurvive to the age of 37; therefore, 108 perſons have died between the two ages. Thus, 252 is the chance of the ſaid perſon's living to the age of 37, and 360 the chance of the ſaid perſon's dying before he attains the age of 37, and the probability of life of that perſon is expreſſed by the fraction $\frac{252}{360}$ or $\frac{7}{10}$; therefore, the odds in that perſon's favour, or the chance that he ſhall live to that age, is 7 to 3.

2. The *probability of dying* is expreſſed by a fraction, which is the difference between the former fraction and unity. Thus, the probability that the aforeſaid perſon ſhall die before the age of 37 is expreſſed by $\frac{108}{360}$ or $\frac{3}{10}$, which

ſhows that the chance of that perſon's dying before the ſaid age is as 3 to 7.

3. The *extremity of life* is the period beyond which there is no probability of ſurviving. In the Northampton tables this is 96 years.

4. The *complement of life* is the number of years which any perſon's age wants of the full extremity of life; this in the Northampton table, for a life aged 80, is 17 years.

5. The *expectation of life* is the number of years due to the life of a perſon of a certain age, upon an equality of chance. And it is the number of years purchaſe, which an annuity for life is worth in ready money, without allowing any intereſt. And in ſingle lives it is always equal to the ſum of all the probabilities of ſurviving to the extremity of life.

6. The *number of years purchaſe* of annuities, at any rate of intereſt, is that number which, if multiplied by the annuity, is equal to the preſent value thereof, according to ſuch rate of intereſt; therefore, it is the preſent value of an annuity of 1*l.* according to a given rate of intereſt, as ſeen in Table V.

7. The *reverſion* of a life annuity is where two or more lives are in joint poſſeſſion, and the expectation depends upon the probability of one particular life ſurviving the reſt.

PROBLEM I.

To find the Value of an Annuity for the Life of any Perſon, at a given Rate of Intereſt.

Rule. Seek the age of the perſon in the firſt column of Table V. and againſt it, under the proper intereſt, is the number of years purchaſe for either the London or Northampton life; or, which is the ſame thing, the preſent value of an annuity of 1*l.* during ſuch life. Multiply this by the annuity, and the product is the anſwer.

Example. Suppoſe the given age be 49, the rate of intereſt 3 *per cent.* and the annuity 20*l.* Here again 49, and under 3 *per cent.* ſtands 11.6 according to the London bills, which, multiplied by 20*l.* gives 232*l.* for the value of a London life. And in the next column ſtands 12.69, the value according to the Northampton bills, which, multiplied by 20*l.* as before, produces 253*l.* 16*s.* for the value of a Northampton life.

SECT. II.

OF THE VALUE OF AN ANNUITY DURING JOINT LIVES.

PROBLEM I.

To find the Value of an Annuity for the joint Continuance of two Lives; that is, one Life failing, the Annuity to ceaſe.

CASE I.

When both Perſons are of the ſame Age.

Rule. Find the value of any one of the lives from Table V. Multiply this value by the intereſt of 1*l.* for a year at the given rate; ſubtract the product from 2, divide the aforeſaid value by this remainder, and the quotient will be the value of 1*l.* annuity, or the number of years purchaſe.

Example. What is the value of 100*l.* annuity, for the joint lives of two perſons, aged 40 years each, according to the London tables, reckoning intereſt at 5 *per cent.*? Here, by the table, one life for 40 years is,

	10.3
Multiply by	.05
Subtract this product	.515
From	2.000
Remains	1.485

And 1.485)10.3(6.9 value of 1*l.* annuity, which multiplied by 100 is 690*l.* the value of the annuity ſought.

CASE II.

When the two Perſons are of different Ages.

Rule. Find the values of the two lives in Table V. Multiply them one into the other, and call the reſult the firſt product; then multiply the ſaid firſt product by the intereſt of 1*l.* for a year, at the given rate, calling the reſult the ſecond product: add the values of the two lives together, and from the ſum ſubtract the ſecond product; divide the firſt product by the remainder, and the quotient will be the value of 1*l.* annuity, or the number of years purchaſe.

Example. What is the value of 50*l.* annuity, for the joint lives of two perſons, whereof one is 20, and the other 30 years of age, according to the Northampton tables, intereſt at 4 *per cent.*?

The value of 20 years is	16.03
And the value of 30 years is	14.78
Firſt product	236.9234
	.04
Second product	9.476936
Sum of the two lives	30.810000
Remainder	21.333064

And 21.333064)236.9234(11.1 value of 1*l.* annuity.

50

555.0 Value required.

PROBLEM

PROBLEM III.

To find the Value of an Annuity during the Life of the longeſt Survivor of two Lives; that is, as long as either of the two Parties live.

Rule. From the ſum of the values of the ſingle lives, ſubtract the value of the joint lives, and the remainder will be the value ſought.

Example. What is the value of an annuity of 1 *l.* to continue during the longeſt of two lives: the one perſon being 30, and the other 40 years of age; intereſt at 4 *per cent.* the life of 30 years of age valued according to the London bills, and that of 40 years of age according to the Northampton bills?

By the table, the value of 30 years is	13.1
The value of 40 years is . .	13.20
	26.30
The value of their joint lives, by Problem II. Caſe 2, is . . .	8.9
The value ſought . . .	17.4

If the annuity be any other than 1 *l.* multiply the above found value, by the given annuity; and if the two perſons be of equal ages, the value of their joint lives muſt be found by Caſe 1, of Problem II.

PROBLEM IV.

To find the Value of an Annuity for the joint Continuance of three Lives; that is, one Life failing, the Annuity to ceaſe.

Rule. Multiply the value of the three ſingle lives continually into each other, calling the reſult the product of the three lives; multiply that product by the intereſt of 1 *l.* and that product again by 2, calling the reſult the double product; then from the ſum of the ſeveral products of the ſaid lives, taken two and two, ſubtract the double product; divide the product of the three lives by the remainder, and the quotient will be the value of the three joint lives.

Example. What is the value of an annuity of 1*l.* during the joint lives of three perſons, whereof A is 10 years of age, B 20, and C 30, at 4 *per cent.* according to the London tables?

Here, by the table, the value of A's life is 16.4, that of B's 14.8, and C's 13.1, which three multiplied together is 3179.63; this multiplied by .04, the intereſt of 1*l.* gives 127.18528, which, multiplied again by 2, gives 254.370 for the double product. Then

The product of A and B is	242.72
And the product of A and C is	214.84
The product of B and C is	193.88
The ſum of all taken two and two	651.44
Double product to ſubtract	254.37
Remainder	397.07

And 397.07)3179.632(8.007 Value ſought.

PROBLEM V.

To find the Value of the Annuity for the longeſt Life of three or more Perſons.

Rule. Find an age anſwerable to the value of the longeſt life of any two lives, which ſubſtitute in lieu of the two, and then find the value of the longeſt life of that life and the other, and that will be the value of the longeſt life among three lives. If there are four or more lives, ſubſtitute the age correſponding to this value in lieu of the three lives, and find the value of the longeſt life of this age, and the other remaining age, and it will be the value of the longeſt of four lives: proceed in the ſame manner for five or more lives, having regard to the rate of intereſt.

The examples in the foregoing Problems will be found ſufficient to inſtruct the learner how to perform all the following Problems. I ſhall, therefore, give a few Problems with their rules, leaving their operation for the exerciſe of the learner.

SECT. III.

THE VALUE OF CONTINGENT REMAINDERS AND REVERSIONS.

PROBLEM I.

To find the Value of the Reverſion of an aſſigned Life after a given Term.

Rule. Subtract the value of the annuity for the given term of years, from the value of the propoſed life, on the contingency of its ceaſing, upon the extinction of the aforeſaid life, and the remainder will be the anſwer.

PROBLEM II.

To find the Value of the Reverſion of an Annuity for the Remainder of a given Term of Years after an aſſigned Life.

Rule. From the value of an annuity, certain for the given term of years, ſubtract the value of the annuity for the ſaid term, on the contingency of its ceaſing, upon the failing of the propoſed life; and the remainder will be the value of the reverſion.

PROBLEM

PROBLEM III.

To find the Value of the Reverſion of one Life after another.

Rule. Subtract the value of the two joint lives from the value of the life in expectation, and the remainder will be the value of the reverſion.

PROBLEM IV.

To find the Value of the Reverſion of two Lives after one.

Rule. Subtract the value of the life in poſſeſſion from the value of the longeſt of the three lives, and the remainder will be the value of the reverſion.

PROBLEM V.

To find the Value of the Reverſion of one Life after two Lives.

Rule. If the two lives are joint lives, ſubtract the value of the three joint lives from the value of the life in expectation, and the remainder will be the anſwer. But if the reverſion takes place after the extinction of either life, ſubtract the value of the two lives in poſſeſſion from the value of the three lives, and the remainder will be the value of the reverſion.

PROBLEM VI.

To find the preſent Value of any Number of Lives in Succeſſion.

Rule. Multiply the value of each life by the intereſt of 1*l.* for one year, and ſubtract each product from unity or 1; multiply all the remainders continually together, and ſubtract this product from unity; then the remainder multiplied by the perpetuity *, will be the value of all the ſucceſſive lives.

PROBLEM VII.

A given ſum of money is to be received as a legacy on the deceaſe of D, who is at a given age; what is the value thereof in preſent money?

Rule. Subtract the value of the life of D from the perpetuity; then ſay, as the perpetuity is to the remainder, ſo is the propoſed ſum to the preſent value.

PROBLEM VIII.

To find the Value of a Sum of Money to be received at the Deceaſe of B, in caſe A is then deceaſed alſo.

Rule. Subtract the value of the oldeſt life from the value of an annuity for as many years as are expreſſed by the complement of B's age; then ſay, as the complement of the younger life is to the remainder, ſo is the propoſed ſum to its preſent value.

ANNUITIES UPON TONTINES.

PROBLEM IX.

To find the Value of an Annuity for either Perſon of two, who have a joint Annuity, which at the Deceaſe of either one, is to become the ſole Property of the Survivor.

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* The perpetuity, or value of an annuity to continue for ever, is found by dividing 100*l.* by the rate of intereſt per cent. or by dividing 1*l.* by the intereſt of 1*l.* for a year; and the quotient is the perpetuity, or value of an annuity of 1*l.* to continue for ever. The perpetuity is alſo equal to the number of years purchaſe, which a perpetual annuity is worth, without allowing any intereſt.

Rule. From the value of the life of either of the two perſons, ſubtract half the value of the two joint lives, and the remainder will be the value of the other perſon's life.

PROBLEM X.

What is the preſent value of an annuity to be poſſeſſed by D and his heirs, as ſoon as any two of the three lives, A, B, and C, become extinct; D and his heirs holding the ſame during the life of the ſurvivor of the three lives, A, B, C?

Rule. Add thrice the value of the three joint lives, A, B, and C, to the ſum of the value of the three ſingle lives, deducting therefrom twice the ſum of the value of each two joint lives: viz.—of A and B, A and C, and B, C; and the remainder will be the anſwer.

PROBLEM XI.

What is the value of the right of any one of the three following perſons, viz.—A, B, and C, who enjoy an annuity equally among them, which, upon the deceaſe of any one, is to become the property of the two ſurvivors, during their joint lives, and on the deceaſe of the next perſon to become the property of the laſt ſurvivor during his life?

Rule. Subtract half the ſum of the values of the joint lives, A and B, and the joint lives of A and C, from the value of the life of A; then to the remainder add one third of the value of the three joint lives, and the ſum will be the value of the right of A. The value of the right of either of the other parties may be found by a ſimilar method.

PROBLEM XII.

What is the value of the two ſucceſſive lives, A and B, A having an annuity for life, and to have the nomination of a ſucceſſor, who is to hold the annuity for his own life, after the deceaſe of A?

Rule. Multiply the value of the life of A, by the value of the life put in at his deceaſe; divide the product by the perpetuity, and ſubtract the quotient from the ſum of the ſaid values, and the remainder will be the anſwer.

SECT. IV.

OF ASSURING LIVES.

By aſſuring a life, is meant, obtaining ſecurity for receiving a certain ſum of money, ſhould the aſſured life fail in a certain given time: in conſideration of which, a premium is given to the aſſurer, which is a ſufficient compenſation for the loſs he is likely to ſuſtain, in caſe the life ſhould drop. This compenſation, called the *premium*, is varied according to the two following cauſes:—Firſt, The rate of intereſt at which the money is ſuppoſed to be improved; and, ſecondly, the probability of the duration of the life to be aſſured. If the intereſt be high, and the probability of life high alſo, the value of the aſſurance will be low in proportion: on the contrary, if the intereſt be low, and the probability of life alſo low, the value of the aſſurance will be proportionably high. For example:—Let 100*l.* be ſuppoſed to be aſſured on a life, for 1 year; that is, let 100*l.* be payable a year hence, provided a perſon of a given age dies in that time.

Now,

Now, if the intereſt of the money be 5 *per cent.* and the life *ſure* * of failing, the value of the aſſurance would be the ſame as the preſent value of 100*l.* payable at one year's end, reckoning intereſt at 5 *per cent.* and would be that ſum, which being put out to intereſt now, at 5 *per cent.* would produce the 100*l.* at the end of the year, which ſum is 95*l.* 4*s.* 9*d.*

If it be an even chance, or the odds are equal, whether the life does or does not fail in the year, which is the caſe when one half of a given number of lives fail in a given time, the value of the aſſurance will be half as much as the former value, or 47*l.* 12*s.* 4½*d.*

If the odds againſt the perſon's life failing are two to one, which is the caſe when one third of a given number of lives fail in the time, the value of the aſſurance will alſo be one third of the firſt value (if the intereſt be the ſame), or 31*l.* 14*s.* 11*d.*

If the odds are nineteen to one againſt the life failing, which is the caſe when the twentieth part of the lives fail in the given time, the value of the aſſurance will be a twentieth part of the firſt value, or 4*l.* 15*s.* 2¼*d.*

If the odds are forty-nine to one againſt the life failing, or when only one out of fifty of ſuch lives fails in the given time, the aſſurance will be only a fiftieth part of the firſt value, or 1*l.* 18*s.* 1*d.*

Now the odds of two to one, according to Dr. Halley's table, are, that a life aged 85 years will not drop in a year. The odds of nineteen to one are, that a life aged 64 will not drop in a year. And the odds of forty-nine to one are, that a life aged 39 will not drop in a year. Therefore, the value of the aſſurance of 100*l.* for one year, on a life aged 85, is

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* By this word is to be underſtood a certainty, according to the calculations from the tables.

31*l.* 14*s.* 11*d.*;—on a life aged 64, 4*l.* 15*s.* 2¼*d.*;—on a life aged 39, 1*l.* 18*s.* 1*d.* at 5 *per cent.* intereſt. But if intereſt be reckoned at 3 *per cent.* theſe three values will be 32*l.* 7*s.*—4*l.* 17*s.*—1*l.* 18*s.* 10*d.* reſpectively.

This calculation ſuppoſes the value of the aſſurance to be paid in one ſingle preſent payment. But the value may be paid in *annual payments*, and be continued till the failure of the life, ſhould that happen within the given term; or, if not, till the determination of the time.

The value of an aſſurance upon a life cannot be diſcovered by any one ignorant of the method of calculating the value of life annuities, delivered in the former part of this chapter. But thoſe who underſtand what has been delivered, may form any calculations upon this ſubject, from the following examples:

Example. What money in hand, and alſo in annual payments during life, ought a perſon of a given age to pay for a given ſum of money, payable at his death to his heir?

Rule. Subtract the value of the given life at the rate *per cent.* as given in Table V. from the perpetuity: multiply the remainder by the product of the ſum to be aſſured, and the rate of intereſt for a year; divide this laſt product by 100*l.* increaſed by its intereſt for a year, and the quotient will be the anſwer, or the money which ought to be given in a ſingle preſent payment: and this payment divided by the value of the life, will quote the ſum that ought to be paid in annual payments during the whole continuance of life.

CASE I.

Where the Premium is to be paid immediately, in a ſingle Payment.

Question 1. What premium ſhould be given to ſecure 100*l.* at the deceaſe of a perſon aged 45 years, intereſt 3 *per cent.*?

Operation.

Operation.

The perpetuity	33.333
The value of the life by Table V. at 3 *per cent.*	12.3
Remainder	21.033
The given ſum multiplied by its intereſt	300

103)6309.9(61.26116 Anſwer.

Thus it appears that 61.26116, or 61*l.* 5*s.* 2½*d.* is the premium which ought to be immediately paid to ſecure 100*l.* on the deceaſe of a perſon aged 45 years, at 3 *per cent. per annum*, according to the probability of life for London.

CASE II.

When the Premium is to be paid in fixed annual Payments, during the whole Continuance of Life.

QUESTION 2. What money ſhould be given in equal annual payments, during the life of a perſon aged 45 years, to ſecure 100*l.* on the deceaſe of the ſaid perſon; intereſt at 3 *per cent. per annum?*

In this caſe, the value of the aſſurance in one preſent payment is to be found as in the foregoing caſe, which value divided by the value of the life, quotes the ſum to be paid annually during the life of the perſon:—Thus, 61.26116 divided by 12.3, quotes 4.98, or 4*l.* 19*s.* 7*d.* which is the ſum to be paid annually during life, in order to ſecure the ſum of 100*l.* at the extinction of the ſaid life.

If the foregoing queſtions be repeated, reckoning the intereſt at 3½ *per cent. per annum*, the premium will be leſs, viz. in one preſent payment at 3½ *per cent.* it will be 57*l.* 11*s.* and the annual payments 4*l.* 19*s.* 7½*d.*

Thus it appears upon what very eaſy terms a large ſum of money might be ſecured at the deceaſe of any perſon, if the premium be paid by annual payments. Hence the great advantage of inſtitutions for the aſſurance of lives, provided they be properly conducted, and managed by perſons ſufficiently ſkilled in numbers to avoid errors in making their calculations, which are moſt detrimental to ſocieties of this nature, and from which there are hardly any of theſe inſtitutions exempt.

Aſſurances of this nature might be extended conſiderably more than they are at preſent; and rendered not only ſubſervient to the parochial poor; but alſo of infinite advantage to the nation at large, particularly to the revenue in a financial reſpect, were the ſubject to meet the approbation of the legiſlature; and perhaps more pecuniary aſſiſtance might be derived from eſtabliſhments of this nature, under proper modifications, than from any other mode of funding, and creating permanent debts, as I ſhall prove in another treatiſe.

When an eſtate, or a perpetual annuity, is to be aſſured for the duration of another life, after the failure of the aſſured life, inſtead of aſſuring a groſs ſum, the value of a ſingle payment will be the value of the life ſubtracted from the perpetuity, and the remainder multiplied by the annuity, or by the rent of the eſtate. And the value in annual payments to begin immediately, will be the ſingle payment divided by the value of the life, increaſed by unity. Therefore, an aſſurance of an eſtate or annuity, after any given life or lives, is worth as much more than the aſſurance of a correſponding ſum, as 100*l.* increaſed by its intereſt for a year, is greater than 100*l.* Thus the preſent values, in ſingle and annual payments, of the aſſurance of an eſtate of 5*l.* *per annum* for ever, and of 100*l.* in money, are to one another as 105*l.* is to 100*l.* The reaſon of the difference is, that the algebraical calculations, by which theſe values are determined, ſuppoſe the groſs ſum and the firſt yearly payment of the annuity are to be received at the ſame time, after the expiration of the life or lives.

The examples here given will be found ſufficient to inſtruct any perſon in the method of finding the value of annuities, in all caſes of reverſions: as alſo in the principles of aſſurances upon lives.

CHAP. XI.

OF LOGARITHMS.

SECT. I.

OF THE ORIGIN AND NATURE OF LOGARITHMS.

LOGARITHMS are certain artificial numbers, which are the ratios of other natural numbers; and are the indices of the ratio of numbers to one another; or, a ſeries of numbers in an arithmetical proportion, anſwering to as many others in a geometrical proportion, and in ſuch a manner, that 0 in the arithmeticals is the index of 1 in the geometricals. Logarithms were invented for the eaſe of arithmetical calculations, where the numbers, or operations, are large.

The nature of logarithms depends upon theſe axioms: if a ſeries of quantities increaſe, or decreaſe, according to the ſame *ratio*, it is called a *geometrical progreſſion*, as the numbers 1, 2, 4, 8, 16, 32, which are multiplied by 2: if the ſeries or quantities increaſe, or decreaſe, according to the ſame *difference*, it is called an *arithmetical progreſſion*, as the numbers 3, 6, 9, 12, 15, 18, &c. which increaſe by 3, which is therefore called their common difference. Now, if underneath the numbers proceeding in a geometrical progreſſion, be placed as many other numbers, proceeding in an arithmetical progreſſion, theſe laſt are called the logarithms of the firſt; as in the following:

Terms -	1.	2.	4.	8.	16.	32.	64.	128.	256.	512.
Logarithms	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.

In this progreſſion, o is the logarithm of 1, the firſt term: 1 the logarithm of the ſecond, which is 2; and 2 the logarithm of the third term, 4, &c.

Theſe indices, or logarithms, may be adapted to any ſeries in a geometrical progreſſion; and, therefore, there may be as many different kinds of indices, or logarithms, as there can be different kinds of geometrical progreſſions; as may be ſeen in the following ſeries:—

Log. -	.0	1.	2.	3.	4.	5.	&c.	
	1.	2.	4.	8.	16.	32.	&c.	The Geometrical Progreſſion.
or	2^0.	2^1.	2^2.	2^3.	2^4.	2^5.	&c.	
	1.	3.	9.	27.	81.	243.	&c.	
or	3^0.	3^1.	3^2.	3^3.	3^4.	3^5.	&c.	
	1.	10.	100.	1000.	10000.	100000.	&c.	
or	10^0.	10^1.	10^2.	10^3.	10^4.	10^5.	&c.	

Here the ſame indices, or logarithms, ſerve for any of the ſix under-written geometrical ſeries, from which it appears, that there may be an endleſs variety of ſets of logarithms, adapted to the ſame common numbers, by varying the ſecond term of the geometrical ſeries, as this will change the original ſeries of terms, whoſe indices are the numbers 1. 2. 3. &c. And by interpolation the whole ſyſtem of numbers may be made to enter the geometric ſeries, and receive their proportionable logarithms, whether they be integers or decimals.

The logarithm of any number is the index of ſuch a power of ſome other number, as is equal to the given one involved to the power denoted by the index of the other number. Thus, if N be equal to r^n, then the logarithm of N is n, which may be either poſitive or negative, and r any number whatever, according to the different ſyſtems of logarithms. When N is one, then n is o, whatever the value of r may be; and, conſequently, the logarithm of 1 is always o in every ſyſtem of logarithms. But in the common logarithms, r is equal to 10; ſo that the common logarithm of any number is the index of that power of 10, which is equal to the ſaid number: thus the common logarithm of $N = 10^n$, is n the index of the power of 10.—For example:—1000 being the third power of 10, has 3 for its logarithm: and if 50 be = $10^{1.69897}$, then 1.69897 is the common logarithm of 50; from which it will follow, that the following decimal ſeries of terms will have the following logarithms reſpectively:

The Geometric Series.	1000,	100,	10,	1,	.1,	.01,	.001
or	10^3,	10^2,	10^1,	10^0,	10^{-1}	10^{-2}	10^{-3}
Logarithms	3,	2,	1,	0,	—1,	—2,	—3.

The logarithm of a number, which is contained between any two terms of the firſt ſeries, is included between the two correſponding terms of the latter ſeries; therefore, that logarithm will have the ſame index, whether poſitive or negative, as the ſmaller of theſe two terms, together with a decimal fraction, which will always be poſitive. Thus, the number 50 falling between the numbers 10 and 100, its logarithm will fall between 1 and 2, being equal to 1.69897 nearly; and the number .05 falling between .1 and .01, its logarithm will fall between —1 and —2, and is equal to —2 +.69897, the index of the leſs term, together with the decimal .69897. The index is ſometimes called the characteriſtic of the logarithms, and is always an integer, either poſitive or negative, or elſe 0, and ſhows what place is occupied by the firſt ſignificant figure of the given number, either above or below the place of units, being in the former caſe poſitive, and in the latter negative.

When the characteriſtic of a logarithm is negative, the ſign — is commonly ſet over it to diſtinguiſh it from the decimal part, which being the logarithm found in the tables, is always poſitive; thus, the logarithm of .05 or —2 +.69897 is written thus, $\bar{2}$ 69897. But when it is required to reduce the whole expreſſion to a negative form, it is done by making the characteriſtic leſs by 1, and taking the arithmetical complement of the decimal; that is, beginning at the left hand, ſubtract each figure from 0, except the laſt ſignificant figure, which is ſubtracted from 10; then will the remainder form a logarithm, wholly negative: thus, the aforementioned logarithm $\overline{2}.69897$, or $-2+69897$ is expreſſed by -1.30103, which is all negative. Sometimes it is convenient to expreſs the logarithm as poſitive, which is done by joining to the tabular decimal, the complement of the index to 10: thus, the above logarithm is expreſſed by 8.69897, which is only increaſing the indices in the ſcale by 10.

From the foregoing definitions of logarithms, conſidered either as the indices of a geometric ſeries, or as the indices of the powers of the ſame root, it appears, that numbers may be multiplied together by the addition of their logarithms: and they may be divided by the ſubtraction of their logarithms. Alſo a number may be raiſed to any power by multiplying the logarithm of the root by the index of the power: and the extraction of roots may be performed by dividing the logarithm of the given number by the index of the root required to be extracted.

Logarithms conſidered in their theory, are of very ancient origin, and were known to moſt of the ancients; but the celebrated John Napier, Baron of Merchiſton, in Scotland, was the firſt who applied the uſe of them to Trigonometry; but the form of their conſtruction was not made known till the opinion of the mathematicians was had concerning them. His ſon, Robert Napier, in the year 1619, publiſhed a new edition of his father's work, with the conſtruction of logarithms. And in the ſame year Mr. John Speldell publiſhed an Improvement of Napier's Logarithms.

Other Tables were ſoon after publiſhed by John Kepler, and ſome others; all which tables were of that kind, called Hyperbolical; becauſe the numbers expreſſed are as between the aſymptote and curve of the hyperbola.

Henry Briggs, Profeſſor of Geometry in Greſham College, ſoon after publiſhed the logarithms of the firſt one thouſand numbers on a new ſcale: viz.—In which the logarithm of the ratio of 10 to 1 is 1; whereas the logarithms of the ſame ratio in Napier's ſyſtem is 2.302 58, &c. And in 1624 he alſo publiſhed his *Arithmetica Logarithmica*, containing the logarithms of 30,000 natural numbers, to 14 places of figures, beſides the index, which form was recommended to him by Napier, and which is the form in preſent uſe. In 1633, he publiſhed, to 14 places of figures, his *Trigonometria Britannica*, which contained the natural and logarithmic ſigns, tangents, ſecants, &c.

From the time of Robert Napier, to the preſent period, ſeveral mathematicians publiſhed logarithmic tables, with various improvements; the principal of which were Gunter, Wingate, Henrion, Miller, and Norwood, Cavalerius, Vlacq, and Rowe, Frobenius, Newton, Caramuel, Sherwin, Gardiner, and Dodſon.

In addition to what has been ſaid, it may be obſerved, that the indices or characteriſtic of logarithms correſpond to the denominative part of the natural number, as the other member of the logarithm does the numerative part of the number; that is, the index ſhows the denomination of, or place of the left-hand figure of the number, and conſequently of all the reſt. Thus, 0 affixed to a logarithm, denotes the firſt figure of the number to which the logarithm anſwers to be nothing diſtant from the place of units. The index 1 ſhows the firſt figure of the number to be diſtant 1 place from the place of units; that is, in the place of tens, and conſequently the number itſelf to be either 10, or ſome number between 10 and 100; and the ſame may be obſerved of all the other indices. Therefore, all numbers that have the ſame denominative, but not the ſame numerative parts, as all numbers from 1 to 10, from 10 to 100, &c. will have logarithms whoſe indices are the ſame, but the other members different. And again, all numbers, which have the ſame numerative, but not the ſame denominative parts, will have different indices; but the reſt of the logarithms will be the ſame. If a number be purely decimal, to its logarithm is affixed a negative index, ſhowing the diſtance of its firſt ſignificative figure from the place of units. Thus the logarithm of the decimal .256 is $\bar{1}.408240$; that of the decimal .0256 is $\bar{2}.408240$. Inſtead of theſe negative indices, ſome uſe their complements to 10, ſet down with a point on each ſide, thus, .9. and .8. that is, ſuch a figure is made the index, as, when ſubtracted from 9, leaves a remainder expreſſing the number of cyphers prefixed to the decimal, as before obſerved.

SECT. II.

OF THE ARITHMETIC OF LOGARITHMS.

FROM what has been ſaid, the learner may have acquired a tolerable idea of the nature of logarithms. The conſtruction of the logarithmic canon requires more room than is conſiſtent with the preſent treatiſe. The table of logarithms is generally printed in a table conſiſting of nine columns, beſides the column of natural numbers and that of the differences, except that part of the table which contains the logarithms of numbers from 1 to 100.

To find the Logarithm of a Number, or the Number correſponding to a given Logarithm.

If the given number be under 100, the logarithm is found at once in the table, ſtanding oppoſite the number: if the given number be above 100, and under 1000, its logarithm will be found oppoſite to it in the column under 0: if the number be between 1000 and 10000, the firſt three figures of the number are to be found in the column marked No, and the fourth figure at the top of one of the other columns, and in the column under the ſaid fourth figure, and even with the firſt three figures, will be found the logarithm required, changing the index 2 into 3.

If the given number be greater than any in the common canon, but leſs than 10000000, cut off four figures on the left of the given number, and find the logarithms thereof as before directed; then multiply the common difference which ſtands againſt the logarithm in the column marked *Difference*, by the remaining figure or figures of the given number, and from the product cut off as many figures from the right-hand ſide as you multiplied by, and add the remainder to the logarithm of the firſt four figures of the given number, fitting it with a proper index. Thus, if it were required to find the logarithm of the number 92375: the logarithm of the firſt four figures 9237 is 3.965531, then 47 multiplied by 5=235, therefore, cut off 5 and add 23, and the required logarithm (proportioning its index) is 4.965554.

To find the Number correſponding to any Logarithm.

Seek the logarithm in the table, beginning in the column under 0 (neglecting the index); and if the logarithm, or the two or three firſt figures thereof, cannot be found in this column, ſeek them in the other columns; and having found it on the next leſs logarithm thereto, lineally againſt it in the column of numbers are the three firſt figures of the number ſought, to which joining the figure over the column of the logarithm, you have the required number; when the number conſiſts of four places of figures, i. e. when the index of the logarithm is 3; but when the index is 1, the two laſt figures of the number are decimal, and only the two firſt numbers integral: when the index is 0, the firſt number ouly is an integral, &c.

Thus, to find the number correſponding to a logarithm is the converſe of finding the logarithm of a number.

To multiply or divide Numbers by Logarithms.

RULE. To multiply two numbers together, add their two logarithms together, and their ſum will be the logarithm of the product, adding to the affirmative indices what is carried from the decimal parts; and if the indices be one affirmative, and the other negative, the difference is to be taken for the index to the logarithm of the product.

The reaſon of this rule is evident, for as unity is to one of the factors, ſo is the other factor to the product. Thus, the logarithm of the product is a fourth equi-different term to the logarithm of unity and thoſe of the factors; but the logarithm of unity being 0, the ſum of the logarithms of the factors muſt be the logarithm of the factum, or product.

Therefore, as the factors of a ſquare are equal to each other, that is, a ſquare is the factum or product of its root multiplied into itſelf; the logarithm of a ſquare will be double the logarithm of the root.

Therefore, unity is to the exponent of the power, as the logarithm of a root to the logarithm of the power.

From hence it appears that the logarithm of the cube is triple the logarithm of the root; the logarithm of the biquadrate, quadruple that of the root; the logarithm of the fifth power quintuple that of the root; that of the ſixth power ſextuple, &c.

To perform diviſion by logarithms, ſubtract the logarithm of the diviſor from the logarithm of the dividend, and the remainder will be the logarithm of the quotient, firſt changing the ſign of the logarithm of the index of the diviſor, and if they be of different ſigns, the difference is to be taken, but if of the ſame kind, their ſum.

For as the diviſor is to the dividend, ſo is unity to the quotient; therefore the logarithm of the quotient is a fourth equi-different number to the logarithms of the diviſor, the dividend, and the logarithm of unity. The logarithm of unity, therefore, being 0, the difference of the logarithm of the diviſor, and that of the dividend, is the logarithm of the quotient. For the difference between 7 and 5, or the remainder, is 2, which is the logarithm of the quotient 4, which quotient is obtained by dividing 128 by 32. Alſo, ſubtracting 3 from 8, the remainder 5 is the logarithm of the quotient 32, which is obtained by dividing 256 by 8.

Examples

Examples in Multiplication.

Example 1.

	Num.	Log.
Multiply	68	1.832509
By	12	1.079181
Products	816	2.911690

Example 2.

	Num.	Log.
Multiply	9	0.954242
By	9	0.954242
	81	1.908484

Example 3. Multiply 3.902, and 597.16, and .0314728, continually together.

Example 4. And alſo multiply 35.86, and 2.1046, and 0.8372, and 0.0294, continually together.

Num.	Log.
3.902	0.5912873
597.16	2.7760907
.0314728	$\bar{2}$.4979353
73.33533	1.8653133

Num.	Log.
35.86	0.5546103
2.1046	0.3231696
0.8372	$\bar{1}$.9228292
0.0294	$\bar{2}$.4683473
.1857618	$\bar{1}$.2689564

In the laſt example the 2 carried from the decimals cancels the $\bar{2}$, and there remains $\bar{1}$ to be ſet down; and in the third example, the $\bar{2}$ cancels the 2, and the 1 carried from the decimals is ſet down.

Examples in Diviſion.

Example 1.

	Num.	Log.
Div.	24163—	4.3831509
By	4567—	3.6596310
Quo.	5.290782—	0.7235199

Example 2.

	Num.	Log.
Div.	37.149—	1.5699471
By	523.76—	2.7191323
Quo.	07092752—	$\bar{2}$.8508148

In the laſt example, the 1 carried from the decimals is added to the 2, which, with its ſign, changed, becomes $\bar{3}$; conſequently the remainder, or index of the quotient, is $\bar{2}$.

EXAMPLE 3.

	Num.	*Log.*
Div.	.06314—	$\bar{2}$.8003046
By	.007241—	$\bar{3}$.8597985
Quo.	8.719792—	0.9405061

EXAMPLE 4.

	Num.	*Log.*
Div.	.7438—	$\bar{1}$.8714562
By	12.9476—	1.1121893
Quo.	05744694—	$\bar{2}$.7592669

In the third example, there is 1 carried from the decimals and added to the $\bar{3}$, which makes it $\bar{2}$; and which taken from the other $\bar{2}$ leaves 0. In the fourth example, the 1 is taken from the $\bar{1}$, which leaves a remainder $\bar{2}$.

To find the Logarithm of a Fraction.

RULE. Subtract the logarithm of the numerator from that of the denominator, and to the remainder prefix the negative ſign. Thus, ſuppoſe it was required to find the logarithm of the fraction $\frac{3}{7}$.

0.845098	Logarithm of 7.
0.477121	Logarithm of 3.
—0.367977	= Logarithm of $\frac{3}{7}$.

The reaſon of this rule is evident: for a fraction being the quotient of the numerator divided by the denominator, its logarithm muſt be the difference of the logarithms of theſe two: ſo that the numerator being ſubtracted from the denominator, the difference becomes negative. And the logarithms of proper fractions muſt always be negative, if the logarithm of unity be 0; becauſe a proper fraction is leſs than an unit.

Or the logarithm of the denominator, though greater than that of the numerator, may be ſubtracted from that of the numerator, regard being had to the ſign of the index, which alone in this caſe is negative. Thus,

0.477121 Logarithm of 3.
0.845098 Logarithm of 7.

$\bar{1}.632023$ = Logarithm of $\frac{3}{7}$.

This produces the ſame effect in any operation as the logarithm before found, viz.—0.367977, this being to be ſubtracted, and the other added.

Or the fraction may be reduced to a decimal, and its logarithm found; which logarithm differs from that of a whole number only in the index, which is to be negative. For an improper fraction, ſubtract the logarithm of the denominator from that of the numerator, and the remainder is the logarithm of the fraction, as in the fraction $\frac{9}{5}$.

0.9542425 Logarithm of 9.
0.6989700 Logarithm of 5.

0.2552725 = Logarithm of $\frac{9}{5}$.

In the ſame manner the logarithm of any mixed number may be found, by reducing the mixed number into an improper fraction.

Or laſtly, an improper fraction may be reduced to a mixed number, and its logarithm muſt be found as if it were a whole number; and its index taken according to the integral part.

In addition, ſubtraction, &c. of logarithms, with negative and affirmative indices, the ſame rules are to be obſerved as thoſe given in algebra, for like and unlike ſigns.

In addition of logarithms of this nature, all the figures, except the index, are reckoned poſitive; and, therefore, the figure to be carried to the index from the other part of the logarithms, takes away ſo much from the negative index. Thus, if 1.863326 be added to $\bar{3}.698972$, the ſum is $\bar{1}.562298$.

And in ſubtraction, if either one or both of the logarithms have negative indices, you muſt change the ſign of the index of the ſubtrahend, after you have carried to it what may ariſe from the decimal part, and then add the indices together; thus, if 1.863326 be ſubtracted from $\overline{1}.562298$, the remainder will be $\overline{3}.698972$. In multiplication, what is carried from the product of the other parts of the logarithms muſt be ſubtracted from the product of the indices; thus, if $\overline{2}.477121$ be multiplied by 5, the product will be $\overline{8}.385605$. In diviſion, if the diviſor will exactly meaſure the index, proceed as in common arithmetic. Thus, $\overline{4}.924782$ divided by 2, quotes $\overline{2}.462391$; but if the diviſor will not exactly meaſure the index, add units to the index till you can exactly divide it, and carry theſe units to the next firſt number. Thus, if $\overline{8}.385605$ be divided by 5, it quotes 2.477121.

Involution and Evolution.

Involution is performed by multiplying the logarithm of the number given by the propoſed index of the power; and if the index of the logarithm be negative, the product will be negative; but what is carried from the decimal part of the logarithm will be affirmative; therefore the difference of this carried number and the product of the index will be the index of the product.

EXAMPLE 1. To find the cube or third power of 3.07146.

EXAMPLE 2. To find the fourth power of .09163.

	Num.	*Log.*		*Num.*	*Log.*
Root	3.07146—	0.4873449	Root	.09163—	$\overline{2}.9620377$
		3			4
Pow.	28.97575—	1.4620347	Pow.	.0000704938—	$\overline{5}.8481508$

Evolution is performed by dividing the logarithm of the power or given number by its power, and the quotient will be the logarithm of the root; and when the index of the logarithm is negative, and the diviſor is not exactly contained in it, without a remainder, increaſe the index of the logarithm by ſuch a number as will make it exactly diviſible; and carry the units borrowed, as ſo many tens, to the decimals; then divide the reſults by the index of the root.

Example 1. To find the cube root of 12345.

Example 2. To find the cube root of .00048.

	Num.	*Log.*		*Num.*	*Log.*
Pow.	12345—	3)4.0914911	Pow.	.00048—	3)$\bar{4}$.6812412
Root	32.11162—	1.3638304	Root	.07829735—	2.8937471

In the laſt example, 2 is to be added to the $\bar{4}$, to make it become $\bar{6}$, in which the diviſor 3 is contained $\bar{2}$ times; and the 2 borrowed is carried to the other figure 681, &c.; and the logarithm of the root is $\bar{2}$.8937471.

SECT. III.

OF THE RULE OF PROPORTION BY LOGARITHMS.

To perform the Rule of Proportion by Logarithms.

Rule. Add the logarithm of the ſecond and third terms together, and from the ſum ſubtract the logarithm of the firſt; and the remainder is the logarithm of the fourth number.

Example. Find a fourth proportional to the numbers 4, 68, and 3.

1.832509

1.832509	Logarithm of 68.
0.477121	Logarithm of 3.
2.309630	Sum.
0.602060	Logarithm of 4.
1.707570	Logarithm of 51, the anſwer.

This rule is founded on the ſame reaſon as the rule of proportion in common arithmetic; for adding the logarithms of the ſecond and third numbers together, and ſubtracting the logarithm of the firſt from the ſum, is the ſame thing as multiplying the ſecond and third numbers together, and dividing the product by the firſt.

Or the operation may be performed by the following rule, viz. Againſt the firſt term write the arithmetical complement of its logarithm, and againſt the ſecond and third terms write the logarithms themſelves; and the ſum of theſe three logarithms, abating 10 in the index, will be the logarithm of the fourth term; thus, in reſolving the aforeſaid queſtion, the operation will ſtand thus:

9.397940	Arithmetical complement of log. of 4.
1.832509	Logarithm of 68.
0.477121	Logarithm of 3.
1.707570	Logarithm of 51. Anſwer.

The reſolution of problems of this nature is of eminent ſervice in trigonometry, as will be ſeen hereafter.

CHAP. XII.

OF TRIGONOMETRY.

Definitions.

1. TRIGONOMETRY is the art of finding all the ſides and angles of a triangle, from having any three of theſe, one of which, at leaſt, muſt be a ſide. Or, to find the ratio of the ſides, when the angles are given; and converſely the ratio of the angles, when the ſides are given. And it is founded on the mutual proportion which ſubſiſts between the ſides and angles of triangles, which proportions are known by finding the relations between the radius of the circle and certain lines drawn in and about the circle, called *chords*, *ſines*, *tangents*, *ſecants*, &c.

For this purpoſe, the circumference of a circle is divided into 360 parts, called degrees; and every degree ſubdivided into 60 other parts, called minutes, and every minute into 60 ſeconds, and every ſecond into 60 thirds, &c.; and any angle is ſaid to conſiſt of ſo many degrees, minutes, and ſeconds, as are contained in the arc that meaſures the angle, or that is intercepted between the legs or ſides of the angle; the point of the ſaid angle being at the centre of the circle.

2. The *complement* of an arc is the difference between the arc and a quadrant.

3. The *ſupplement* of an arc is the difference between the arc and a ſemicircle.

4. The *right ſine* of an arc, commonly called the *ſine*, is a perpendicular falling from one end of the arc, to the radius, drawn through the other end of the ſame arc, as D E (fig. 1, plate 9) is the ſine of the arc D B, and it is always equal to half the ſubtenſe of double the arc. Thus, D E is equal to half of D O, which is the ſubtenſe or chord of the arc D O; and the arc D O is double the arc D B. Hence the ſign of an arc of 30 degrees is equal to one half the radius, becauſe the chord of 60 degrees is equal to the radius.

5. The *ſine complement* of an arc is that part of the radius intercepted between the centre and the right ſine, as C E, and is alſo the ſine of the complement of the arc to a quadrant; for C E is equal to F D, which is the ſine of the arc D H.

6. The *coſine* of an arc is the ſame as the ſine complement.

7. The *verſed ſine* is that part of the radius intercepted between the right ſine and the circumference of the circle, as E B.

8. The *tangent* of an arc is a perpendicular drawn from the extremity of the radius to the ſecant, as B G, which is the tangent of the arc D B.

9. The *ſecant* of an arc is a line drawn from the centre of the circle, through one end of the arc, till it meets the tangent; as C G.

10. The *coſecant* and *cotangent* of an arc is the ſecant and tangent of that arc, which is the complement of the former arc. And the chord of an arc, and the chord of its complement to a circle, is the ſame; ſo likewiſe the ſine, tangent, and ſecant of an arc are the ſame as the ſine, tangent, and ſecant of its ſupplement or complement to a ſemicircle. Thus, the ſine E D, the tangent B G, and ſecant C G, is

the ſine, tangent, and ſecant of the arc, D A, which is the ſupplement of the former arc D B.

11. The *ſinus totus* is the greateſt ſine, being the ſine of an arc of 90 degrees, or one quarter of a circle, and is equal to the radius of the circle.

Thus, the ſines always increaſe from B, at which place they are nothing, till they come to the radius C H, which is the greateſt, being the ſine totus. From hence they decreaſe all the way along the ſecond quadrant from H to A, and at length vaniſh at the point A; whereby we ſee that the ſine of the ſemicircle B H A, is nothing. After this, the ſines are negative, as they proceed along the next ſemicircle A O B, being drawn on the oppoſite ſide, or downwards, from the diameter A B.

As D E is the coſine of D H: the ſine, coſine, and radius of any arc form a right-angled triangle; as, C D E, or C D F; of which, the radius C D is the hypothenuſe: and therefore the ſquare of the radius is equal to the ſum of the ſquares of the ſine and coſine of any arc.

The ſines, coſines, tangents, &c. of every degree and minute, in a quadrant, are calculated to a radius of 1, and ranged in tables for uſe. But to ſhorten the operation in calculations in trigonometry, we uſe the logarithms of them, inſtead of the natural numbers, which are called the artificial ſines, tangents, &c. and theſe numbers ſo ranged in tables, form the trigonometrical canon; and contain every ſpecies of right-angled triangles; ſo that no triangle can be propoſed, but one ſimilar to it can be found there by compariſon, with which the propoſed one may be computed by analogy of proportion. Laſtly, ſometimes the proportion is not expreſſed in numbers: but the ſeveral ſines, tangents, &c. are actually laid down upon lines of ſcales; from whence the line of ſines, of tangents, &c. on the plane ſcale, the conſtruction and uſe of which follow:—

The *plane ſcale* is a mathematical inſtrument of moſt extenſive uſe, commonly two feet in length. The lines uſually drawn upon it, are the following:—1. Lines of equal parts. 2. Of Chords.—3. Rhumbs.—4. Sines.—5. Tangents.—6. Secants.—7. Semi-tangents.—8. Longitude.—And, 9. Latitude. (Fig. 2.)

1. The lines of equal parts are of two kinds: viz.—ſimply divided, and diagonally divided. The firſt of theſe are formed by drawing three parallel lines, and dividing them into any number of equal parts, by ſhort lines drawn acroſs them; and in like manner ſubdividing the firſt diviſion into ten other equal ſmaller parts, by which numbers or dimenſions of two figures may be taken off. Upon ſome rules ſeveral of theſe ſcales of equal parts are ranged parallel to each other, with figures ſet to them, to ſhow into how many equal parts they divide the inch; as 20, 25, 30, &c. 2. The diagonal diviſions are formed by drawing eleven long parallel lines, equidiſtant from each other, which are divided into equal parts, and croſſed by other ſhort lines, as the former; then the firſt of theſe equal parts have the two outermoſt of the eleven parallel lines divided into ten equal parts, and the points of diviſion connected by diagonal lines, as ſhown in Menſuration. The whole ſcale is thus divided into dimenſions of three places of figures.

The other lines upon the ſcale are commonly uſed in Trigonometry, Navigation, Aſtronomy, Dialling, &c. &c. and are all conſtructed from the diviſions of a circle, as follows:—

2. Deſcribe a circle* with any convenient radius, and divide it in four equal parts, by two diameters, drawn at right angles to each other, (*fig.* 2.) Continue one diameter C D towards F, and draw the tangent line E A, parallel thereto; then draw the chords D A and D B.

3. To conſtruct the line of chords, divide the quadrant A D into 90 equal parts: then on A, as a centre, with the

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* Only half the circle is drawn in the figure for want of room; but in general a complete circle is formed.

compaſſes, transfer theſe diviſions to the chord line A D, which mark with the correſponding numbers, and it will become the line of chords, which may be transferred to the ruler.

4. For the line of rhumbs, divide the quadrant B D into eight equal parts: then with the compaſſes, from the centre B, transfer the diviſions to the line B D, which will be the line of rhumbs.

5. For the line of ſines, through each of the diviſions of the arc A D draw right lines parallel to the radius D C, which will divide the radius A C into the ſines, or verſed ſines; numbering it from C to A for the ſines, and from A to C for the verſed ſines.

6. For the line of tangents, lay a ruler on C, and the ſeveral diviſions of the arc A D; and it will interſect the line E A, which will become a line of tangents, transferring the numbers from the arc A D to that line.

7. For the line of ſecants transfer the diviſions from the tangent line to the line F D with the compaſſes, and from C as the centre, marking the diviſions with the correſponding numbers on the tangent line.

8. For a line of ſemi-tangents, lay a ruler on B and the ſeveral diviſions of the arc A D, which will interſect the radius C D in the ſeveral diviſions of the ſemi-tangents, which are to be numbered according to the arc A D.

9. For the line of longitude, divide the radius A C into 60 equal parts, through each of theſe draw lines parallel to the radius C D; the points where theſe lines interſect the arc A D are to be transferred with the compaſſes from A as a centre to the chord A D, and numbered thereon, which will give the line of longitude.

10. For the line of latitude, the ſemicircle A D B muſt be completed to a circle, then a ruler laid on the point D, and on the ſeveral diviſions of the line of ſines, A C will interſect the next quadrant of the circle, in as many points; when from the oppoſite part of the circle to D, as the centre,

the interſections of the arc are to be transferred to its chord, and numbered according to the numbers on the line of ſines.

The chief uſes of the lines of ſines, tangents, ſecants, and ſemi-tangents, are to find the poles, and centres of the ſeveral circles, repreſented in a projection of the ſphere.

I have been more particular in deſcribing the conſtruction of this ſcale, as it is an inſtrument in moſt general uſe in mathematics; and by the foregoing directions the learner may conſtruct any lines on the ſcale himſelf, where there happens not to be a mathematical inſtrument maker nigh at hand, and place them on a rule, as ſeen *fig.* 3.

SECT. I.

OF PLANE TRIGONOMETRY.

The three methods of reſolving triangles, or caſes in trigonometry, are:—1. By geometrical conſtruction. 2. Arithmetical computation. And, 3. Inſtrumental operation. In the firſt method, the triangle is conſtructed, by drawing, and laying down the ſeveral parts, viz.—the ſides from a ſcale of equal parts, and the angles from a ſcale of chords, or other inſtrument: then the unknown parts are meaſured by the ſame ſcales; and thus they become known.

In the ſecond method, the terms of the proportion are ſtated according to rule; which terms conſiſt partly of the numbers of the given ſides, and partly of the ſines, &c. of angles taken from the tables; the proportion is then reſolved like all other proportions, in which a fourth term is to be found from three given terms, viz. by the Rule of Three.

In the third method of reſolving the triangle, by inſtrumental operation, recourſe muſt be had to the logarithmic lines, on one ſide of the two foot ſcales; extending the compaſſes from the firſt term to the ſecond or third, which happens to be of the ſame kind with it; then that extent will reach from the other term to the fourth term. In this operation for the ſides of triangles, is uſed the line of numbers, and for the angles the line of ſines or tangents, according as the proportion reſpects ſines or tangents.

In every caſe in plane trigonometry, there muſt be given three parts, one of which, at leaſt, muſt be a ſide. And every triangle that can be propoſed, will fall under one of the three following caſes:

CASE I.

When two of the three given Parts are a Side, and its oppoſite Angle.

CASE II.

When there are given two Sides, and their contained Angle.

CASE III.

When the three Sides are given.

Rule. For the firſt caſe, viz.—That the ſides are proportional to the ſines of their oppoſite angles: that is, as the one ſide given, is to the ſine of its oppoſite angle, ſo is another ſide given to the ſine of its oppoſite angle. Or, as the ſine of a given angle is to its oppoſite ſide, ſo is the ſine of another given angle to its oppoſite ſide. Thus, to find an angle, we muſt begin the proportion with a given ſide, that is, oppoſite to a given angle; and to find a ſide, we muſt begin with an angle oppoſite to a given ſide.

Example. In the triangle, B D C, (*fig.* 4) having the ſide B D equal to 106, the ſide B C equal to 65, and the angle B D C, 31 degrees, 49 minutes, to find the angle B C D, and the ſide C D.

1. *By geometrical Conſtruction.*

Draw a line B D equal to 106; at D, make an angle of 31° 49′ by drawing D C; take 65 in the compaſſes, and with one foot at B, extend the other foot to C, in the line D C, then draw the line B C, and it is done: for the angle C will be 120° 43′; the angle D, 31° 49′; and the angle B, 27° 28′; and the ſide D C, 56.

2. *By arithmetical Computation.*

	Log.
As the ſide B C, 65	1.81291
Is to the ſine angle D, 31° 49′	9.72198
So is the ſide B D, 106	2.02531
	11.74729
	1.81291
To ſine angle C, 120° 43′	9.93438

To find the ſide D C.

	Log.
As the ſine angle D, 31° 49′	9.72198
Is to ſide B C, 65	1.81291
So is ſine angle B, 27° 28′	9.66392
	11.47683
	9.72198
To the ſide D C, 56.88	1.75485

Or it may be wrought as follows:—

180° 0′	The ſum of three angles
59° 17′	Supplement of angle C
120° 43′	Angle C
31° 49′	Angle D
152° 32′	Their ſum
180° 0′	
152° 32′	
27° 28′	Angle B.

Here it muſt be noted, that when the given angle is obtuſe, the angle ſought will be acute; but when the given angle is acute, and oppoſite to a leſs given ſide, then the required angle is doubtful, whether acute or obtuſe; it ought therefore to be determined before the operation be performed.

For the above proportion gives 59° 17′ for the required angle; but as it is obtuſe, its ſupplement to 180° muſt be taken, viz. 120° 43′.

3. *By Gunter's Line, or inſtrumental Operation.*

Rule. Extend the compaſſes from 65 to 106 on the lines of numbers, and that extent will reach from 31° 49′ to 59° 17′ on the line of ſines.

Secondly. The extent from 31° 49′ to 27° 28′ on the line of ſines, will reach from 65 to 56.88 on the line of numbers.

CASE II.

When the three given Parts are two Sides and their contained Angle.

Rule. As the ſum of the two given ſides is to the difference of the ſides, ſo is the tangent of half the ſum of the two oppoſite angles or cotangent of half the given angle to the tangent of half the difference of thoſe angles.

Then the half difference added to the half ſum gives the greater of the two unknown angles, and ſubtracted, leaves the leſs of the two angles.

Thus, having all the angles, the remaining third ſide will be found by the former caſe.

Example. Having the ſide B C, equal to 109, B D equal to 76, (fig. 5,) and the angle C B D, 101° 30′, to find the angle B D C, or B C D, and the ſide C D.

1. *By geometrical Conſtruction.*

Draw the line B C equal to 109 and B D, ſo as to make an angle with B C, of 101° 30′, and make B D equal to 76; join B C and B D with a right line, and it is done; for the angle D being meaſured, is found to be equal to 47° 32′, the angle C 30° 58′, and the ſide D C 144.8.

2. *Arithmetically by Logarithms.*

Side B C	109	109		180° 0′	
Side B D	76	76		101° 30′	
Their Sum	185	their 33	difference	78° 30′	Sum of the angle D & C.
				39° 15′	Half the ſum.

Then to find the angles D and C.

	Log.
As the ſum of the ſides B C and B D=185	2.26717
Is to their difference 33	1.51851
So is tan. of ½ ſum of the angles C & D 39° 15′	9.91224
	11.43075
	2.26717
To tangent of ½ the diff. of the angles C & D 8° 17′	9.16358

To ½ the ſum of the angles D and C	39° 15′
Add half the difference of the angles C and D	8° 17′
Gives the greater angle D	47° 32′
Subtracted, gives the leſſer angle C	30° 58′

To find D C.

	Log.
As ſine angle D 47° 32′	9.86786
Is to the ſide B C 109	2.03743
So is ſine angle B 101° 30′	9.99119
	12.02862
	9.86786
To the ſide D C, required 144.8	2.16076

3. *By Gunter.*

Firſt. The extent from 185 to 33 on the line of numbers will reach from 39° 15′ to 8° 17′ on the line of tangents. Secondly, the extent from the angle D 47° 32′ to 78° 30′, (the ſupplement of angle B,) on the line of ſines, will reach from the ſide B C 109 to 144.8, the ſide D C required on the line of numbers.

CASE III.

When the three Sides are given, to find the three Angles.

Rule. Let fall a perpendicular from the greateſt angle upon the oppoſite ſide or baſe, dividing it into two ſegments, and the whole triangle into ſmaller right-angled triangles: then the proportion will be; as the baſe, or ſum of the two ſegments, is to the ſum of the other two ſides; ſo is the difference of thoſe ſides, to the difference of the ſegments of the baſe.

Then half the difference of the two ſegments added to half the baſe, or half the ſum of the two ſegments, gives the greater ſegment; and ſubtracted gives the leſs. Thus, in each of the two right-angled triangles, there are given the hypothenuſe and the baſe, beſides the right angle; therefore, the other angles may be found by the firſt caſe.

Example. Having the ſides B C equal to 105 (*fig.* 6), B D equal to 85, and C D equal to 50; to find the three angles D, C, and B.

1. *Geometrically by Conſtruction.*

1. Draw the line B C, equal to 105; with the compaſſes open to 50, and having one foot on the point C, deſcribe an arc; then with the compaſſes open to 85 and one foot in B, cut the former arc in D, join B D and C D, and it is done; for the angles meaſured, B will be found equal to 28° 4′, and C 53° 7′, which being added together, and ſubtracted from 180°, leaves 98° 49′, for the angle D.

2. *Arithmetically by Logarithms.*

The two ſhorteſt ſides of the triangle B D and C D added together, is 135, and their difference 35. The ſegments of the baſe B C are found in the following manner:

	Log.
As the ſide B C equal to 105	2.02119
Is to the ſum of the ſides B D and D C 135	2.13033
So is their difference 35	1.54407
To the difference of the ſegments of B C 45	1.65321

Thus, having the ſum and difference of the ſegments of the baſe, it is only neceſſary to add half their ſum 52½ to half the difference 22½, and it will give the greater ſegment, which is 75; and which ſubtracted from 105 leaves 30, the leſſer ſegment.

To find the Angle B D A.

	Log.
As the hypothenuſe B D 85	1.92942
Is to the radius	10.00000
So is the greater ſegment 75	1.87506
To the ſum of the angle B D A	9.94564

Therefore, the angle B D A is equal to 61° 56′.

To find the Angle A D C.

	Log.
As the hypothenuſe D C 50	1.69897
Is to the radius	10.00000
So is the leſs ſegment 30	1.47712
To the ſine of A D C	9.77815

Therefore, the angle A D C is equal to 36° 53′, and the whole angle B D C equal to 98° 49′.

To find the angle B, it is only neceſſary to ſubtract the angle B D A, or 61° 56′, from 90°, and the remainder 28° 4′, is the angle B, and the angle C is equal to 53° 7′.

3. *By Gunter.*

1. The extent from 105 to 135 will reach from 35 to 45 on the line of numbers. Secondly, The extent from 85 to 75 on the line of numbers, will reach from the radius to 61° 56′, or the angle B D A, on the line of ſines. Thirdly, The extent from 50 to 30 on the line of numbers, will reach from the radius to 36° 53′, the angle A D C on the line of ſines.

The three foregoing caſes of plane triangles, contain all the varieties of both right and oblique triangles. But there are ſome other theorems, ſuited to ſome particular forms of triangles, which are often more expeditious in uſe, than the foregoing general ones; particularly the following theorem, for right-angled triangles, being a caſe which frequently occurs.

CASE IV.

When in a right-angled Triangle there are given the Angles, and one Leg, to find the other Leg, or Hypothenuſe.

Rule. As the radius is to the given leg A B (*fig.* 7), ſo is the tangent of the adjacent angle A, to the oppoſite leg B C; and ſo is the ſecant of the ſame angle A, to the hypothenuſe A C.

Example. In the triangle A B C, having the leg A B equal to 162, and the angle A equal to 53° 7′ 48″, and conſequently the angle C 36° 52′ 12″; to find the ſides B C and A C.

1. *Geometrically.*

Draw A B equal to 162, and erect the indefinite perpendicular B C; make the angle A 53° 7′ 48″; then the ſide A C will cut B C in the point C, and form the triangle A B C, which, by meaſuring, A C is found equal to 270, and B C to 216.

2. *Arithmetically.*

	Log.
As radius	10.0000000
Is to A B 162	2.2095150
So is the tangent A 53° 7′ 48″	10.1249372
	12.3344522
	10.0000000
To B C 216	2.3344522
So is the ſecant A 53° 7′ 48″	10.2218477
To A C 270	2.4313627

3. *By Gunter.*

Extend the compaſſes from 45° at the end of the tangents, (the radius,) to the tangent of $53^\circ\frac{1}{8}$; and that extent will reach from 162 to 216 on the line of numbers for B C. Then extend the compaſſes from 36° 52′ to 90 on the ſines; and that extent will reach from 162 to 270 on the line of numbers for A C.

There is alſo another method of frequent uſe in trigonometry, called, *making every ſide radius*, which is as follows:

Let A B C (*fig.* 8) be a given triangle; make the hypothenuſe A C radius firſt; that is, with the extent of A C as a radius, and on A and C, as two centres, deſcribe the two arcs C D and A E; then each leg A B, B C, will repreſent the ſine of its oppoſite angle: viz. The leg B C will be the ſine of the arc C D, or of the angle A; and the leg A B the ſine of the arc A E, or of the angle C.

Again, making either leg radius, the other leg will repreſent the tangent of its oppoſite angle, and the hypothenuſe the ſecant of the ſame angle: thus, with the radius A B and centre A, deſcribe the arc B K; and B C will repreſent the tangent of that arc, or of the angle A, and the hypothenuſe A C the ſecant of the ſame; or, with the radius B C and centre C, deſcribe the arc B G; then the other leg A B is the tangent of that arc B G, or of the angle C, and the hypothenuſe C A is the ſecant of the ſame.

Then the proportions are obvious; for the ſides in this figure bear the ſame proportions to each other as the parts they repreſent.

SECT. II.

OF SPHERICAL TRIGONOMETRY.

SPHERICAL Trigonometry teaches the reſolution of a *ſpherical triangle*, having three given parts: and, like plane trigonometry, may be either right-angled, or oblique-angled; but before the learner can proceed to the analogies of a ſpherical triangle, it is neceſſary to be acquainted with the ſix following theorems:—

Theorem I.

In all right-angled ſpherical triangles the ſine of the hypothenuſe is to the radius, as the ſine of a leg is to the ſine of its oppoſite angle. And as the ſine of a leg is to the radius, ſo is the tangent of the other leg to the tangent of its oppoſite angle.

Demonſtrated thus. Let E D A F G (*fig.* 9) repreſent the eighth part of a ſphere, where the quadrantal planes E D F G, and E D B C, are both perpendicular to the quadrantal plane A D F B, and the quadrantal plane A D G C is perpendicular to the plane E D F G; and the ſpherical triangle A B C is right-angled at B, and therefore C A is the hypothenuſe, and B A, B C, are the legs.

Draw the tangents H F and O B to the arches G F and C B, and the ſines G M, C I, on the radii D F and D B; alſo draw B L the ſine of the arc A B, and C K the ſine of A C: then join I K and O L. Now H F, O B, G M, and C I, are all perpendicular to the plane A D F B. And H D G K, and O L, lie all in the ſame plane A D G C; alſo F D, I K, B L, lie all in the ſame plane A D G C. Therefore, the right-angled triangles H F D, C I K, and O D L, having the equa-angles H D F, C K I, O L B, are ſimilar. And as C K is to D G, ſo is C I to G M; that is, as the ſine of the hypothenuſe is to the radius, ſo is the ſine of a leg to the ſine of its oppoſite angle. For G M is the ſine of the arc G F, which meaſures the angle C A B. Alſo as L B is to D F, ſo is B O to F H; that is, as the ſine of a leg is to the radius, ſo is the tangent of the other leg to the tangent of its oppoſite angle. Q. E. D.

From this it follows, that the ſines of the angles of any oblique ſpherical triangle, as A C D (*fig.* 10), are to one another directly as the ſines of the oppoſite ſides; therefore, in every right-angled ſpherical triangle, having the ſame perpendicular, the ſines of the baſes will be to each other inverſely as the tangents of the angles at the baſes.

Theorem II.

In every right-angled ſpherical triangle, as A B C (*fig.* 11), the proportion is, as radius is to the coſine of one leg, ſo is the coſine of the other leg to the coſine of the hypothenuſe.

Therefore, if two right-angled ſpherical triangles A B C, C B D (*fig.* 10), have the ſame perpendicular B C, the coſines of their hypothenuſes will be to each other directly as the coſines of their baſes.

Theorem III.

In any ſpherical triangle, the proportion is, As radius is to the ſine of either angle, ſo is the coſine of the adjacent leg to the coſine of the oppoſite angle.

Therefore, in right-angled ſpherical triangles having the ſame perpendicular, the coſines of the angles, at the baſe, will be to each other, directly, as the ſines of the vertical angles.

Theorem IV.

In any right-angled ſpherical triangles, the proportion is, As radius is to the coſine of the hypothenuſe, ſo is the tangent of either angle to the cotangent of the other angle.

Thus, As the ſum of the ſines of two unequal arches is to their difference, ſo is the tangent of half the ſum of thoſe arches, to the tangent of half their difference; and, As the ſum of the coſines is to their difference, ſo is the cotangent of half the ſum of the arches to the tangent of half the difference of the ſame arches.

Theorem V.

In any ſpherical triangles A B C (*fig.* 12 and 13), the proportion is, As the cotangent of half the ſum of half the difference, ſo is the cotangent of half the baſe, to the tangent of the diſtance (D E) of the perpendicular, from the middle of the baſe.

Theorem VI.

In any ſpherical triangle A B C (*fig.* 12), As the cotangent of half the ſum of the angles at the baſe, is to the tangent of half their difference, ſo is the tangent of half the vertical angle, to the tangent of the angle which the perpendicular C D makes with the line C F, biſecting the vertical angle.

The Solution of the Caſes of right-angled ſpherical Triangles, (Fig. 11.)

Caſe.	Given.	Sought.	Solution.
1	The hypo. AC, and one angle A.	The oppoſite leg B C.	As radius : ſine hypothenuſe A C :: ſine A : ſine BC. (by Theor. 1.)
2	The hypo. AC, and one angle A.	The adjacent leg A B.	As radius : coſine of A :: tangent A C : tangent A B. (by Theor. 1.)
3	The hypothenuſe A C, and one angle A.	The other angle C.	As radius : coſine of A C :: tangent A : cotangent C. (by Theor. 4.)
4	The hypo. AC, and one leg A B.	The other leg B C.	As coſine A B : radius :: coſine A C : coſine B C. (by Theor. 2.)
5	The hypo. AC, and one leg A B.	The oppoſite angle C.	As ſine A C : radius :: ſine A B : ſine C. (by Theor. 1.)
6	The hypo. AC, and one leg A B.	The adjacent angle A.	As tangent A C : tangent A B :: radius : coſine A. (by Theor. 1.)
7	One leg AB, and the adjacent angle A.	The other leg B C.	As radius : ſine A B :: tangent A : tangent B C. (by Theor. 4.)
8	One leg AB, and the adjacent angle A.	The oppoſite angle C.	As radius : ſine A :: coſine A B : coſine C. (by Theor. 3.)
9	One leg AB, and the adjacent angle A.	The hypothenuſe A C.	As coſine A : radius :: tangent A B : tangent A C. (by Theor. 1.)
10	One leg BC, and the oppoſite angle A.	The other leg A B.	As tangent A : tangent B C :: radius : ſine A B. (by Theor. 4.)
11	One leg BC, and the oppoſite angle A.	The adjacent angle C.	As coſine B C : radius :: coſine of A : ſine C. (by Theor. 3.)
12	One leg BC, and the oppoſite angle A.	The hypothenuſe A C.	As ſine A : ſine B C :: radius : ſine A C. (by Theor. 1.)
13	Both legs A B and B C.	The hypothenuſe A C.	As radius : coſine A B :: coſine B C : coſine A C. (by Theor. 2.)
14	Both legs A B and B C.	Any angle, as A.	As ſine A B : radius :: tangent B C : tangent A. (by Theor. 4.)
15	Both angles A and C.	Any leg, as AB.	As ſine A : coſine C :: radius : coſine A B. (by Theor. 3.)
16	Both angles A and C.	The hypothenuſe A C.	As tangent A : cotangent C :: radius : coſine A C. (by Theor. 4.)

Note. The 10th, 11th, and 12th caſes are ambiguous, as it cannot be determined by the data, whether A B C, and A C, be greater or leſs than 90 degrees each.

Solution of the Caſes of oblique ſpherical Triangles, (Fig. 12 and 13.)

Given.	Sought.	Solution.
ides AC, BC, ~~the angle~~ A ofite to one of m.	The angle B, oppoſite to the other.	As ſine B C : ſine A :: ſine A C : ſine B. *Note*. When BC is leſs than AC, it cannot be determined whether B be acute or obtuſe.
ides AC, BC, the angle A oſite to one of m.	The angle ACB.	Let fall the perpendicular C D upon A B, produced (if neceſſary); then radius : coſine AC :: tangent A : cotangent ACD.
ides AC, BC, the angle A oſite to one of m.	The other ſide AB.	As radius : coſine A :: tangent AC : tangent AD. (by Theor. 1.) This and the laſt caſe are both ambiguous, when the firſt is ſo.
ſides AC, AB, l the included gle A.	The other ſide BC.	As radius : coſine A :: tangent AC : tangent AB. (by Theor. 1.) Whence AD is alſo known.
ſides AC, AB, l the included gle A.	Either of the other angles, as B.	As radius : coſine A :: tangent AC : tangent AD. (by Theor. 1.) Whence BD is known; then as ſine BD : ſine AD :: tangent A : tangent B. (by Theor. 4.)
angles A, B, and the ſide betwixt them	The other angle B.	As radius : coſine AB :: tangent A : cotangent ACD. (by Theor. 4.) Whence BCD is alſo known; then as ſine ACD : ſine BCD :: coſine A : coſine B. (by Theor. 3.)
angles A, B, and the ſide betwixt them	Either of the other ſides, as BC.	As radius : coſine AC :: tangent A : cotangent A C D. (by Theor. 4.) Whence B C D is alſo known; then as coſine BCD : coſine ACD :: tangent AC : tangent BC. (by Theor. 1.)
angles A, B, d the ſide AC poſite to one them.	The ſide BC, oppoſite to the other.	As ſine B : ſine AC :: ſine A : ſine BC.
angles A, B, d the ſide AC poſite to one them.	The ſide AB betwixt them.	As radius : coſine A :: tangent AC : tangent AD. (by Theor. 1.) and as tangent B : tangent A :: ſine A D : ſine B D, whence AB is alſo known.
angles A, B, d the ſide AC poſite to one them.	The other angle ACB.	As radius : coſine AC :: tangent A : cotangent ACD. (by Theor. 4.) and as coſine A : coſine B :: ſine ACD : ſine BCD. (by Theor. 3.)
he three ſides 3, AC, and BC.	Any angle, as A.	As tangent ½ A B tangent $\frac{AC+BC}{2}$ :: tangent $\frac{AC-BC}{2}$: tangent DE the diſtance of the perpendicular from the middle of the baſe, whence A D is known. Then as tangent AC : tangent AD :: rad. : coſine A. (by Theor. 1.)
he three angles , B, and ACB.	Any ſide, as AC.	As cotangent $\frac{ABC+A}{2}$: tangent $\frac{ABC-A}{2}$ :: tangent $\frac{ACB}{2}$: tangent of the angle between the perpendicular and a line biſecting the vertical angles, whence ACD is alſo known. Then as tangent A : to tangent ACD :: radius : coſine AC.

The following propoſitions concerning ſpherical triangles, will render them more intelligible.

1. A ſpherical triangle is either equilateral, iſoſcelar, or ſcalene, according as it has the three angles all equal, or two of them equal, or all three unequal; and *vice verſa.*

2. The greateſt ſide is always oppoſite the greateſt angle, and the ſmalleſt ſide oppoſite the ſmalleſt angle.

3. Any two ſides, taken together, are greater than the third.

4. If the three angles of a ſpherical triangle be all acute, or all right, or all obtuſe, angles, the three ſides will be accordingly all leſs than 90 degrees, or equal to 90 degrees, or greater than 90 degrees; and *vice verſa.*

5. If from the three angles A, B, C, (*fig.* 14) of a ſpherical triangle, A, B, C, as poles, there be deſcribed upon the ſurface of the ſphere, three arches of a great circle, D E, D F, and F E, forming by their interſections another ſpherical triangle D E F; each ſide of the new triangle will be the ſupplement of the angle at its pole; and each angle of the ſame triangle will be the ſupplement of the ſide oppoſite to it in the triangle A B C.

6. In any triangle A B C, or A b C, right angled at A, (*fig.* 15,) the angles at the hypothenuſe are always of the ſame kind as their oppoſite ſides. And the hypothenuſe is greater or leſs than a quadrant, according as the ſides, including the right angle, are of different kinds, or of the ſame kind; that is to ſay, according as theſe ſame ſides are either both obtuſe, or both acute, or as one is obtuſe, and the other acute, and *vice verſa.* Firſt, the ſides, including the right angle, are always of the ſame kind as their oppoſite angles. Secondly, the ſides, including the right angle, will be of the ſame or different kinds, according as the hypothenuſe is leſs or more than 90 degrees; but one of them at leaſt will be of 90 degrees, if the hypothenuſe be ſo.

CHAP.

CHAP. XIII.

OF GEOGRAPHY.

SECT. I.

GEOGRAPHICAL DEFINITIONS.

1. GEOGRAPHY is the knowledge of the earth, or a deſcription of the terreſtrial globe, particularly of the ſurface and known habitable parts thereof, with all its different diviſions.

2. The earth is a globular body, ſurrounded with an atmoſphere of air, by which all terreſtrial bodies are confined to its ſurface, being attracted thereto by the laws of gravity.

That the earth is of a globular form, has been demonſtrated by a number of experiments, particularly by obſervations of the eclipſes of the moon; in which it appears, that the ſhadow of the earth always appears circular, whichever way it is projected. Alſo, by the obſervation of ſhips at ſea, which, after their departure from any coaſt, gradually diſappear to an obſerver on land, from the bottom upwards; that is, the firſt part which diſappears from the ſight, is the keel, or lower part of the ſhip; then thoſe parts which are higher up, and ſo on; the top of the maſt being the part that is laſt ſeen: this is owing to the convexity of the waters, which have the ſame globular figure as the earth *.

3. The earth alſo has a diurnal motion on its own axis, performing one revolution in 24 hours; thereby occaſioning the changes of the day and night, as will be ſeen in Aſtronomy.

4. The circumference of the globe is ſuppoſed to be divided into 360 parts, called degrees, and each degree ſubdivided into 60 minutes, and each minute into 60 ſeconds, &c. Every degree contains 60 geographic miles; conſequently, the circumference of the globe is 21,600 ſuch miles; and the diameter 6900 miles. But as 60 geographical miles are above 69 Britiſh meaſure, the circuit of the globe is therefore 24,840 Engliſh miles, and the diameter almoſt a third, or 7900 in round numbers.

5. The globe of the earth conſiſts of land and water: the proportion of the land to the water is not accurately known, but it is generally believed to be near one third.

6. The waters are divided into three oceans, (beſides the ſmaller ſeas:) viz. The Atlantic, Pacific, and the Indian ocean. 1. The Atlantic or Weſtern ocean divides Europe from America, and is 3000 miles wide. 2. The Pacific ocean divides America from Aſia, and New Holland, and is 10,000 miles wide. 3. The Indian ocean lies between the Eaſt Indies and Africa, and is 3000 miles wide. The parts or

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* The philoſophers of the laſt age differed greatly in their deſcriptions of the true ſpherical figure of the earth, of which there were two general opinions. The one maintained that the earth was a prolate ſpheroid: of this number was Caſſini. The other party maintained it to be an oblate ſpheroid: of this Sir Iſaac Newton was the chief, who inſiſted that the polar was ſhorter than the equatorial diameter by 36 miles. A party of philoſophers from France was ſent by the king of that country, to meaſure a degree on the polar circle, and alſo on the equator; the reſult of their experiment turned out exactly in favour of Sir Iſaac Newton's theory. But this inequality of the ſhape of the earth, as well as the inequalities occaſioned by the mountains, &c. make no *ſenſible* difference in the form of the earth.

branches of theſe oceans, called ſeas, as the Mediterranean ſea, &c. receive their names generally from the countries they border upon.

7. A *bay*, or *gulf*, is a part of the ſea, almoſt ſurrounded by land; as the gulf of Mexico, the bay of Biſcay, &c.

8. A *ſtrait* is a narrow paſſage out of one ſea into another; as the ſtrait of Gibraltar.

9. A *lake* is a water ſurrounded by land; as the lake of Geneva.

10. *Rivers* are ſtreams of water, iſſuing from ſprings in high grounds, and falling into the ſea, or other rivers; and are wider near their mouths than towards their heads or ſprings: they are deſcribed in maps by black lines.

11. The land is divided into two great continents: viz. The eaſtern and weſtern continents, beſides iſlands. The eaſtern continent is ſubdivided into three parts: viz. Europe, which is the north-weſt part; Aſia, the north-eaſt; and Africa, the ſouth. The weſtern continent conſiſts of America only, divided into North and South America.

12. An *iſland* is a piece of land entirely ſurrounded by water.

13. A *peninſula* is a country, or piece of land, ſurrounded by water, except on one ſide, where it joins to ſome other land.

14. An *iſthmus* is a narrow neck of land which joins a peninſula to ſome other country; as the iſthmus of Suez, which joins Africa to Aſia; the iſthmus of Darien, which joins North and South America.

15. A *cape*, called ſometimes a promontory, or head-land, is a point of land extending ſome way into the ſea.

16. The ſurface of the earth is ſuppoſed to be divided by ſeveral imaginary circles, for the better determining the ſituation and boundaries of the ſeveral countries and parts of the world, of which the moſt conſiderable circles are the following:—1. The *equator*, called alſo the equinoctial line, which divides the globe of the earth into two equal parts, or

hemiſpheres, the one north and the other ſouth. This circle is every where equally diſtant from the two poles; and upon this circle the degrees of longitude are marked. 2. The two *tropical circles*: viz. The *tropic of Cancer*, or the northern tropic, which encompaſſes the globe at the diſtance of 23½ degrees from the equator; and the *tropic of Capricorn*, or ſouthern tropic, which encompaſſes the globe at the ſame diſtance on the ſouth ſide of the equator. The ſpace between theſe two tropics is called the *Torrid Zone*. 3. The two polar circles; viz. the *arctic circle*, which ſurrounds the north pole; and the *antarctic circle*, which ſurrounds the ſouth pole; each at the diſtance of 23½ degrees from its reſpective pole. The ſpace included between the tropic of Cancer and the arctic circle is called the northern *Temperate Zone*, and that ſpace between the arctic circle and the north pole, is called the north *Frigid Zone*; and the correſponding ſpaces on the ſouthern hemiſphere have ſimilar names, as the ſouthern *Temperate Zone*, and the ſouthern *Frigid Zone*. 4. The *meridional lines*, which are lines drawn at right angles to the equator, and coinciding at the poles. Theſe lines run directly north and ſouth: and when the ſun appears full ſouth of any place, he is then ſaid to be on the meridian of that place; and it is then twelve o'clock at noon at that place. The latitude of places is always numbered on theſe lines.

17. The *longitude* is the diſtance of one country from another, and is either eaſt or weſt, and meaſured on the equator. The longitude of a place is always meaſured from the capital of the country where the author or traveller is; thus, when a perſon in England mentions the longitude of any other place, it is always underſtood that the longitude is reckoned from London; that is, the degrees of longitude are meaſured on the equator, and from that part of the equator, where the meridian paſſing through London, interſects the equator, at that part of the equator which is cut by the meridian of the other place meaſured to.

18. The *latitude* of a place is the diſtance of that place from the equator, meaſured on the meridional line; and is either north or ſouth.

19. The inhabitants of the earth are diſtinguiſhed from each other by their relative ſituations; of which there are reckoned three ſorts, *Periœci*, *Antœci*, and *Antipodes* :— 1. The *Periœci* are thoſe people who live at the ſame diſtance from the equator, but under oppoſite meridians: the length of their days and ſeaſons is the ſame; but when it is mid-day with one, it is midnight with the other. 2. The *Antiœci* live under the ſame meridian, but oppoſite parallels, and live equally diſtant from the equator; the one being in the ſouth latitude, and the other in the north. Theſe have the ſun at the ſame hour at noon; but the longeſt day of the one is the ſhorteſt day of the other, and their ſeaſons of the year are different; for when it is ſummer with one, it is winter with the other. 3. The *Antipodes* are ſituated directly on oppoſite ſides of the globe to each other, the feet of the one being directly oppoſite to the feet of the other. Theſe lie under oppoſite meridians, and oppoſite parallels: it is noon-day with the one, when it is midnight with the other; the longeſt day with the one, is the ſhorteſt day with the other; and when it is ſummer with the one, it is winter with the other.

20. The inhabitants of the earth are ſometimes diſtinguiſhed from each other (in geography) by the direction of their ſhadows at noon-day; and are called *Amphiſcii*, *Aſcii*, *Heteroſcii*, or *Periſcii*. 2. The *Amphiſcii* are thoſe ſituated in the torrid zone, and have their ſhadows, one part of the year, directed towards the north at noon-day, and at another part of the year, towards the ſouth, at noon-day, according to the part of the ecliptic the ſun is in; conſequently, the ſun is vertical to theſe people twice a year. They are then called: —2. *Aſcii*, ſhowing no ſhadow at noon-day. 3. The *Heteroſcii*, are thoſe who inhabit the temperate zones, and whoſe ſhadows at mid-day always fall one way: viz. The ſhadows

of thoſe in the northern temperate zóne, falling always towards the north, at noon-day; and thoſe in the ſouthern zone, always ſouth at noon-day. 4. The *Periſcii* are thoſe who inhabit either of the frigid zones. Theſe have their ſhadows moved entirely round them every 24 hours, when the ſun is in their hemiſpheres, and ſo far declined towards their pole, as not to ſet for ſeveral days.

21. The *horizon* is properly a double circle; one of the horizons being called the *ſenſible*, and the other the *rational horizon*. The former comprehends only that ſpace which we can ſee around us, upon any part of the earth; and is very different, according to the difference of our ſituation. The other, called *rational*, is parallel to the former, but paſſing through the centre of the earth, and ſuppoſed to be continued as far as the celeſtial ſphere itſelf; whereas the former is ſuppoſed to paſs over the ſurface of the earth, where the ſpectator ſtands: but in geography, when the horizon is mentioned, the rational horizon is always underſtood. By reaſon of the round figure of the earth, every different part has a different horizon. The poles of the horizon, that is, the points directly above the head, and oppoſite the feet of the obſerver, are called the *zenith* and *nadir*.

22. The *zenith* is that pole of the horizon directly over the obſerver's head.

23. The *nadir* is the oppoſite pole of the horizon, or that directly under the obſerver's feet.

A TABLE,

SHOWING

The Number of Miles in a Degree of Longitude, in every Degree of Latitude, from the Equator.

Degrees of Latitude.	Miles.	Degrees of Latitude.	Miles.	Degrees of Latitude.	Miles.
1	59.96	31	51.43	61	29.04
2	59.94	32	50.88	62	28.17
3	59.92	33	50.32	63	27.24
4	59.86	34	49.74	64	26.30
5	59.77	35	49.15	65	25.36
6	59.67	36	48.54	66	24.41
7	59.56	37	47.92	67	23.45
8	59.40	38	47.28	68	22.48
9	59.20	39	46.62	69	21.51
10	59.08	40	46.00	70	20.52
11	58.89	41	45.28	71	19.54
12	58.68	42	44.95	72	18.55
13	58.46	43	43.88	73	17.54
14	58.22	44	43.16	74	16.53
15	58.00	45	42.43	75	15.52
16	57.60	46	41.68	76	14.51
17	57.30	47	41.00	77	13.50
18	57.04	48	40.15	78	12.48
19	56.73	49	39.36	79	11.45
20	56.38	50	38.57	80	10.42
21	56.00	51	37.73	81	9.38
22	55.63	52	37.00	82	8.35
23	55.23	53	36.18	83	7.32
24	54.81	54	35.26	84	6.28
25	54.38	55	34.41	85	5.23
26	54.00	56	33.55	86	4.18
27	53.44	57	32.67	87	3.14
28	53.00	58	31.70	88	2.09
29	52.48	59	30.90	89	1.05
30	51.96	60	30.00	90	0.00

Deſcription and Uſe of the Globes and Armillary Sphere.

By means of maps, the true ſituations of the different places of the earth, with regard to one another, and every other particular relative to them, may be eaſily known; conſequently the hour of the day, ſeaſon of the year, &c. for any particular place may be diſcovered. But theſe problems, to be reſolved by maps, would be tedious and complex; therefore, thoſe machines called the celeſtial and terreſtrial globes, and the armillary ſphere, have been invented, by which many calculations are ſaved, and every problem in geography may be ſolved mechanically, in the moſt eaſy and expeditious manner.

If a map of the world be accurately delineated on a ſpherical ball, the ſurface thereof will repreſent the ſurface of the earth; for the higheſt hills bear no greater proportion to the bulk of the whole earth than ſo many grains of ſand do to a common mathematical globe of twelve or eighteen inches diameter; the diameter of the earth being near 8000 miles, and no hill upon its ſurface is above three miles in perpendicular height.

The armillary ſphere is a large hollow ſphere of glaſs, having as many bright ſtuds fixed on its inſide, as there are viſible ſtars in the heavens, and of the ſame magnitude, and at the ſame angular diſtances from each other. This ſphere is a true repreſentation of the heavens, to an eye ſuppoſed to be placed in the centre; for to an obſerver placed any where within the ſurface of an indefinite ſphere, all objects will appear equally diſtant, though ſome be much nearer than others: and if a ſmall globe, having a map of the earth upon it, be placed on an axis in the centre of this ſphere, and the ſphere be made to turn round its axis, it will repreſent the *apparent* motion of the heavens round the earth: but if the globe be turned round its axis, while the ſphere remains fixed, it will repreſent the *true* motion of the earth.

If there be drawn a great circle upon this ſphere, equally diſtant from its poles, and having the plane of the circle perpendicular to the axis of the ſphere, it will repreſent the celeſtial equinox, which divides the heavens into two equal parts or hemiſpheres; and the two axes of the ſphere will repreſent the two poles of the heavens.

If there be another great circle drawn upon the ſphere, cutting the equinoctial at an angle of 23⅓ degrees, in two oppoſite points, this circle will repreſent the ecliptic, or circle of the ſun's apparent annual motion; one half of which is on the north ſide, and the other half on the ſouth ſide of the equinoctial.

If there be made a large ſtud to move eaſtward in this ecliptic, and with ſuch a motion as to go quite round it in the time that the ſphere is turned round weſtward upon its axis 366 times; this ſtud will repreſent the ſun changing its place every day in the ecliptic, a 365th part; and going round weſtward in the ſame direction as the ſtars, but with a motion ſo much ſlower than that of the ſtars, that they will make 366 revolutions in the time that the ſun makes only 365, about the axis of the ſphere.

If the terreſtrial globe in this machine be about one inch in diameter, and the diameter of the ſtarry ſphere about five or ſix feet, a ſmall inſect, placed upon the globe, would ſee only a very ſmall portion of its ſurface; but it would ſee one half of the ſurface of the ſtarry ſphere, the convexity of the globe hiding the other half from its view. If the ſphere be ſet in motion as before directed, and the globe alſo revolving on its own axis, the inſect will ſee all the phenomena obſerved by the inhabitants of this world, in the diurnal rotation of the earth round its axis.

The exterior parts of this machine are ſeveral braſs rings, which repreſent the principal circles in the heavens: viz. 1. The *equinoctial*: 2. the *ecliptic*, divided into the ſigns and degrees, and alſo into the months and the days of the year, to ſhow in what point of the ecliptic the ſun is on any given day in the year: 3. the *two tropics:* 4. the *arctic* and *antarctic circles:* 5. the *equinoctial colure*, which is a great circle paſſing through the north and ſouth poles of the heavens, and through the equinoctial circle at the points where the equinoctial is cut by the ecliptic: 6. the *ſolſtitial colure*, which is a great circle paſſing through the poles of the heavens, and at right angles to the equinoctial colure. Hence the ſolſtitial colure paſſes through the equinoctial at the points where the equinoctial is at the greateſt diſtance from the ecliptic. Theſe points in the equinoctial are called the *ſolſtitial points.*

In the north pole of the ecliptic is a nut, to which is fixed one end of a quadrantal wire, having at the other end a ſmall ſun, which is carried round the ecliptic, by turning the nut; and in the ſouth pole of the ecliptic, another quadrantal wire is fixed, with a ſmall moon upon it, which may be moved round by the hand. There is alſo a particular contrivance, for cauſing the moon to move in her own orbit.

On the axis of the ſmall globe is fixed a flat celeſtial meridian, which may be ſet directly over the meridian of any place on the globe; and then turned round with the globe, ſo as to keep over the ſame meridian. This globe has alſo a moveable horizon, which turns upon two wires, which proceed from it on the eaſt and weſt points of the globe, and entering the globe at the oppoſite points in the equator, which is a moveable braſs ring, let into the globe in a groove. The whole fabric is ſupported on a pedeſtal, and may be elevated or depreſſed to any number of degrees, from 0 to 90.

Deſcription of the Terreſtrial Globe.

On the terreſtrial globe are drawn all the principal circles before mentioned, as the equator, ecliptic, tropics, polar circles, and meridians. The ecliptic is divided into twelve ſigns, and each ſign into thirty degrees. Each tropic is 23½ degrees from the equator; and each polar circle 23½ degrees from its reſpective pole. There are alſo circles drawn parallel to the equator, at every 10 degrees diſtance from it, on each ſide towards the poles: theſe circles are called *parallels of latitude*. There are, alſo, ſeveral other circles, drawn perpendicularly to the equator, and interſecting each other at the poles; theſe circles are called *meridians*, and ſometimes circles of *longitude*, or *hour circles*; and on large globes they are drawn through every tenth degree of the equator; but on globes of leſs than 12 inches diameter they are drawn through every fifteenth degree.

The globe is hung in a braſs ring, called the *brazen meridian*, turning upon a wire in each pole, ſunk into one ſide of the meridian ring. This meridian is divided into 360 degrees; one half of theſe degrees are numbered from the equator to the poles, to ſhow the latitude of places; the other half are numbered from the poles to the equator, to ſhow how to elevate either of the poles above the horizon. This ring divides the globe into two equal parts, called the *eaſtern* and *weſtern hemiſpheres*; as the equator divides it into the *northern* and *ſouthern hemiſpheres*.

The brazen meridian is let into two notches, made in a broad flat ring, called the *wooden horizon*; the upper ſurface of which divides the globe into two equal parts, called the *upper* and *lower hemiſpheres*. This horizon correſponds to the true rational horizon; and upon it are ſeveral concentric circles, which contain the months of the year, the ſigns and degrees anſwering to the ſun's place for each month and day, the thirty-two points of the compaſs, and the circles of amplitude and azimuth, with ſome other circles.

There is a ſmall horary circle, fixed to the north part of the brazen meridian, and having the wire in the north pole of the globe in its centre; on which wire is an index, which goes over all the twenty-four hours of the circle, as the globe is turned round its axis. Sometimes there are two horary circles, one at each pole.

There is a thin ſlip of braſs, called the *quadrant of altitude*, divided into 90 degrees. This is occaſionally fixed to the uppermoſt point of the brazen meridian, by a nut and ſcrew, about which the quadrant turns round.

There is alſo, to ſome globes, a magnetic needle, which moves over a circle, divided into 360 degrees, alſo over the thirty-two points of the compaſs. This needle ſerves to fix the globe according to the meridian of the place, as the needle makes nearly a conſtant certain angle with the meridian, which angle is called the *variation of the compaſs*; and which being known, the globe may be rectified to the true meridian of the place: thus, at London, the variation of the needle is 23 degrees northward; therefore, by moving the frame of the globe about, till the needle ſettles itſelf near the twenty-third degree, reckoning weſtward from the north point, we then have the braſs meridian of the globe coinciding with the true meridian of the place.

SECT. II.

OF THE SOLUTION OF GEOGRAPHICAL PROBLEMS.

Directions for uſing the Globe.

In uſing the globe, the eaſt ſide of the horizon muſt always be kept towards you (except the globe require turning by the Problems), for then the graduated ſign of the meridian will be towards you, the quadrant of altitude before you, and the globe divided exactly into two hemiſpheres, by the graduated ſide of the meridian.

When it is required to turn the globe and frame entirely round, as is the caſe in working ſome Problems, the ball of the globe will ſometimes be moved from the degree to which it is ſet, by ſuch motion; to avoid this, the feather end of a quill may be thruſt between the ball of the globe and the meridian, which, without hurting the ball, will keep it from turning in the meridian, while you turn the horizon about.

PROBLEM I.

To find the Latitude and Longitude of any given Place upon the Globe.

Turn the globe on its axis, until the given place comes exactly under the graduated ſide of the brazen meridian, and obſerve what degree of the meridian the place then lies under, which is its *latitude*, and is either north or ſouth.

The globe remaining in this poſition, the degree of the equator, which is under the graduated ſide of the brazen meridian, is the *longitude* of the place; and is either eaſt or weſt, as the place lies on the eaſt or weſt ſide of that which is accounted the *firſt meridian*. Thus, all the Atlantic Ocean, and the continent of America, is on the weſt ſide of the meridian of London; and the greater part of Europe and of Africa, together with all Aſia, is on the eaſt ſide of the meridian of London, which is reckoned the firſt meridian of the globe, by Britiſh geographers and aſtronomers; though the exact meridian for England is that of Greenwich.

PROBLEM II.

To find any Place on the Globe, having the Latitude and Longitude.

This is the reverſe of the former Problem, and is found by bringing the point of longitude in the equator to the brazen meridian: then count from the equator on the brazen meridian to the degree of the given latitude; and under that degree of latitude will be the place required.

PROBLEM III.

To find the Difference of Longitude, or Difference of Latitude, between any two given Places.

Bring each of the two given places to the brazen meridian, and mark their latitudes; then, if both places are on the ſame ſide of the equator, the leſſer latitude ſubtracted from the greater will give the difference; but if they are on different ſides of the equator, both latitudes muſt be added together. And the difference of longitude is found by bringing each place to the meridian, and reckoning on the equator the difference of degrees between the meridians of the two places, if it be leſs than 180 degrees, or half a circle; but if the difference be greater, it muſt be ſubtracted from 360, and the remainder is the difference of longitude.

PROBLEM IV.

To find all thoſe Places that have the ſame Latitude and Longitude with any given Place.

Bring the given place to the brazen meridian; then all thoſe places which lie under the ſaid meridian will have the ſame longitude. Then turn the globe round on its axis, and all thoſe places which paſs under the ſame degree of latitude in the brazen meridian, that the given place does, have the ſame latitude.

PROBLEM V.

To find the Diſtance between any two Places on the Globe.

Lay the graduated edge of the braſs quadrant of altitude, over both the places, and count the number of degrees intercepted,

tercepted between them on the quadrant, which will be the diſtance in degrees; and which multiplied by 60, will give the diſtance in geographical miles; but multiplied by 69½ gives the diſtance in Engliſh miles. Or, the diſtance between the two places may be taken with a pair of compaſſes, and that extent applied to the equator, will ſhow the number of degrees diſtant.

PROBLEM VI.

The Hour of the Day at any Place being given, to find what is the Hour at any other Place.

Bring the given place to the brazen meridian, and ſet the index to the given hour; then turn the globe, until the place where the hour is required, comes to the meridian; and the index will point to the hour at that place.

PROBLEM VII.

To find the Sun's Place in the Ecliptic, and his Declination, for any given Day in the Year.

Look on the wooden horizon for the given day, and againſt it there is placed the degree of the ſign in which the ſun is on that day at noon. Find the ſame degree of this ſign in the ecliptic line upon the globe, and having brought it to the brazen meridian, obſerve what degree of the meridian ſtands over it; and that is the ſun's declination, reckoned from the equator.

PROBLEM. VIII.

To find all thoſe Places in the north Frigid Zone where the Sun begins to ſhine conſtantly, without ſetting, on any given Day: which muſt be between the 21ſt of March and the 23d of September. (See Fig. 3, Plate 15.)

Having brought the ſun's place for the given day to the brazen meridian, and found his declination, (by Problem VII.)

VII.) count as many degrees on the meridian from the north pole as are equal to the ſun's declination, and mark that degree from the pole on the meridian; then turning the globe round on its axis, obſerve what places in the north frigid zone paſs directly under that mark; for they are the places required.

The ſame problem may be reſolved for places within the ſouth frigid zone, for the other half of the year.

PROBLEM IX.

To find the Place over which the Sun is vertical at any Hour of a given Day.

Find the ſun's declination for the given day, (by Problem VII.) which mark on the brazen meridian; then bring the place where you are, (ſuppoſe London,) to the brazen meridian, and ſet the index to the given hour; then turn the globe on its axis until the index point to 12 at noon; and the place on the globe which is directly under the point of the ſun's declination, marked upon the meridian, has the ſun that moment in the zenith, or directly vertical.

Note. The hour 12 at noon on the hour circle is the uppermoſt 12.

PROBLEM X.

Having the Day and Hour of a Lunar Eclipſe, to find all thoſe Places of the Earth to which it will be viſible.

When the moon is eclipſed, ſhe is always at the full, and, conſequently, oppoſite to the ſun; therefore, whatever part of the earth the ſun is vertical to, the moon muſt be vertical to the antipodes of that part; conſequently, the ſun will be viſible then to one half the earth, and the moon to the other half.

Therefore, find the place to which the ſun is vertical at the given hour, (by Problem IX.) elevate the pole to the latitude of that place, and bring the place to the upper part of the brazen meridian: then as the ſun will be viſible to all thoſe parts of the globe which are above the horizon, the moon will be viſible to all thoſe parts below it, at the middle of the eclipſe.

PROBLEM XI.

To rectify the Globe for the Latitude, the Zenith, and the Sun's Place.

Find the latitude of the place, (by Problem I.) and if the place be in north latitude, raiſe the north pole as many degrees above the horizon, counting upon the meridian from the north pole to the horizon; but if the place be in ſouth latitude, raiſe the ſouth pole as many degrees: then turn the globe till the place comes under its latitude on the brazen meridian, and faſten the quadrant of altitude to the meridian, ſo that the chamfered edge of its nut may be joined to the zenith. Then bring the ſun's place in the ecliptic for the given day, to the graduated ſide of the brazen meridian, and ſet the hour index to 12 at noon, and the globe will be rectified.

PROBLEM XII.

Having the Latitude of any Place between the two Polar Circles, to find the Time of the Sun's Riſing and Setting, or the Length of the Day and Night, for any given Day in the Year.

Rectify the globe for the latitude, and the ſun's place, by the foregoing problem; then bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon, and the hour index will ſhow the time of ſun-riſing; then turn the globe on its axis, until the ſun's place comes to the weſtern ſide of the horizon, and the index will then ſhow the time of ſun-ſetting.

The hour of ſun-ſetting being doubled, gives the length of the day; and the hour of the ſun-riſing doubled, gives the length of the night.

PROBLEM XIII.

Having the Latitude of a Place, and the Day of the Month, to find when the Morning Twilight begins, and Evening Twilight ends.

Rectify the globe, and bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon; then mark that point of the ecliptic which is in the weſtern ſide of the horizon, which is the point oppoſite to the ſun's place; and lay the quadrant of altitude over the ſaid point, and turn the globe eaſtward, keeping the quadrant at the ſame mark, until the ſaid point on the ecliptic is 18° high on the quadrant, and the index will point out the time when the morning twilight begins; for the ſun's place will be 18° below the eaſtern ſide of the horizon. Then, to find the time when the evening twilight ends, bring the ſun's place to the weſtern ſide of the horizon, and the point oppoſite to it, which was marked, will be riſing in the eaſt; bring the quadrant over that point, and keeping it thereon, turn the globe weſtward, until the ſaid point be 18° above the horizon on the quadrant, and the index will ſhow the time when the evening twilight ends; as the ſun's place will be then 18° below the weſtern ſide of the horizon.

When the ſun does not go 18° below the horizon of any place, the twilight continues the whole night in that place: and between 49° of latitude, and the polar circles, the twilight continues for ſeveral nights together, in the ſummer ſeaſon: and the nearer the place is to the polar circle, the greater is the number of theſe nights.

PROBLEM XIV.

To find what Day of the Year the Sun begins to ſhine conſtantly without ſetting, on any given Place in the Frigid Zone, and how long it continues to do ſo.

Rectify the globe for the latitude of the place, and turn it about till ſome point of the ecliptic, between *Aries* and *Cancer*, (if the given place be in the north frigid zone,) coincides with the north point of the horizon, where the brazen meridian cuts it. Then find on the wooden horizon, what day of the year the ſun is in that point of the ecliptic; for that is the day on which the ſun begins to ſhine conſtantly on the given place, without ſetting. Then turn the globe until ſome point of the ecliptic, between *Cancer* and *Libra*, coincide with the north point of the horizon, where the brazen meridian cuts it; and find on the wooden horizon on what day the ſun is in that point of the ecliptic; which is the day the ſun leaves off conſtantly ſhining on the ſaid place, and riſes and ſets to it, as to other places on the globe. The number of natural days, or complete revolutions of the ſun about the earth, between the two days above found, is the time that the ſun keeps conſtantly above the horizon without ſetting; for all that portion of the ecliptic which lies between the two points which interſect the horizon in the very north, never ſets below it; and there is juſt as much in the oppoſite part of the ecliptic that never riſes; therefore, the ſun will keep as long conſtantly below the horizon of every place upon the globe in winter, as he is above it in ſummer.

PROBLEM XV.

Having the Latitude, the Sun's Place, and his Altitude, to find the Hour of the Day, and the Sun's Azimuth, or Number of Degrees that he is diſtant from the Meridian.

Having rectified the globe, and brought the ſun's place to the given height upon the quadrant of altitude, which muſt be on the eaſtern ſide of the horizon, if the time be in the forenoon; and on the weſtern ſide if it be afternoon; then the index will ſhow the hour of the day; and the number of degrees in the horizon intercepted between the quadrant of altitude and the ſouth point, will be the ſun's true azimuth at that time.

PROBLEM XVI.

To find what Hour of the Day it is, in any Part of the World.

Rectify the globe for the latitude of the place; and having ſet the index to the hour of the day, turn the globe, and bring the places of which the hour is required, ſucceſſively to the brazen meridian, and the index will point to the ſeveral hours. For example: if the place be London, and the hour 12 at noon, the globe being rectified for London, and London brought to the meridian, and the index ſet to the hour 12, turn the globe, till Naples comes to the brazen meridian, and the index then will point to the hour 1; Naples being 15° eaſtward of London. Then continue to turn the globe 15° farther, and Peterſburgh, Conſtantinople, and Grand Cairo, will come under the brazen meridian, or very near it, then the index will point to the hour 2; theſe three cities having the noon-day ſun about two hours before us in London. And turning the globe 15° further, the index will point to the hour 3; and all the places under the meridian will have the ſun vertical to them. And thus for every 15° of longitude eaſtward, the inhabitants of thoſe places have the ſun an hour ſooner. On the contrary, all the inhabitants ſituated to the weſtward of London, have the ſun later in the ſame proportion; that is, an hour later for every 15° of weſtern longitude.

Moſt of the foregoing Problems may be reſolved by a map as well as a globe, though the operation may be ſomewhat more tedious, particularly by Plate 10, where the two hemiſpheres of the world repreſent the ſurface of a terreſtrial globe in plano.

WINDS.

Winds are generally divided into two parts, according to the different parts of the earth on which they blow; being the tropical winds, or thoſe which blow between the two tropics, and thoſe which blow without the tropics.

The tropical winds generally extend to 30° on each ſide the equator, and are of three kinds: 1. The *general trade-winds.* 2. The *monſoons.* 3. The *ſea and land breezes.*

1. The trade-winds blow from north-eaſt on the north ſide of the equator, and from the ſouth-eaſt on the ſouth ſide of the equator, and near the equator almoſt due eaſt; but under the equator, and from 2 to 5° on each ſide of the equator, the winds are variable; and ſometimes it is calm for a month together.

2. The monſoons are periodical winds, which blow ſix months in one direction, and the other ſix months in the oppoſite direction. At the change or ſhifting of the monſoons, are generally violent ſtorms of wind, thunder, lightning, and rain, which always happen about the equinoxes. The monſoons extend about two hundred leagues from land, and are chiefly in the Indian ſeas.

3. The ſea and land breezes are alſo periodical winds, which blow from the land in the night, and beſt part of the morning; and from the ſea from ten in the forenoon till ſix in the evening. Theſe do not extend above two or three leagues from ſhore.

Near the coaſt of Guinea, in Africa, the wind blows almoſt conſtantly from the weſt.

On the coaſt of Peru, in South America, the wind blows conſtantly from the ſouth-weſt.

Between the third and tenth degree of ſouth latitude, the ſouth-eaſt trade-wind continues from April to October; but during the reſt of the year, the wind blows from the north-weſt. Between Sumatra and New Holland this wind blows from the ſouth from March to September; but from September to April blows in the oppoſite direction. Between Africa and Madagaſcar its direction is influenced by the coaſt; for it blows from the north-eaſt from October to April; and from the ſouth-weſt the reſt of the year.

In the Indian ocean, to the northward of the third degree of ſouth latitude, the north-eaſt wind blows from October to April, and the oppoſite wind the reſt of the year. This wind blows nearly from the ſouth in the ſummer months, from the iſle of Borneo, along the coaſt of Malacca, as far as China; and in the winter months it blows from the north by eaſt.

In temperate zones the winds are very irregular, and no certain rule can be formed of their changes. But when winds are violent and continue long, it is generally found that they extend over a large tract of country; particularly if they blow from the north or eaſt. By the multiplication and compariſons of meteorological tables, the following theorems have been deduced.

In Virginia, the prevailing winds are between the ſouth-weſt, weſt, north, and north-weſt; but the moſt frequent is the ſouth-weſt. At Ipſwich, in New England, the prevailing winds are the ſame, but the moſt frequent is the north-weſt. At Cambridge, in the ſame province, the moſt frequent is the ſouth-eaſt. The predominant winds of New York are the north and weſt. And in Nova Scotia, north-weſt. And at Hudſon's Bay, weſt.

It appears from theſe obſervations, that the weſterly winds are the moſt frequent over the whole eaſtern coaſts of North America; but in the ſouthern provinces the ſouth-weſt wind is predominant; and the north-weſt wind becomes gradually more frequent as we approach the frigid zone.

In Egypt, from May to September, incluſive, the wind blows almoſt conſtantly from the north, varying in a few points from eaſt to weſt, in the months of June and July.

In the Mediterranean ſea the wind blows nearly nine months of the year from the north; and at the equinoxes there is always an eaſterly wind in that ſea. But in the ſtraits of Gibraltar the winds are either from eaſt or weſt.

In Italy the prevailing winds differ conſiderably, according to the ſituation of the places: at Rome and Padua they are northerly; at Milan eaſterly.

The prevailing wind in Spain and Portugal is the weſt; particularly on the weſtern coaſts of theſe countries; but at Madrid it is north-eaſt.

In France, along the whole ſouth coaſt of that country, the wind blows moſt frequently from the north, or north-eaſt and north-weſt. On the weſtern coaſt of the Netherlands, as far north as Rotterdam, the prevailing wind is the ſouth-weſt.

From the regiſter, kept for the ſpace of ten years, by order of the Royal Society at London, the average of the winds at that place, blow in the following order:—

Winds.	Days.	Winds.	Days.
South-weſt	112	South-eaſt	32
North-eaſt	58	Eaſt	26
North-weſt	50	South	18
Weſt	53	North	16

It appears from this regiſter, that the ſouth-weſt wind blows at an average more frequently than any other winds, during every month of the year, but particularly in July and Auguſt; and the north-eaſt blows moſt conſtantly during the months January, March, April, May, and June, and moſt ſeldom during February, July, September, and December; and the north-weſt blows more frequently from November to March; and more ſeldom in September and October than in any other months.

Tides.

The tide is that riſe and fall of the water obſerved on all maritime coaſts.

It is obſervable, that on the ſhores of the ocean, and in all bays, creeks, harbours, &c. which have a free communication with the ocean, the waters riſe up above a certain mean rate twice a day, and as often ſink below: this is what is called the *flood* and *ebb*; or an high and low water. The whole interval between high and low water is called a *tide*: the water is ſaid to *flow* and to *ebb*; and the riſing is called the *flood tide*, and the falling the *ebb tide*.

This riſe and fall of the waters is very variable in quantity. Thus, at Plymouth it is ſometimes twenty-one feet between the greateſt and leaſt depth of the water in one day, or between high and low water; and ſometimes it is only twelve feet.

The greateſt flow of tide in any place is called a *ſpring tide*, and the leaſt flow is called a *neap tide*; and the different heights of the tide gradually increaſe every day from a neap to a ſpring tide; and then gradually decreaſe from a ſpring to a neap tide.

The whole time between the ſpring and neap tide is about fifteen days; and two of theſe intervals will make an exact lunation, or change of the moon. For the ſpring tide is obſerved to happen at a certain interval of time (generally between two and three days) after the new or full moon; and a neap tide at a certain interval after the half moon. Thus, the high water happens at new and full moon, when the moon has a certain determined poſition with reſpect to the meridian of the place of obſervation, preceding or following the moon's ſouthing a certain interval of time; which is always conſtant with reſpect to that place, but very different in different places.

The interval between two ſucceeding high waters is very variable. It is leaſt of all about new and full moon, and greateſt when the moon is at her quadratures. As two high waters happen every day, we may call the double of their interval, a tide day. Now, this tide day is ſhorteſt about new and full moon, being then about 24 hours and 37 minutes; but longeſt at the moon's quadratures, being then 25 hours and 27 minutes.

The tides, being in ſimilar circumſtances, are greateſt when the moon is at her leaſt diſtance from the earth; and leaſt, when ſhe is at her greateſt diſtance from the earth.

The ſame may be remarked with reſpect to the ſun's diſtance. Thus, the greateſt tides are obſerved during the winter months in Europe, or when the ſun is at his leaſt diſtance.

The tides in every part of the ocean increaſe as the moon, by changing her declination, approaches the zenith of that place.

The tides which happen while the moon is above the horizon are greater than thoſe of the ſame day, when the moon is below the horizon.

Theſe are all the regular phenomena of the tides. They are of the utmoſt importance to all commercial nations, and have therefore been much attended to by all navigators and aſtronomers.

SECT. III.

THE GRAND DIVISIONS OF THE EARTH.

The principal diviſions of the earth, as before mentioned, are, into land and water.

The land is divided into two great continents, beſides iſlands: viz. The eaſtern and weſtern continents. The eaſtern continent is ſubdivided into the following parts: viz. Europe on the north-weſt, Aſia on the north-eaſt, and Africa on the ſouth, being joined to Aſia by the Iſthmus of Suez, which is 60 miles over. The weſtern continent conſiſts of North and South America, joined by the Iſthmus of Darien, between 60 and 70 miles in breadth.

Europe is again ſubdivided into the following principal parts, and is ſituated between the tenth degree weſt longitude and the ſixth-fifth degree eaſt longitude, and between the thirty-ſixth and ſeventy-ſecond degree of north latitude. It is bounded on the north by the Frozen Ocean, on the eaſt by Aſia, on the ſouth by the Mediterranean Sea, which divides it from Africa; and on the weſt by the Atlantic Ocean, which ſeparates it from America; being 3000 miles in length, from Cape St. Vincent in the weſt to the mouth of the river Oby in the north-eaſt; and 2500 miles broad from north to ſouth, from North Cape in Norway to Cape Cayha in the Morea, the moſt ſouthern point of Europe. It contains the following ſtates and kingdoms:

A TABLE

A TABLE OF EUROPE.

Nations.	Length.	Breadth	Chief Cities.	North Latitude.			Longitude from Greenwich.			Difference of Time from Greenwich.		
				D.	M.	S.	D.	M.	S.	H.	M.	S.
England	360	300	London	51	31	0	0	5	37W	0	0	22½aft
Scotland	300	150	Edinburgh	55	57	57	3	12	15W	0	12	49 aft.
Ireland	285	160	Dublin	53	21	12	6	6	30W	0	24	26 aft.
Norway	1000	300	Bergen	60	11	0	5	45	0E	0	24	0bef.
Denmark	240	180	Copenhagen	55	40	45	12	35	15E	0	50	21bef.
Sweden	800	500	Stockholm	59	20	35	18	3	55E	1	12	16bef.
Ruſſia	1500	1100	Peterſburgh	59	56	0	30	19	15E	2	1	17bef.
Poland	700	680	Warſaw	52	14	0	21	0	30E	1	24	2bef.
Pruſſia	609	350	Berlin	52	32	30	13	26	15E	0	53	45bef.
Germany	600	500	Vienna	48	12	40	16	22	30E	1	5	30bef.
Bohemia	300	250	Prague	50	4	30	14	45	0E	0	59	0bef.
Holland	150	100	Amſterdam	52	22	45	4	45	30E	0	19	2bef.
Flanders	200	200	Bruſſels	50	51	0	4	21	45E	0	17	27bef.
France	600	500	Paris	48	50	14	2	20	0E	0	9	20bef.
Spain	700	500	Madrid	40	25	0	3	25	45W	0	13	43 aft.
Portugal	300	100	Liſbon	38	42	25	9	9	59W	0	36	40 aft.
Swiſſerland	260	100	Bern	40	0	0	7	40	0E	0	28	0bef.
Italy	750	400	Rome	41	53	54	12	29	15E	0	49	50bef.
Hurgary	300	200	Buda	47	20	0	19	22	0E	1	17	0bef.
Turkey in Europe	1400	730	Conſtantinople	41	0	24	28	53	49E	1	55	35bef.

Beſides the foregoing ſtates, Europe contains ſeveral iſlands, of which the following are the principal.

Iſlands.		*Chief Towns.*	*Subject to*
In the Northern Ocean	Iceland	Skalholt	Denmark
In the Baltic Sea	Zealand, Funen, Alſen, Falſter, Langland, Laland, Femeren, Mona, Bornholm	-	Denmark
	Gothland, Aland, Rugen	-	Sweden
	Oſel Dagho	-	Ruſſia
	Uſedom, Wollin	-	Pruſſia
Mediterranean Sea	Ivica	Ivica	Spain
	Majorca	Majorca	Spain
	Minorca	PortMahon	Spain
	Corſica	Baſtia	France
	Sardinia	Cagliari	K. Sardinia
	Sicily	Palermo	K. of 2 Sici.
Gulf of Venice	Luſiena, Corfu, Cephalonia, Zant, Leucadia	-	Venice
Archipelago & Levant Seas	Candia, Rhodes, Negropont, Lemnos, Tenedos, Scyros, Mytelene, Scio, Samos, Patmos, Paros, Cerigo, Santorin, &c. being part of Ancient and Modern Greece	-	Turkey

Aſia is ſituated between 25 and 180 degrees of eaſt longitude, and between the equator and 80 degrees north latitude; being about 4740 miles in length, from the Dardanelles on the weſt to the eaſtern ſhore of Tartary; and about 4380 in breadth, from the moſt ſouthern part of Malacca to the moſt northern cape of Nova Zembla. It is bounded by the Frozen Ocean on the north, on the weſt it is ſeparated from Africa by the Red Sea, and from Europe by the Levant or Mediterranean, the Archipelago, the Helleſpont, the Sea of Marmora, the Boſphorus, the Black Sea, the river Don, and a line drawn from it to the river Tobel, and from thence to the river Oby, which falls into the Frozen Ocean; on the eaſt it is bounded by the Pacific Ocean or South Sea, which ſeparates it from America; and on the ſouth, by the Indian Ocean: thus, it is almoſt ſurrounded by the ſea. The principal diviſions are as follow:

A TABLE OF ASIA.

Nations.	*Length.*	*Breadth.*	*Chief Cities.*	*North Latitude.*			*Eaſt Longitude from Greenwich.*			*Difference of Time from Greenwich.*		
				D.	M.	S.	D.	M.	S.	H.	M.	S.
Tartary. Independent	*Boundaries of theſe countries variable.*		Samarcand	39	50	0	69	0	0	4	36	0bef.
Mogulean			Tibet	37	0	0	8	0	0	5	40	0bef.
Chineſe			Chynian	48	0	0	12	0	0	8	4	0bef.
Ruſſian			Tobolſki	58	12	18	68	12	45	4	38	41bef.
Perſia	1300	1100	Iſpahan	32	25	0	52	40	0	3	31	20bef.
India	2000	1000	Siam or Pegu	14	18	0	100	50	0	6	43	20bef.
Empire of Mogul	2000	1500	Delhi	28	20	0	79	25	0	5	16	0bef.
China	1440	1260	Pekin	39	54	30	116	14	15	7	45	37bef.
Georgia	210	140	Teflis	43	0	0	46	15	0	3	10	0bef.
Turcomania	360	300	Erzerum	39	56	35	48	35	45	3	14	23bef.
Diarbec or Meſopotamia	560	310	Bagdat	33	20	0	43	46	30	2	55	6bef.
Natolia	750	388	Burſa or Smyrna	38	28	7	35	20	0	2	21	20bef.
Paleſtine	210	90	Jeruſalem	31	55	8	27	19	45	1	49	15bef.
Syria	270	160	Aleppo	35	45	23	37	20	0	2	29	20bef.
Part of Arabia	1300	1200	Mecca.	21	45	0	40	55	0	2	52	0bef.

Aſiatic Iſlands in the Indian and Pacific Ocean.

Iſlands.	*Towns.*	*Belonging to*
New Holland	Sydney Cove	Engliſh
The Japaneſe Iſles	Jeddo and Meaco	Dutch
The Ladrones	Guam	Spain
Formoſa	Tai-ouan-fou	China
Anian	Kiontcheow	China
The Philippines	Manilla	Spain
The Molucca or Clove Iſles	Victoria Fort	Dutch
The Banda or Nutmeg Iſles	Lantor	Dutch
Amboyna	Amboyna	Dutch
Celebes	Macaſſer	Dutch
Gilolo	Gilolo	Dutch
The Sunda Iſles Borneo,	Borneo Caytongee	Several Nations
Sumatra,	Achen & Bencoolen	Engliſh & Dutch
Java, &c.	Batavia and Bantam	Dutch
The Andaman and Nicobar Iſles	Andaman, Nicobar	Several Nations
Ceylon	Candia	Engliſh
The Maldives	Caridon	Engliſh
Bombay	Bombay	Engliſh
The Kurile Iſles in the Sea of Kamſchatka, diſcovered by the Ruſſians	—	Ruſſia

New Guinea, New Britain, New Ireland, New Hebrides, New Caledonia, New Zealand, and the Friendly, Sandwich, and Society Iſlands, are uncolonized.

Africa, the third grand diviſion of the globe, is generally repreſented as bearing ſome reſemblance to the form of a pyramid, whoſe vertex or point is the Cape of Good Hope, and its baſe the ſhores of the Mediterranean Sea. It is a peninſula of great extent, joined to Aſia by the Iſthmus of Suez: its greateſt length from north to ſouth, from Cape Bona in the Mediterranean, to the Cape of Good Hope, is 4600 miles; and the breadth, from Cape Verd to Cape Guardafui, is 3500 miles. It is bounded on the north by the Mediterranean Sea, which ſeparates it from Europe; on the eaſt by the

Iſthmus of Suez, the Red Sea, and the Indian Ocean, which divides it from Aſia; on the ſouth by the Southern Ocean; and on the weſt by the great Atlantic Ocean, which ſeparates it from America.

Very few travellers have penetrated into the interior part of this quarter of the world; conſequently we ſtill remain ignorant of the bounds, and even of the names of many of the inland parts; but, according to the beſt accounts, it is divided according to the following table:

Nations.	*Length.*	*Breadth.*	*Chief Cities.*	*Latitude.*	*Longitude from Greenwich.*	*Difference of Time from Greenwich.*
				D. M. S.	D. M. S.	H. M.
Morocco, Tafilet, &c.	500	480	Fez	33 40 0N	6 0 0W	0 24 aft.
Algiers	600	400	Algiers	36 49 0N	2 12 0E	0 9 bef.
Tunis	400	250	Tunis	36 40 0N	10 0 0E	0 39 bef.
Tripoli	400	300	Tripoli	32 50 0N	21 30 0E	1 26 bef.
Barca	700	240	Polemeta	32 53 0N	13 5 0E	0 52 bef.
gypt	600	250	Grand Cairo	30 2 0N	31 18 0E	2 5 bef.
iledulgerid	2500	350	Dara	–	8 0 0W	0 32 aft.
aara	2400	660	Tegeſa	21 40 0N	6 0 0W	0 24 aft.
egroland	2200	840	Madinga	–	–	0 38 aft.
uinea	1800	360	Benin	7 40 0N	5 4 0E	0 20 bef.
Ethiop. Nubia	940	600	Nubia	17 0 0N	33 0 0E	2 12 bef.
Abyſſinia	900	800	Gondar	13 10 0N	35 0 0E	2 20 bef.
Abex	540	130	Doncala	15 6 0N	39 0 0E	2 36 bef.

The middle parts, called Lower Ethiopa, are very little known to Europeans, but are computed at one million two hundred thouſand miles.

oango	510	300	Loango	5 0 0N	11 0 0E	0 44 bef.
ongo	640	420	St. Salvador	5 0 0S	15 0 0E	1 1 bef.
ngola	460	250	Loando	8 30 0S	14 30 0E	0 58 bef.
enguela	430	180	Benguela	11 0 0S	14 30 0E	0 58 bef.
lataman	450	240	No towns			
jan	900	300	Brava	1 0 0N	45 0 0E	3 0 bef.
anguebar	1400	350	Mozambique	15 0 0S	40 0 0E	2 40 bef.
Monomotapa	960	660	Monomotape	–	–	1 18 bef.
Monemugi	900	660	Chicova	–	–	1 44 bef.
Sofola	480	300	Sofola	20 0 0S	36 40 0E	2 26 bef.
Terra de Natal	600	330	No town			
Caffraria, or Hottentots	708	660	Cape of G. Hope	33 55 0S	18 23 0E	1 13 bef.

Iſlands.	Towns.	Belong to
Babel Mandel	Babel Mandel	
Zocotra in the Indian Ocean	Calanſia	
The Comora Iſles, ditto	Joanna	
Madagaſcar, ditto	St. Auſtin	
Mauritius, ditto	Mauritius	French.
Bourbon, ditto	Bourbon	Ditto.
St. Helena, in the Atlan. Ocean	St. Helena	Engliſh.
Aſcenſion Iſle, Ditto	—	Uninhabited.
St. Thomas	St. Thomas	
St. Matthew, ditto	—	Uninhabited.
Anaboa, Princeſs Iſlands	Anaboa	Portugueſe.
Fernandogo, ditto		
Cape Verd Iſlands, ditto	St. Domingo	Portugueſe.
Goree, ditto	Fort St. Michael	French.
Canaries, ditto	Palma, St. Chriſtopher	Spain.
Madeiras, ditto	Santa Cruz	Portugueſe.
The Azores, or Weſtern Iſles, which are at an equal diſtance from Europe, Africa, and America, ditto	Angra, St. Michael	Portugueſe.

America, the great weſtern continent, called the New World, runs north and ſouth through every habitable climate upon the earth; extending from the eightieth degree of north latitude to the fifty-ſixth degree of ſouth latitude; and its breadth, where it is known, extends from the thirty-fifth to the one hundred and thirty-ſixth degree of weſt longitude from London; being near 9000 miles in length, and 3690 in breadth. Extending into both the hemiſpheres, it has conſequently two ſummers and two winters, and has all the variety of climates to be met with on the face of the earth. On the eaſt it is bounded by the great Atlantic Ocean, which divides it from the eaſtern continent. On the weſt it has the Pacific Ocean, or great South Sea, which ſeparates it from Aſia. It is compoſed of two parts, called North and South America, joined together by a narrow neck of land, calle the Iſthmus of Darien, in the kingdom of Mexico, 1500 miles long, and at one part being only 60 miles in breadth; ſo that to effect a communication between the two oceans is by no means difficult. In the great gulf, which is formed between the iſthmus and the northern and ſouthern continents, lie a multitude of iſlands, denominated the Weſt Indies, in contradiſtinction to the iſlands of Aſia, beyond the Cape of Good Hope, which are called the Eaſt Indies. The grand diviſions of North America are as follows:

North America.

Countries.	*Length.*	*Breadth.*	*Chief Towns.*	*Latitude.*	*Longitude from Greenwich*	*Difference of Time from Greenwich.*
New Britain	*Not known.*			D. M. S.	D. M. S.	H. M. S.
Canada	800	200	Quebec	46 55 0 N	69 53 0 W	4 39 30 aft.
Nova Scotia	350	250	Halifax	44 45 0 N	64 30 0 W	4 18 0 aft.
United States	1390	700	Philadelphia	39 57 0 N	75 8 0 W	5 0 0 aft.
Eaſt Florida	500	440	St. Auguſtine	30 8 0 N	81 10 0 W	5 25 0 aft.
Weſt Florida			Penſacola	30 32 0 N	87 20 0 W	5 49 0 aft.
Louiſiana	*Not known.*		New Orleans	30 0 0 N	87 5 0 W	5 48 0 aft.
N. Mexico and California	2000	1600	Santa Fee St. Juan	35 32 0 N	105 0 0 W	7 0 0 aft.
Mexico, or New Spain	2000	600	Mexico	19 54 0 N	100 5 0 W	6 40 0 aft.

South America.

Countries.	*Length.*	*Breadth.*	*Chief Towns.*	*Latitude.*	*Longitude from Greenwich*	*Difference of Time from Greenwich.*
Terra Firma	1400	700	Panama	8 48 0 N	80 28 0 W	5 22 0 aft.
Peru	1800	500	Lima	12 1 0 S	76 49 0 W	5 7 0 aft.
Amazonia	1200	960	*Little known.*			
Guiana	780	480	Surinam	6 0 0 N	55 30 0 W	3 42 0 aft.
			Cayenne	4 56 0 N	52 15 0 W	3 29 0 aft.
Brazil	2500	700	St. Sebaſtian	22 59 0 S	44 16 0 W	2 57 0 aft.
			St. Salvadore	12 0 0 S	38 0 0 W	2 32 0 aft.
Paraguay, or La Plata	1500	1000	Aſſumption	34 10 0 S	60 40 0 W	4 3 0 aft.
			and Buenos Ayres	34 35 0 S	58 31 0 W	3 54 0 aft.
Chili	1200	500	St. Jago	33 40 0 S	77 0 0 W	5 8 0 aft.
Terra Magellanica, or Patagonia	700	300	Uncoloniſed by Europeans.			

Grand Diviſions of South America.

Nations.	*Length.*	*Breadth*	*Chief Towns.*	*Diſtances & Bearing from London.*	*Belongs to*
Terra Firma	1400	700	Panama	4650 S. W.	Spain.
Peru	1800	600	Lima	5520 S. W.	Ditto.
Amazonia	1200	960			
Guiana	780	680	Surinam Cayenne	3840 S. W.	Dutch. French.
Brazil	2500	700	St. Sebaſtian	6000 S. W.	Portugal.
La Plata	1500	1000	Buenos Ayres	6040 S. W.	Spain.
Chili	1200	500	St. Jago	6600 S. W.	Ditto.
Terra Magellanica, or Patagonia	1400	460			

The

The principal Iſlands in North America belonging to the Europeans.

	Iſlands.	Length.	Breadth.	Chief Towns.	Belongs to
In the Gulf of St. Lawrence.	Newfoundland	350	200	Placentia	England
	Cape Breton	110	80	Louiſburg	Ditto
	St. John	60	30	Charlotte	Ditto
In the Atlantic.	The Bermuda Iſles	400 in number	-	St. George	Ditto
	The Bahama Iſles	-	-	Naſſau	Ditto
The Weſt India Iſlands, lying in the Atlantic, between N. and S. America.	Jamaica	140	60	Kingſton	Ditto
	Barbadoes	21	14	Bridgetown	Ditto
	St. Chriſtopher's	20	7	Baſſe Terre	Ditto
	Antigua	20	20	St. John's	Ditto
	Nevis	6	3	Charleſtown	Ditto
	Montſerrat	5	4	Plymouth	Ditto
	Barbuda	20	12	—	Ditto
	Anguilla	30	10	—	Ditto
	Dominica	28	13	—	Ditto
	St. Vincent	24	18	Kingſton	Ditto
	Granada	30	15	St. George	Ditto
	Cuba	700	90	Havannah	Spain
	Hiſpaniola	450	150	St. Domingo	France
	Porto Rico	100	49	Porto Rico	Spain
	Trinidad	90	60	St. Joſeph	England
	Margarita	40	23	—	Spain
	Martinico	60	30	St. Peter	France
	Guadaloupe	45	38	Baſſe Terre	Ditto
	St. Lucia	23	12	—	England
	Tobago	32	9	—	Ditto
	St. Bartholomew	-	-	—	Sweden
	Deſeada	-	-	—	France
	Marigalante	-	-	—	Ditto
	St. Euſtatia	7	4	The Bay	Dutch
	Curaſſou	30	10	—	Ditto
	St. Thomas	5	3	—	Denmark
	St. Croix	30	10	Baſſe End	Ditto

I ſhall here ſubjoin a table of the ſuperficial content of the ſeveral parts of the globe in ſquare miles, accounting 60 miles to a degree on the equator.

	Square Miles.	*Iſlands.*	*Square Miles.*	*Iſlands.*	*S.M.*
The Globe	199,512,595	Cuba	38,400	Funen	768
Seas and unknown Parts	160,522,026	Java	38,250	Yvica	625
		Hiſpaniola	36,000	Minorca	520
The habitable World	38,990,567	Newfoundland	35,500	Rhodes	480
		Ceylon	27,730	Cephalonia	420
Europe	4,456,065	Ireland	27,457	Amboyna	400
Aſia	10,768,822	Formoſa	17,000	(Orkney)	
Africa	9,654,807	Anian	11,900	Pomona	324
America	14,110,874	Gilolo	10,400	Scio	300
Perſian Emp. under Darius	1,650,800	Sicily	9,400	Martinico	260
		Timor	7,800	Lemnos	220
Roman Emp. in its greateſt height	1,610,000	Sardinia	6,600	Corfu	194
		Cyprus	6,300	Providence	168
		Jamaica	6,000	Man	160
Ruſſian	4,161,685	Flores	6,000	Bornholm	160
Chineſe	1,749,000	Ceram	5,400	Wight	150
Great Mogul	1,116,000	Breton	4,000	Malta	150
Turkiſh	950,057	Socotra	3,600	Barbadoes	140
Preſent Perſia	800,000	Candia	3,220	Zant	120
ISLANDS.		Porto Rico	3,200	Antigua	100
Borneo	228,000	Corſica	2,520	St. Chriſtopher	
Madagaſcar	168,000	Zealand	1,935		80
Sumatra	129,000	Majorca	1,406	St. Helena	80
Japan	118,000	St. Jago	1,400	Guernſey	50
Great Britain	72,926	Negropont	1,300	Jerſey	43
Celebes	68,400	Teneriff	1,272	Bermudas	40
Manilla	58,500	Gothland	1,000	Rhodes	36
Iceland	46,000	Madeira	950		
Terra del Fuego	42,075	St. Michael	920		
		Skye	900		
Mindanao	39,200	Lewis	880		

There are alſo ſeveral other conſiderable iſlands, chiefly in the South Seas, the exact dimenſions of which are not certainly known, but they may be ranged according to their magnitude in the following order: New Holland being nearly equal in ſize to the whole continent of Europe.

New Holland.	Otaheite, or King George's Iſland.
New Guinea.	Friendly Iſlands.
New Zealand.	Marqueſas.
New Caledonia.	Eaſter, or Davis's Iſland.
New Hebrides	

SECT. IV.

OF THE DIFFERENT GOVERNMENTS OF THE WORLD.

MANKIND were no ſooner united into civil ſocieties, than they diſcovered an inclination to oppreſs each other. That ſyſtem of equality, in which they were left by nature, gave the ſtrongeſt, and the moſt crafty, the advantage over his weaker and undeſigning neighbours. From hence aroſe the neceſſity of forming conjunctions of ſeveral individuals, or families together, who ſhould implicitly follow the dictates or commands of ſome choſen ſuperior, or leader. And, to prevent the altercations, ſtrife, and conſequently bloodſhed, that inevitably followed the nomination of every new leader, or prince, they cauſed the office to be made hereditary. Conſequently, abſolute and hereditary monarchy was the firſt original form of government; as appears from ſacred writ; where Nimrod is repreſented by his courage and dexterity to have acquired a ſuperiority of fame and power above his contemporaries; and he founded, at Babylon, the firſt monarchy whoſe origin is mentioned in hiſtory.

In the year 1496 before Chriſt, the Greeks were the firſt people who, by the advice and public-ſpirited endeavours of Cecrops, and Cranaus his ſucceſſor, formed a regular council. For Amphictyon, one of thoſe diſintereſted characters who live for the good of the community of which he is a member, endeavored to find an expedient to unite the ſeveral independent kingdoms of Greece into one body; and thus to put a ſtop to thoſe fatal conſequences of inteſtine diviſion, and civil diſcord, which rendered them a prey to each other, and an eaſy conqueſt to the invader. He, therefore, engaged the kings, or leaders, of twelve different cities, to unite together for their mutual ſecurity and welfare. Two deputies from each city aſſembled twice a year at Thermopylæ, and formed the Amphictyonic Council. In this aſſembly the general intereſts of the ſtates were diſcuſſed. Amphictyon, in order to render thoſe ſeveral connexions more durable, connected them with religious charge, intruſting the care of the temple at Delphi, with the riches that accrued to this place from thoſe who conſulted the oracles, to the care alſo of theſe deputies. This aſſembly was the firſt political eſtabliſhment of a plurality of power, that we have any authentic account of in hiſtory; and gave an energy of action to Greece, which enabled them to defend their liberty and independence againſt the great force of the Perſians.

This was the firſt deviation from abſolute monarchy, recorded in profane hiſtory; from that time, various have been the modes and forms of government in different nations; though, if we except ſome part of the Roman hiſtory, Greece, and a few nations of leſs note, the monarchical form of government was the moſt prevailing for the next two thouſand years.

Athens is an inſtance of the pernicious effects of diviſion in a ſtate; and alſo diſplays the benefits of unanimity. Theſeus, king of Attica, about the year before Chriſt 1234, perceiving the danger to which his country was expoſed by this twelvefold diviſion, endeavoured to form a conjunction of the ſtates; for this purpoſe he detached the leaders of the different tribes as much as poſſible from the people they governed; he aboliſhed the different courts eſtabliſhed in different parts of

Attica, and appointed one council hall, common to all the Athenians. He eſtabliſhed a common form of religion, with certain religious ceremonies to be performed at Athens, the more effectually to ſtrengthen civil allegiance; and by inviting ſtrangers from all parts of the world, by the promiſe of privileges and protection, he raiſed the city to the higheſt pitch of fame and popularity. The ſplendour of Athens eclipſed that of all the other ſtates of Greece.

This monarchy ſoon gave place to an overbearing influence. Theſeus had formed his kingdom into three diſtinct claſſes; the nobles, the artizans, and the huſbandmen. And to prevent the increaſing power of the nobles, he granted many immunities and privileges to the two other claſſes. This ſyſtem of politics, in a few years, gave the two inferior claſſes an opportunity of acquiring conſiderable property: and, conſequently, they became important members of the ſtate; and, by their riches and independence, upon the death of Codrus, a prince of great merit, in the year B. C. 1070, they had power and influence enough to aboliſh the regal power, under pretence of finding no one worthy of filling the throne of Codrus, who had devoted himſelf to death for the ſafety of his people. Thus they proclaimed Jupiter king, declaring none elſe was fit to govern Athens. This was the firſt inſtance of a republican form of government in Greece.

From this period, ſo various have been the modes and forms of government, that it is impoſſible to diſtinguiſh them all. Governments are generally divided into three diſtinct forms, each of which has its partizans, viz. the monarchical, ariſtocratical, and democratical.

The monarchical form of government is, where a nation is governed by a king, or monarch; and is divided into two parts, called abſolute, and limited, monarchy. Abſolute monarchy is, where the ſovereign is entirely unreſtrained, having the legiſlative as well as the executive power. A limited ed monarchy is, where the ſovereign is reſtrained by certain laws, beyond which he cannot paſs.

An ariſtocracy is, where the legiſlative and executive authority is veſted in the hands of a ſelect number of perſons, generally titled nobility; and in whom the office is moſtly hereditary.

A democracy is that government in which the legiſlative and executive authority is veſted in a certain number of individuals, who hold their office by election; and generally elected by the majority of the nation at large.

From the various modifications of theſe different forms of government, all the governments of the earth are formed; ſome approaching nearer to one, and ſome to another form. For there is hardly a government exiſting, that is entirely either an abſolute monarchy, a perfect ariſtocracy, or a complete republic.

SECT. V.

OF RELIGION.

Religion is coeval with the origin of mankind: without it the preſent order of the univerſe would be entirely overturned; and mankind, from their natural depravity, be rendered worſe than the moſt voracious of the brute creation.

The diſtinguiſhing religions in antiquity were Judaiſm, and Polytheiſm, or Paganiſm.

But in modern times the prevailing religions may be divided into the four following, viz. the Jewiſh, Chriſtian, Mahometan, and Pagan.

Before treating of the four foregoing ſyſtems, it may be neceſſary to premiſe the following general axiom, viz. That all ſyſtems of religion contribute more or leſs to the welfare of ſociety. From hence we deduce the following theorem; that all religion muſt have ſomewhat in its origin of a divine nature, however it may be transformed, corrupted, or miſapplied, by the ignorance or artifice of its propagators.

In conſidering the Jewiſh code of religion, it does not appear as a complete ſyſtem of religion, adapted to all countries and ages, but ſeems particularly deſigned by the all-wiſe Creator, for the people to whom it was ſent; for the age they lived in, being over-run by idolatry; the circumſtances in which they had lived in Egypt, and the means by which they were to form their new ſettlement in the land of Canaan.

From hence they were enjoined the obſervation of the ſabbath, in honour to that Being who created the heavens and the earth, with all the hoſt of heaven; which hoſt, ſun, moon, ſtars, &c. were worſhipped by the Egyptians as *eternal beings*. To prevent their communication with the neighbouring idolatrous nations, they were proſcribed the uſe of certain animals for food, and permitted others; that, by being forbidden the uſe of thoſe animals for food, ſuch as the hog, &c. which the Gentile nations conſidered as the greateſt luxury, a perpetual bar might be kept up between the Jews and Gentiles. And by being permitted to eat other animals, ſuch as goats, ſheep, oxen, &c. which were worſhipped in Egypt, and from which the Egyptians religiouſly withheld all violence, the Jews would ſoon overcome any religious prejudices they might have acquired from the Egyptian idolatry.—The reſtitution of property, in the year of Jubilee, which would anſwer no purpoſe in another ſtate, was deſigned to preſerve the order of rank, and that diviſion of property, originally eſtabliſhed.

In condeſcenſion to their rude and groſs notions of Deity, the Creator permitted them, in their wanderings through the Wilderneſs, to have a tabernacle, or portable temple, in which he ſometimes deigned to diſplay ſome rays of his glory.

From this general view of the Jewiſh religion, it appears happily adapted to promote the welfare of its followers. In comparing it with other religions, it is neceſſary to reflect on the peculiar purpoſes for which it was eſtabliſhed; which were principally two; firſt, to preſerve the Jews a ſeparate people; and ſecondly, to guard them from the idolatry with which they were every where ſurrounded. The religion of the Jews was not formed, nor deſigned, to be propagated through all the earth; that would have been inconſiſtent with the purpoſes for which it was inſtituted: therefore we ſee the Jewiſh religion, though near four thouſand years old, wants that eſſential attribute for propagation, to be found in all other religions, viz. a difference of ſentiment, and, conſequently, a diviſion and ſubdiviſion into different ſects.

The Chriſtian religion is to be conſidered as an improvement of the Jewiſh. The effects of the Jewiſh religion were indeed beneficial, but were confined almoſt to them alone; whereas the effects of the Chriſtian religion are extended to all mankind; repreſenting them with true philanthropy as children of the ſame God, and heirs of the ſame ſalvation. It levels all diſtinctions of rich and poor, native and foreigner, as accidental and inſignificant diſtinctions with that impartial Being, who rewards or puniſhes according to the demerits of his creatures.

The precepts of the Chriſtian religion are more happily calculated to promote the happineſs of mankind, than thoſe of any other religion. Its whole deſign is to inſpire mankind with mild, benevolent, and peaceable diſpoſitions. Its diſtinguiſhing rule, by which it excels all other religions, is, *to do unto others, as we would they ſhould do unto us*; and ſuch is its purity, that it does not allow an impure thought. It requires its followers to abandon their vices, however dear; and to join the cautious wiſdom of the ſerpent with the innocent ſimplicity of the dove. And to prevent perſeverance in immorality, it offers a pardon for the paſt, provided the offender forſake his vicious practices. The practice and belief of the Goſpel have a peculiar tendency to raiſe the mind of man above the trifling purſuits of time; and to render its followers incorruptible by wealth, honour, or pleaſures. It not only requires the Chriſtian to abſtain from injuring his neighbour, but even enjoins him to forgive any unmerited injuries which he himſelf ſuffers, upon the principle of his being forgiven by his offended Creator. It repreſents the Deity and his attributes in the faireſt light, ſo as to render our ideas of him conſiſtent with the correct principles of reaſon and philoſophy. The rites of this goſpel are few and ſimple; eaſy to perform, expreſſive, and edifying. It inculcates no duties, but what are founded in the principles of human nature, and on the relation on which man ſtands to God, as his Creator, Redeemer, and Sanctifier. The aſſiſtance of the Spirit of God is there promiſed to thoſe who labour to diſcharge the duties which it enjoins. It teaches us that worldly afflictions are caſual accidents; incident to both bad and good men: *a doctrine highly encouraging to virtue, conſoling in affliction, preventing deſpair, and encouraging in difficulty.*

Such are the precepts and ſpirit of the Chriſtian religion. And even thoſe who have refuſed to give credit to its hiſtory, and follow its doctrines, have acknowledged the excellency of its precepts. Bolingbroke, one of its moſt zealous oppoſers, ſays, that “ no religion ever yet appeared in the world, of which the natural tendency was ſo much directed to promote the peace and happineſs of mankind, as the Chriſtian; and that the Goſpel of Chriſt is one continued leſſon of the ſtricteſt morality, of juſtice, benevolence, and univerſal charity.” Thus we can pronounce, with confidence, that the precepts of a religion, which is ſo happily formed to promote all that is juſt and beneficial to mankind, cannot but be in the higheſt degree divine. By reviewing the effects which it has produced, we ſhall be more confirmed in our aſſertion.

Chriſtianity has produced the moſt beneficial change in the circumſtances of domeſtic life. It has greatly contributed towards the abolition of ſlavery, and towards the mitigation of the rigours of ſervitude. We meet with no laws in Chriſtian countries ſo inhuman as thoſe practiſed at Rome; where maſters were allowed to remove their ſick or infirm ſlaves to an iſland in the Tiber, there to periſh without any aſſiſtance. The rigours of ſlavery are eaſed and aboliſhed; not by any particular precept of the Goſpel, but by the gentle and humane ſpirit which breathes through the general tenour of the whole ſyſtem. And though it may be objected, that a trade in ſlaves is at preſent carried on by people who preſume to call themſelves Chriſtians, and ſanctioned by the legiſlature of ſome Chriſtian ſtates; yet it muſt be remembered, that the ſpirit of the Chriſtian code condemns the practice; and the true Chriſtian will not engage in it.

Chriſtianity is alſo gradually ſoftening barbarous nations into humanity. The influence of ſelfiſhneſs has been checked and reſtrained. And even war, with all the pernicious improvements, by which mankind has ſought to render it more terrible, has aſſumed much more the ſpirit of mildneſs and peace, than ever entered into it under the influence of Paganiſm.

Theſe are a few of the excellencies of the Chriſtian ſyſtem. Its laſt diſtinction I ſhall mention, is that of its extending its benefits to thoſe nations who have not received its doctrines and precepts. The virtues aſcribed to Julian the apoſtate, are, no doubt, owing to his acquaintance with Chriſtianity; and after the propagation of Chriſtianity through the Roman empire, even while the purity of its doctrines was deſpiſed, it had a remarkable effect on the manners of thoſe unconverted Pagans, who, in their religious doctrines and worſhip, became leſs immoral and abſurd.

Upon the whole, we muſt conclude, that Chriſtianity is infinitely ſuperior to every other religious ſyſtem, both in point of its religious doctrines, and the effects it has produced upon ſociety. It is an univerſal religion; formed to exert its happy influence in all ages, and among all nations; and has a tendency to diſpel the ſhades of barbarity and ignorance; to promote the cultivation of the powers of the human underſtanding; and to encourage every virtuous refinement in manners.

As the Chriſtian religion is deſtined to be of an univerſal nature, and to be diſſeminated into all parts of the world; ſo, in order for its more effectual propagation, its all-wiſe Founder has ordained that it ſhall be divided into different ſects and parties; that the leaders of each being governed by a mutual emulation, might endeavour to propagate their reſpective opinions, and thereby form a grand junction for propagating a religion, the fundamentals of which would be ultimately the ſame.

The two principal ſects into which the Chriſtian religion is divided, are the Proteſtant and Romiſh churches.

The Romiſh church differs from the Proteſtant chiefly in the following particulars: 1. In believing every thing that was defined by the Council of Trent, concerning original ſin and juſtification. 2. In believing tranſubſtantiation, or the converſion of the material bread and wine, given at the ſacrament, into the real body and blood of Jeſus Chriſt. 3. In the belief of a purgatory; and that ſouls are kept priſoners there after their departure from the body; and that they receive help by the prayers of the faithful. 4. That the ſaints reign together with Chriſt, and are to be worſhipped as mediators for man. 5. That the images of Chriſt, the Virgin Mary, and other ſaints, ſhall be retained, and due honour and veneration be given unto them. 6. That the power of indulgences was left by Chriſt to the church. 7. That the holy church of Rome is the mother and miſtreſs of all churches; and that the biſhop of Rome, or pope, is the ſucceſſor of St. Peter, the prince of the apoſtles, and vicar of Jeſus Chriſt on earth; and that he is infallible and invincible.

Theſe are the chief tenets which diſtinguiſh the church of Rome from the Proteſtant church. The implicit obedience which the followers of this church pay to their leaders, has been a ſource of a very black corruption and error; of which their numerous perſecutions of the Proteſtants are an ample proof. But, on the other hand, it muſt be allowed, that there is no religion ſo zealous of propagating its doctrines. Their miſſionaries have been ſent to all parts of the earth, ſome of whom, by their perſeverance and abſtemiouſneſs, were as great an honour, as others by their profligacy were a diſgrace, to the cauſe in which they were concerned.

The church of Rome is now divided into two ſects: that already deſcribed, which prevails over moſt parts of Italy, Spain, and France, and ſeveral other parts of the continent of Europe; and the Greek church, which differs from the former in not allowing the pope's ſupremacy, not worſhipping idols, though they have many in their churches, and in not enjoining their prieſts to celibacy.

The Proteſtant religion is divided into numerous ſects and parties: the two principal of which are the Lutherans and the Calviniſts.

The Lutherans maintain, that man is a free agent, perfectly capable of performing good or evil: that according to his actions he ſhall be rewarded or puniſhed hereafter: that he is left at perfect liberty to chooſe the good or evil: and that God has no predilection for any particular perſons: that the ſacrament of the Lord's ſupper is nothing but a mere ordinance.

The Calviniſts, on the contrary, aſſert, that man is not a free agent, that he has no power to perform any good action without the Spirit of God aſſiſting him; that God, according to his fore-knowledge, has elected a certain number of individuals to be ſaved: that he is the former of every good thought; and ſaves the elect, not from any goodneſs in themſelves, but merely from his own unmerited mercy; conſequently, that Chriſt did not die for all the world: that the ſacrament is a ſpiritual rite; that the bread and wine is conſubſtantiated (not tranſubſtantiated) in a ſpiritual manner into the body and blood of Chriſt.

Beſides theſe two diviſions of the Proteſtant religion, one or both of which prevail in moſt Proteſtant countries on the continent of Europe, there are a great number of inferior ſects in England, America, Holland, Germany, and other parts, and ſome of them very numerous; as the Quakers, Baptiſts, Diſſenters, Methodiſts, &c. which are too well known to need any deſcription. Suffice it to ſay, each of them differs from the eſtabliſhed church, on account of ſome trifling errors, which they imgaine they have detected in the national church.

The next diviſion of religion that deſerves our notice is that of Mahomet, which ſtill makes ſuch a conſpicuous figure in the world, extends over a large tract of country, and is profeſſed by very powerful nations. Like the Jewiſh religion, it is not merely a ſyſtem of religious doctrines, and moral precepts; but it forms both the civil legiſlature and religious ſyſtems of the nations by which it is profeſſed. It alſo appears to be rather calculated for one particular period, in the progreſs of mankind from barbarity to refinement, than for all ages, and all parts of the world.

In viewing this ſyſtem of religion, there are many parts of it which ſeem copied from the Chriſtian, the Jewiſh, and the Pagan religions. It is difficult to tell which is the greater, the purity of ſome parts of this doctrine, or the abſurdity of other parts.

The greateſt abſurdities, or that which tends moſt effectually to promote impurity of manners, are the Prophet's ideas of heaven and hell. Paradiſe, or the place of future rewards, he makes to abound with rivers, trees, fruits, and ſhady groves; wine, without its intoxicating quality, is to be there ſerved out to believers, who, as they enjoy perpetual youth, their powers of enjoyment are to be enlarged and invigorated, according to the delights they are to enjoy. Mahomet celebrates the pearls and diamonds, robes of ſilk, palaces of marble, diſhes of gold, numerous attendants, wines, and dainties, with the whole train of ſenſual luxury, reſerved for the faithful in theſe regions. Seventy-two black-eyed damſels of reſplendent beauty, blooming youth, virgin purity, and exquiſite ſenſibility, will be created for the uſe of the meaneſt believer. A moment of pleaſure will be prolonged to one thouſand years; and the faculties will be increaſed a hundred fold, to render him worthy of his felicity. There are alſo certain more refined enjoyments; as, believers are to ſee the face of God morning and evening—a pleaſure which is to exceed all the other pleaſures of Paradiſe.

In hell, the place of future puniſhments, the wicked are to drink nothing but boiling, ſtinking water; eat nothing but briers and thorns, and the fruit of a tree, that grows in the bottom of hell, whoſe branches reſemble the heads of devils, and whoſe fruit ſhall be in their bellies like burning pitch: they are to breathe nothing but hot winds, and dwell for ever in continual burning, fire, and ſmoke.

Thus Mahometiſm appears to be a ſtrange mixture of abſurdities, with a few truths and valuable precepts incongruouſly intermixed. A great part of it is incompatible with virtue, and the progreſs of knowledge and refinement. It ſubſtitutes trifling ſuperſtitious ceremonies in the room of genuine piety and virtue; and preſents ſuch a proſpect of futurity, as renders purity of heart no neceſſary qualification for ſeeing God.

However, Mahometiſm forms in ſome meaſure a regular ſyſtem of religion, as it has borrowed many of its precepts and doctrines from both Judaiſm and Chriſtianity, which are, however, greatly degraded and corrupted. It has, nevertheleſs, conſiderably contributed towards the ſupport of civil government in thoſe countries in which it is eſtabliſhed.

It is divided into a numerous party of ſects, which, however, differ ſo little from each other, as ſcarcely to deſerve mention in this place.

Paganiſm next deſerves our notice. It was the moſt prevailing religion of antiquity, and may be principally divided into two parts:—1. The Pagan religion of the ancient barbarous nations; and, 2. The Polytheiſm of the more civilized Greeks and Romans.

The Paganiſm of the ancient nations preſents us with a moſt ſhocking picture of ignorance, ſuperſtition, and abſurdity. We there behold the moſt abſurd doctrines concerning a future ſtate. Various nations have imagined, that the ſcenes and objects of the world of ſpirits are only a ſhadowy repreſentation of the things of the preſent world. According to them, not only the ſouls of men inhabit thoſe regions, but all the inferior animals and vegetables, and even inanimate bodies that are killed or deſtroyed here, are ſuppoſed to paſs into that viſionary world, and exiſting there in unſubſtantial forms, execute the ſame functions, or ſerve the ſame purpoſes, as on earth. By this belief they were ſtimulated, on the death of a king or other great perſonage, to provide for his accommodation in the world of ſpirits, by burying with his corpſe, meat and drink for his ſubſiſtence, ſlaves for his attendance, and wives for his enjoyment. His faithful ſubjects vied with each other in their offerings upon this occaſion; one brought a ſervant, another a wife, a third a ſon or daughter, to accompany their monarch in his future ſtate. Similar practices, on the ſame occaſion, prevailed in New Spain, in the iſland of Java, in the kingdom of Benin, and among the inhabitants of Hindoſtan. A like belief alſo prevailed among the Japaneſe. They not only bribed their prieſts to ſolicit for them a place in the bliſsful manſions of futurity; but looking upon the preſent life with diſguſt and contempt, when ſet in competition with the joys of futurity, they uſed to daſh themſelves from precipices, or cut their throats, in order to get to Paradiſe as ſoon as poſſible. Various other ſuperſtitions, ſubſiſting among rude nations, might here be adduced, as inſtances of the perverſion of the religious principles of the human heart, which render them injurious to virtue and happineſs. Innumerable are the ways of torture which have been invented and practiſed on themſelves by men ignorantly ſtriving to obtain the favour of Heaven. Theſe are ſufficient proofs of religious ſentiments having been ſo ill directed by the influence of imagination, and unenlightened erring nature, aided by the corrupt deſigns of artful prieſts.

The Polytheiſm of the Greeks and Romans, though more favourable to virtue and civilization, than the Pagan notions of antiquity, is yet a very imperfect, not to ſay a pernicious, code of religion: the vicious characters of their deities, the abſurd notions they entertained concerning the government of the univerſe and a future retribution, the abſurdities of their religious rites and ceremonies, the frivolous practices with which they were intermixed, muſt all together have a great tendency to pervert both the reaſoning and moral principles of the human mind: however, it cannot be denied, that this ſyſtem was friendly to the encouragement of arts; particularly of ſuch as depend on the vigorous exertion of a fine imagination, as muſic, poetry, ſculpture, architecture, and painting: all theſe arts appear to have been conſiderably indebted for that perfection to which they attained, to the ſplendid and fanciful ſyſtem of mythology which was received by thoſe people, particularly by the Greeks.

The effect of this religion, to reform the lives of its votaries, was very imperfect. Sacrifices and prayers, temples and feſtivals, not purity of heart and integrity of life, were the means preſcribed for obtaining the favour of their deities.

There were alſo other means of gaining admiſſion into the Elyſian fields, or the ſeat of the councils of the Gods; but none of theſe means appear to have been thoſe commanded by the Chriſtian religion. And whatever might be the effects of the religion of Greece and Rome in general, upon the civil and political eſtabliſhments, and on the manners of the people, yet it muſt be confeſſed to have been but ill adapted to impreſs the heart with ſuch principles as might in all circumſtances direct to a firm, uniform tenour of righteous conduct.

From this view of religion, it appears, that though ſome particular forms, ſuch as thoſe of the Chriſtian, have had a greater influence in reforming the manners of their followers; yet as they all have often contributed to form the mind to virtue, it muſt be acknowledged, that they have always, and under all their forms, been infinitely more beneficial than hurtful to mankind.

When we view the different ſyſtems in a comparative light, with reſpect to their influence on the welfare of ſociety, no one will heſitate to prefer the Polytheiſm of the Greeks and Romans to the ruder ideas of the more ancient Pagans; and Mahometiſm to the Polytheiſm of the Greeks and Romans: Judaiſm is, however, greatly preferable to Mahometiſm; and Chriſtianity to all of them.

SECT. VI.

OF EUROPE.

Europe, though the leaſt quarter of the globe, is by far the moſt eminent in modern hiſtory; and is at preſent the moſt diſtinguiſhed part of the globe for the literature, arts, and ſciences, to which it has given birth and encouraged, and for the learned men it has produced. It is alſo the moſt civilized quarter of the globe. Here are no public marts for buying and ſelling the human ſpecies, as are found in Aſia and Africa. The Chriſtian religion alſo prevails here almoſt univerſally. Its languages are as mixed as its inhabitants, but all derived from the ſix following: viz. the Celtic, Sclavonian, Teutonic, Greek, Latin, and Gothic. It extends about 3000 miles in length, and 2500 in breadth: and is divided into ſeveral kingdoms and ſtates, as ſeen in the Table, page 133.

The British Isles, lying on the weſtern part of Europe, conſiſt of Great Britain (which compriſes England, Wales, and Scotland), Ireland, and the Iſles of Man, Jerſey, Guernſey, Alderney, Sark, and Wight. England lies between 50 and 56 degrees north latitude, and between 2 degrees eaſt, and 6 degrees 20 minutes weſt longitude; and is divided into forty counties. Its conſtitution is that of a limited monarchy, conſiſting of king, lords, and commons, with certain prerogatives and privileges annexed to each.

The legiſlative authority, or power of making laws and raiſing money, is veſted in theſe three branches of the government; and each branch has a negative voice.

The crown is made hereditary in the Hanover line, by ſeveral acts of Parliament, provided they do not profeſs Popery, marry Papiſts, or ſubvert the conſtitution.

The peers are created by the crown; but their honours are hereditary, and cannot be taken from them any more than their lives or eſtates, unleſs forfeited by the commiſſion of high treaſon; and they can only be tried by the Houſe of Peers, being ſubject to no other juriſdiction. This Houſe is the laſt reſort in all civil caſes, and the higheſt court in the kingdom.

Any bill for making a new law, or altering an old law, may be brought in firſt in the Houſe of Peers; but a bill relating to the revenues, or public taxes, muſt be brought into the Houſe of Commons firſt; and it cannot be *altered* by the Peers, though it may be rejected.

The Houſe of Peers can apprehend and commit any man for a reflection on their judicature.

The Commons are compoſed of 658 members: viz. 80 knights, every county in England ſending two, and elected by the freeholders; 50 citizens, two being ſent from each of the 25 cities in England (London ſending four, and Ely none); 334 burgeſſes, from 167 boroughs, ſending two each; five burgeſſes, from the boroughs of Abingdon, Banbury, Bewdley, Higham Ferrars, and Monmouth; four repreſentatives, from the two univerſities; 16 barons, from the five Cinque Ports, Haſtings, Dover, Sandwich, Romney, and Hythe, and their three dependants, Rye, Winchelſea, and Seaford; 12 knights, from the 12 counties of Wales; 12 burgeſſes, from the 12 boroughs in Wales (Pembroke ſending two, and Merioneth none); 30 knights, from the ſhires of Scotland; 15 burgeſſes, from the Scotch boroughs; and 100 members from Ireland.

Wales is ſituated on the weſt and north-weſt of England, to which it joins; and is divided into 12 counties: it is a principality; and always conſidered as the right of the King's eldeſt ſon, who is therefore titled Prince of Wales. It was peopled in the year 410, by the ancient inhabitants of England, who fled thither from the perſecution of the Picts and Scots.

Ireland is ſituated between ſix and ten degrees of weſt longitude, and between 51 and 55 degrees of north latitude. Bounded by the eaſt by St. George's Channel, or the Iriſh Sea, which divides it from Great Britain. It is divided principally into four provinces: viz. Ulſter, on the north; Leinſter, on the eaſt; Munſter, on the ſouth; and Connaught, on the weſt.

The climate of theſe iſlands is in general mild for the latitude, but very changeable, the weather never continuing a month the ſame, owing to the exhalations from the ſurrounding ſea, which render the air humid. But the ſoil is in general fruitful, and has been of late years greatly improved.

Theſe iſlands have ſeveral very good mines of tin, copper, iron, and lead; gold has alſo been found in Scotland, in ſolid pieces, in the brooks, after a great torrent.

The chief manufacture of England is woollen cloth, which is accounted the ſtaple trade of the kingdom; as linen cloth is that of Ireland.

Denmark, including Norway, is the moſt northern kingdom of Europe, and includes Denmark Proper, the territories in Germany, Norway, part of Lapland, and ſeveral iſlands in the Baltic Sea, and in the German Ocean; and extends from 52 degrees of north latitude, to the fartheſt habitable part of the Arctic Circle. Denmark Proper is bounded on the north by the Cattegate or Skaggerac; on the ſouth by Germany; on the weſt by the German Ocean; and on the eaſt by the Sound.

The eſtabliſhed religion is Lutheriſm. The king is abſolute, though in general mild in his government. It is divided into two parts, called North Jutland and South Jutland.

The air of this country is ſharp, but the exhalations from the ſea abate its ſeverity. The ſummers are very ſhort and hot, but the ſoil is in general fruitful, for a northern latitude, except on the tops of mountains. The manufactures of this country are chiefly hardware; and their artiſts and mechanics, in every branch, are generally ſkilful.

NORWAY is bounded on the ſouth by the Cattegate, on the weſt and north by the Northern Ocean, and on the eaſt by the mountains which ſeparate it from Sweden; and is divided into the north, ſouth, and middle diviſions. The air of Norway is generally healthy and dry in the inland parts of the country, but on the ſea-coaſt it is moiſt. In winter it is exceſſively cold, the whole country being covered with ſnow; it is alſo very hot in the ſummer. Their trade conſiſts of copper, timber, iron, marble, mill-ſtones, fiſh, fowls, tallow, tar, oil, alum, vitriol, &c. Their language is the ſame as that uſed in Denmark; and their religion is that of Lutheriſm.

ICELAND is ſituated in the Northern or Atlantic Ocean; being 726 miles in length from eaſt to weſt, and 300 in breadth: extending from ſixty-three to ſixty-eight degrees of north latitude; and from fourteen to twenty-nine degrees of weſt longitude. It has a milder climate than any other country in the ſame latitude. It is a very mountainous country, but well watered, with ſeveral large rivers. In this country there are ſome large ſprings of boiling hot water, the principal of which is Geyſer, near Skalholt. The water iſſues from this ſpring ſeveral times a day, with a violent noiſe, like that of a great torrent, ſometimes riſing to the height of 60 fathoms, and ſeldom leſs than 90 feet.

There are alſo ſeveral burning mountains in this country, of which the moſt remarkable are, Hecla, Kotlegau, and Oraiſe, the eruptions of which have ſometimes done conſiderable damage. The inhabitants live chiefly by fiſhing, and of theſe different countries. In ſome of the northern parts they live in caverns, not five feet in height; in other parts they lead a wandering life: the natives of ſome parts practiſe agriculture; but in others live on the ſpontaneous productions of the ſoil.

The religion of ſome parts is next to Paganiſm, the natives idolizing inanimate objects, as a ſheep's ſkin; but in other parts they make no public profeſſion of religion. The eſtabliſhed religion of Ruſſia is the Greek church.

The European part of Ruſſia, called Muſcovy, is divided into the following provinces: viz.

In the northern diviſion: Lapland, Samoieda, Bellamorniſckoy, Meſeen, Dwina, Syrianes, Permia, Rubeninſki, Belacſeda.—In the middle diviſion: Pereſlaf, Belozero, Wologda, Jereſlaf, Tweer, Moſcow, Belgorod.—In the eaſtern diviſion: Bulgar, Kanſan, Little Novgorod, Don Coſſacks.—In the weſtern diviſion: Great Novgorod, Rus, Finland, Kexholm, Karelia, Ingria.—In the ſouthern diviſion: Livonia, Smolenſko, Zernigof, Seefſk, Ukrain. Their articles of commerce and manufacture are the ſame as thoſe of Sweden and Denmark; they have, moreover, ſilk, cotton, teas, gold, &c. which they bring from China and India, in caravans, by the way of the Caſpian Sea.

The language is derived from the Sclavonian, to which are added many words from the Greek; their alphabet conſiſts of forty-two characters, which are principally Greek. The people of high rank generally ſpeak French and High Dutch, but their prieſts ſpeak the modern Greek.

POLAND, before its late diſmemberment, was bounded on the north by Livonia, Muſcovy, and the Baltic Sea; on the eaſt, by Muſcovy; on the ſouth, by Hungary, Turkey, and Little Tartary; and on the weſt, by Germany; extending from 47 degrees 40 minutes, to 56 degrees 30 minutes, north latitude; and from 16 to 34 degrees eaſt longitude. It was divided into the provinces of Great and Little Poland, Poliſh Pruſſia, Samogitia, Courland, Lithuania, Maſovia, Podolachia, Poleſia, Red Ruſſia, Podolia, and Volhinia. The ſoil of Poland is in general very fruitful, and the air moſtly temperate, except in the northern parts, where it is very cold. Their paſture land is ſo fruitful, that the height of the graſs often conceals the cattle from the view of a paſſenger at two hundred yards diſtance. Great numbers of beaſts, as horſes, aſſes, oxen, buffaloes, bears, foxes, wolves, &c. run wild in the foreſts. There are ſeveral mines in the country, of gold, ſilver, copper, lead, iron, &c.

The greateſt curioſities in this country are the ſalt-mines, of which that of Wielitſka is the largeſt in the world, and has been wrought above 600 years. It is 743 feet below the ſurface of the ground, and 1115 feet in breadth, and 6691 in length; and appears like a ſpacious plain, with vaulted roofs, ſupported by columns of ſalt, which have been left ſtanding. Many public lights are placed in this mine, for general uſe, which reflect a moſt luminous appearance from every part of the mine. Here are alſo great numbers of huts for the accommodation of the miners and their families, many of whom are born and ſpend their lives in this place, without ever making their appearance on the ſurface of the earth. Through the midſt of the mine is the great road, which paſſes to the mouth of the mine: this road is generally crowded with carriages full of ſalt. A ſtream of freſh water alſo runs through the mine.

The wild men who have been ſeen of late years in the woods of Poland form another curioſity.

The Poles at preſent ſeem almoſt annihilated, and their country divided among the Auſtrians, Pruſſians, and Ruſſians.

PRUSSIA is bounded on the north by Samogitia; on the ſouth by Poland Proper and Maſovia; on the eaſt by part of Lithuania; and on the weſt by Poliſh Pruſſia and the Baltic; but if we take it in its full extent, this kingdom conſiſts of various territories, different parts of Germany, Poland, Swiſſerland, and other northern countries.

The principal diviſions of this kingdom are, Regal Pruſſia, ſituated in Poland; and Upper Saxony, containing Brandenburg, Pruſſian Pomerania, and Swediſh Pomerania, Magdeburg, and Halberſtadt in Lower Saxony; Glatz in Bohemia; Minden, Ravenſburg, Lingen, Cleves, Meures, and Mark, in the dutchy of Weſtphalia; Eaſt Frieſland, Lippe, Gulick, and Tacklenburg, in the circle of Weſtphalia; the margraviate of Anſpach, in the circle of Franconia; Gelder in the Netherlands; Neufchatel in Swiſſerland; and part of Sileſia.

Pruſſia carries on a conſiderable trade, and the balance in favour of Pruſſia is reckoned greater than that of any other European ſtate; great quantities of glaſs, iron works, cloth, camlet, ſilk, linen, paper, powder, copper, and braſs, are annually exported.

Amber is here found in great quantities, from which the crown of Pruſſia receives 26,000 dollars annually; alſo great ſums from the bitumen, of which ſeveral kinds are found in the Baltic Sea.

The religions of Pruſſia are thoſe of the Lutherans and Calviniſts; but all religions are tolerated. His Pruſſian Majeſty is abſolute through all his dominions. The Pruſſian army, even in times of peace, conſiſts of 180,000 men, which are reckoned the beſt diſciplined troops in the world; but in time of war it has been augmented to between 3 and 400,000 men.

GERMANY is bounded on the north by the German Ocean, Denmark, and the Baltic; on the eaſt, by Poland, Hungary, and Bohemia; on the ſouth, by Swiſſerland and the Alps; and on the weſt, by France and the Low Countries. It extends from 45 degrees 4 minutes, to 54 degrees 40 minutes, north latitude; and from 6 degrees to 19 degrees 45 minutes eaſt longitude. Germany is a great empire, having ſeveral dependant ſovereignties under it, under different modifications of government, ſome of which ſcarcely exceed an Engliſh manor in extent. It is divided into nine circles, three of which lie in the north, three in the middle, and three in the ſouth: viz. Upper Saxony, Lower Saxony, Weſtphalia; Upper Rhine, Lower Rhine, Franconia; Auſtria, Bavaria, and Suabia. Theſe circles are ſubdivided into principalities, dutchies, marquiſates, electorates, palatinates, counties, baronies, abbies, biſhoprics, &c.

The climate of Germany is in general healthy and agreeable, except in the moſt northern and ſouthern parts. And the ſoil is particularly fruitful; for though only a ſmall proportion of the country is cultivated, yet proviſions are in general cheaper than in moſt other countries of Europe. They have alſo a greater quantity of domeſtic animals and wild beaſts, as boars, hares, rabbits, foxes, badgers, goats, &c. &c. than other European countries. They alſo abound in moſt of the ſpecies of tame fowl, as well as wild fowl.

There are ſeveral mines in Germany of ſilver, copper, iron, lead, quickſilver, ſulphur, nitre, &c. and coal-pits are found in every part of the empire.

Germany is alſo in great eſteem in all other European countries for its mineral ſprings and baths, the moſt remarkable of which are thoſe of Aix-la-Chapelle, Spa, Pyrmont, Ems, Wiſbaden, Schwalbach, Wildungen, and Brakel, which laſt is encloſed, as the waters are ſo ſtrong as to be capable of intoxication.

The manufactures of Germany conſiſt of velvets, ſilks, cotton and woollen ſtuffs, linen, fuſtian, ribands, lace, tapeſtry, &c. They alſo make beautiful porcelain and lacquered ware, and every kind of hard ware.

The Germans have a conſiderable commerce, owing to their centrical ſituation, and the balance of trade is greatly in their favour. The eſtabliſhed religion is either Romiſh, Lutheran, or Calviniſt, being different in the different parts of the empire; but moſt other religions are tolerated at preſent.

The German language is a dialect of the Teutonic, and is called the High Dutch, being the mother tongue of the whole empire: but every different province has a different dialect.

The government of Germany is in the hands of about 300 civil and eccleſiaſtical princes, each of whom is abſolute in the government of his own ſtate; and the whole of them form a great confederacy, governed by political laws, at the head of which is the Emperor, whoſe power in the collective body is only executive. The Emperor is elected; but the empire for ſome centuries has belonged to the Houſe of Auſtria, as being the moſt powerful of the German princes. The nine electors of the empire have each a particular office in the Imperial court: they have the ſole election of the Emperor, and are as follow:—1. The Archbiſhop of Mentz, who is high chancellor of the empire, when in Germany.—2. The Archbiſhop of Treves, who is high chancellor of France and Arelat (a dignity merely nominal).—3. The Archbiſhop of Cologne.—4. The King of Bohemia, who is cup-bearer.—5. The Elector of Bavaria, who is grand ſewer.—6. The Elector of Saxony, who is great marſhal of the empire.—7. The Elector of Brandenburg (now King of Pruſſia), who is great chamberlain.—8. The Elector Palatine.—9. The Elector of Hanover (King of Great Britain), who claims the poſt of arch-treaſurer.

The revenue of the Emperor, as ſuch, is about 5 or 6000 pounds ſterling per annum, ariſing from the fiefs in the Black Foreſt. The Auſtrian revenues are immenſe, amounting to 12,000,000 pounds ſterling.

The military force of this country amounts to near half a million of men; the ſecular princes bringing upwards of 379,000, the eccleſiaſtical 7450, and the Emperor, as the head of the Houſe of Auſtria, 90,000.

Hungary, Bohemia, and the provinces of Tranſylvania, Sclavonia, Croatia, and Morlachia, may be conſidered as part of the German empire, having been brought under the dominion of the houſe of Auſtria. The eſtabliſhed religion of theſe countries is the church of Rome.—Bohemia lies between 48 and 52 degrees north latitude, and between 12 and 19 degrees eaſt longitude. Bounded on the north by Saxony and Brandenburg; on the eaſt by Poland and Hungary; on the ſouth by Auſtria and Bavaria; and on the weſt by Bavaria.—Tranſylvania lies between 45 and 48 degrees north latitude, and between 22 and 25 degrees eaſt longitude. Bounded on the north by Hungary and Poland; on the ſouth by Walachia; on the eaſt by Moldavia; and on the weſt by Hungary.—Sclavonia lies between 45 and 47 degrees north latitude, and between 16 and 22 degrees eaſt longitude. Bounded by the river Drave on the north; by Auſtria on the ſouth; by the Danube on the eaſt; and by the Save on the weſt.—Croatia lies between 44 and 47 degrees north latitude, and between 15 and 17 degrees eaſt longitude. Bounded on the north by the Save; on the ſouth by Morlachia; on the eaſt of Boſnia; and on the weſt by Carniola.—Morlachia lies between 44 and 46 degrees north latitude, and between 16 and 17 degrees eaſt longitude. Bounded on the north by Carniola; on the ſouth by Dalmatia; on the eaſt by Boſnia; and on the weſt by the Gulf of Venice.

SWISSERLAND is bounded on the north by Suabia; on the eaſt by the lake of Conſtance, Tirol, and Trent; on the ſouth by Italy; and on the weſt by France: extending from 45 to 48 degrees north latitude, and from 6 to 11 degrees eaſt longitude; and is divided into thirteen cantons: viz. Bern, Fribourg, Baſil, Lucern, Soloturn, in the weſt diviſion; Schaffhauſen, Zurich, Appenzel, in the eaſt diviſion; and Zug, Swiſs, Glaris, Uri, and Underwald, in the middle diviſion. Seven of theſe cantons profeſs the Romiſh religion; Fribourg, Lucern, Soloturn, Zug, Swiſs, Uri, and Underwald; the other ſix are Proteſtants. The climate of this country is very various, on account of the inequality of the ſurface of the ground, being ſituated among the Alps; the higheſt mountains in Europe: ſo that it is common for the inhabitants to be reaping on one ſide of the mountain, while thoſe on the other ſide of the ſame mountain are ſowing. The froſts in winter are very ſevere; and in the ſummer the heat is, in ſome parts, intenſe.

The commerce of Swiſſerland conſiſts of their cattle, horſes, cheeſe, butter, hides, ſkins, and the productions of their own manufacture, the principal of which are ſilks, brocades, linen, lace, woollens, ſtuffs, hats, paper, leather, porcelain, toys, watches, and clocks.

Each canton forms a ſeparate republic; but when any controverſy ariſes, it is referred to the general diet, which ſits at Baden, where each canton has a vote, and ſends two deputies.*

The NETHERLANDS lie between 50 and 54 degrees north latitude, and between 2 and 7 degrees eaſt longitude. They are bounded on the north by the German ocean; on the eaſt by Germany; on the weſt by the Britiſh channel; and on the ſouth by France and Lorrain. The Netherlands are divided into ſeventeen provinces; the ſeven northerly ones are called Holland, or the United Provinces, and the other ten are called Flanders, or the Auſtrian and French Netherlands.

The provinces of Holland, are Holland, Zealand, Friezland, Groningen, Overyſſel, Guelderland and Zutphen, and Utrecht.

The air of theſe provinces is very moiſt and foggy; their harbours are generally frozen up four months in the year; and the ſoil is very unfavourable for vegetation; but the induſtry of the inhabitants has greatly improved it, by making canals and ditches to drain the land.

Their commerce is carried on to ſuch an extent, that there is hardly a commodity of traffic on the face of the globe but may be bought here, and almoſt as cheap as in the places where it is produced.

The religion of this country is Calviniſm; but all profeſſions and ſocieties are tolerated, of which there are great numbers.

The government of Holland is a democracy, and has ſo continued for upwards of two hundred years; notwithſtanding they had a prince under the title of ſtadtholder, whoſe powers had very little of the regal nature.

The ten other provinces of the Netherlands, called Flanders, have been divided among the Auſtrians, French, and Dutch, but are now chiefly claimed by the French, and contain the ten following provinces: viz.—Brabant, Antwerp, Malines, Limburgh, Luxemburgh, Namur, Hainault, Cambreſis, Artois, and Flanders.

The ſoil in moſt of theſe provinces is extremely fruitful, and the air generally healthy, except in Brabant, and ſome parts of the ſea-coaſts.

The commerce of theſe provinces conſiſts chiefly of their own manufacture, viz.—fine linens, cambricks, laces, and woollen manufactures.

France extends from 42 to 51 degrees north latitude; and from 5 degrees weſt, to 8 degrees eaſt longitude. It is bounded on the north by the Netherlands and the Engliſh channel; on the eaſt by Germany, Swiſſerland, and Italy; on the ſouth by the Mediterranean ſea and Pyrenean mountains; and on the weſt by the Bay of Biſcay. France was formerly divided into 12 provinces; but at the late revolution it was divided into 84 departments, each department being divided into diſtricts, and each diſtrict into cantons. The eighty-four departments are as follows:—1. Straits of Calais: 2. North. 3. Lower Seine. 4. Somme. 5. Aiſne. 6. Ardennes. 7. Channel. 8. Calvados. 9. Eure. 10. Oiſe. 11. Marne. 12. Meuſe. 13. Moſelle. 14. Lower Rhine. 15. Finiſterre. 16. North coaſt. 17. Iſle and Vilaine. 18. Mayenne. 19. Orne. 20. Eure and Loire. 21. Seine and Oiſe. 22. Paris. 23. Seine and Marne. 24. Aube. 25.

Upper Marne. 26. Meurte. 27. Voſges. 28. Upper Rhine. 29. Morbihan. 30. Lower Loire. 31. Mayenne and Loire. 32. Sarthe. 33. Loire and Cher. 34. Loirett. 35. Yonne. 36. Cote d'Or. 37. Upper Soanne. 38. Doubes. 39. Vendée. 40. Two Sevres. 41. Vienne. 42. Indre and Loire. 43. Indre. 44. Cher. 45. Nievre. 46. Soanne and Loire. 47. Jura. 48. Lower Charente. 49. Charente. 50. Upper Vienne. 51. Creuze. 52. Allier. 53. Rhone and Loire. 54. Ain. 55. Gironde. 56. Dordogne. 57. Correze. 58. Puy de Dome. 59. Upper Loire. 60. Iſere. 61. Landes. 62. Lot and Garonne. 63. Lot. 64. Cantal. 65. Lozere. 66. Ardeche. 67. Dreme. 68. Upper Alps. 69. Lower Pyrenees. 70. Gers. 71. Upper Garonne. 72. Tarne. 73. Aveiron. 74. Herault. 75. Gard. 76. Lower Alps. 77. Upper Pyrenees. 78. Arriege. 79. Aude. 80. Eaſt Pyrenees. 81. Mouths of Rhone. 82. Var. 83. Corſica. 84. Mount Blanc.

The climate of France is reckoned, upon the whole, to be more ſettled than that of any other country in Europe. In the north the winters are very cold; but in the interior parts the air is very temperate and healthy; and in the ſouth it is ſo mild, that invalids retire thither from all the northern countries, to avoid the rigour of their own climates.

The commerce of France conſiſts of wines, brandy, vinegar, drugs, oils, fruits, of which they have great variety, ſilks, cambricks, laces, paper, parchment, hardware, toys, &c. and their trade is very conſiderable and lucrative both to the Eaſt and Weſt Indies; but particularly to the European countries.

The national religion was always Romiſh. And their monarchs were always limited till the three laſt ſovereigns of France. The executive power is now veſted in three conſuls, the chief of whom is created conſul for life, with the power of nominating a ſucceſſor.

Spain lies between 36 and 44 degrees north latitude; and between 10 degrees weſt, and 3 degrees eaſt longitude. It is bounded on the north by the Bay of Biſcay and the Pyrenean mountains; on the ſouth by Gibraltar ſtraits; on the eaſt by the Mediterranean ſea; and on the weſt by Portugal and the Atlantic Ocean. It is divided into the following kingdoms or provinces: Galicia, Aſturia, Biſcay, Navarre, Arragon, Catalonia, Valencia, Murcia, Granada, Andaluſia, Old Caſtile, New Caſtile, Leon, and Eſtremadura.

Spain enjoys a dry, clear, temperate air, except during the equinoctial rains; and in the ſouthern provinces during the ſummer months, where it is very hot. The ſoil is as fruitful as the ſoil of any part of Europe; but the natives are very indolent. In many parts the choiceſt fruits grow ſpontaneouſly. They alſo have a great variety of aromatic herbs. Seville is celebrated for its oranges; and Murcia produces mulberry-trees in ſuch abundance, that the ſilk exported from this part amounts to 200,000 pounds per annum.

The chief articles of commerce in Spain are gold and ſilver, which they derive from their ſettlements in South America. The principal manufactures are ſilk, wool, iron, copper, and hardware.

The national religion of Spain is the profeſſion of the church of Rome. The Inquiſition always reigned in this country, till, by a late edict, it was put under ſome reſtrictions.

The conſtitution of Spain is the moſt abſolute monarchy in Europe. And the revenue from Old Spain only, amounts to upwards of 6,000,000 ſterling: what the exact amount of the whole revenue is, is not accurately known.

The military force of Spain is never leſs than 70,000 men in time of peace; and in time of war the king has raiſed near 200,000.

Portugal joins to Spain, and is bounded by it on the north and eaſt; and on the ſouth and weſt by the Atlantic Ocean,

Ocean. It extends from 36 degrees 50 minutes to 43 degrees north latitude, and from 7 to 10 degrees weſt longitude.

The climate of Portugal is more temperate than that of Spain, on account of its vicinity to the ſea. Their commerce conſiſts chiefly of wines, fruits, ſalt, linen, woollen, and ſome coarſe ſilk. Their religion is that of the church of Rome; and the Inquiſition has greater power here than in any other country. The conſtitution is, like that of Spain, an abſolute monarchy.

Italy, including Sicily, lies between 37 and 47 degrees north latitude, and between 7 and 19 degrees eaſt longitude. On the eaſt, ſouth, and weſt, it is waſhed by the Adriatic and Mediterranean ſeas; and on the north it is ſeparated from the reſt of Europe by the Alps. It contains the following countries: Piedmont, Montferrat, part of Milan, Sardinia Iſle, Naples, Sicily, Milaneſe, Mantua, Tuſcany, the Duke of Parma's territories, Genoeſe territories, Oneglia, the Duke of Modena's territories, Venetian territories, Pope's dominions, Corſica Iſle, Malta Iſle, and ſome other ſmall iſlands. All theſe countries are diſtinct from each other; having different forms of government, different trade, and ſeparate intereſts.

Italy has a fine ſoil, and temperate but warm climate; the ſoil however is greatly neglected, owing to the indolence of the inhabitants.

The religion, univerſally profeſſed throughout Italy, is that of the church of Rome; but people of all other religions generally live unmoleſted in moſt parts of Italy. The commerce and manufactures are various, according to the different ſtates; but wines, fruits, and oil, conſtitute the chief articles. The curioſities to be met with in this extenſive tract of country are almoſt innumerable, it being the ſeat of ſo many nations of antiquity, particularly of ancient Rome: hence, there are innumerable remains of the arts, the works of ancient artiſts: the burning mountains alſo conſtitute one of their greateſt natural curioſities. The Italian language is derived from the Latin; with an intermixture of words from the Goths, and other barbarous nations; but every ſeparate ſtate has a different dialect.

To deſcribe the form of government of each ſtate, would be to enter into too minute a detail, as they are different in every ſtate.

TURKEY extends into both Europe and Aſia.

European Turkey extends from 17 to 40 degrees eaſt longitude, and between 37 and 49 degrees north latitude. It is bounded on the north by Ruſſia, Poland, and Sclavonia; on the eaſt by the Black Sea, the Helleſpont, and the Archipelago; on the ſouth by the Mediterranean; and on the weſt by the Mediterranean, and Venetian and Auſtrian territories.

Turkey in Europe contains ſome of the moſt genial climates in the world; and is divided into the following provinces: Crim and Little Tartary, Budzaic Tartary, Beſſarabia, Moldavia, Wallachia, Bulgaria, Servia, Boſnia, Romania, Macedonia, Janna, Livadia, Epirus, Albania, Dalmatia, Raguſa, Corinth, Argos, Sparta, Olympia, Arcadia, Elis.

The ſoil of Turkey is extremely fruitful, where the leaſt induſtry has been employed: and all the fruits common to all the warm climates are produced here in great perfection; and many valuable drugs are natives of this country.

The commerce and manufactures of Turkey are chiefly ſilks, drugs, dying ſtuffs, in their natural ſtate; with cottons, carpets, leather, velvets, ſoap, &c.; but though the Turks are ſituated in the moſt advantageous part of both Europe and Aſia for traffic, yet they ſhamefully neglect it.

The religion which the Turks univerſally profeſs, is Mahometiſm; but they are divided into as many ſects as the profeſſors of Chriſtianity. The high prieſt, or Mufti, is an officer of ſuch honour, that whenever he comes into court, the Grand Seignior riſes from his ſeat and meets him. Moſt other religions are tolerated here by paying an annual tax.

The government of Turkey is that of an abſolute monarchy; and in this empire there is no hereditary ſucceſſion by law to any property; yet the rights of individuals are rendered ſecure by being annexed to the church, by which means even Jews and Chriſtians may ſecure their property in lands to the lateſt poſterity. The revenue of Turkey amounts to upwards of twenty-five millions per annum, but does not produce four millions to the emperor's treaſury; the reſt being expended in collecting, &c. The forces of the Turkiſh empire are of two ſorts; the one has certain lands for their maintenance, and the other is paid out of the treaſury. The former amount to 268,000 troopers; the latter, called the horſe-guards, are about 12,000; and the janizaries, or foot-guards, 25,000; beſides 100,000 foot ſoldiers in different parts of the empire.

SECT. VII.

OF ASIA.

Asia forms the moſt remarkable quarter of the globe in ancient hiſtory. It was here that the firſt man was created—here the patriarch Noah was preſerved during the flood—and from this quarter the world was repeopled a ſecond time. In Aſia lived all the patriarchs recorded in Scripture—

and this was the ſcene of all the tranſactions recorded in Holy Writ—and, finally, it was here Jeſus Chriſt appeared, and wrought the ſalvation of mankind—and from hence the Chriſtian religion was propagated.

This quarter of the globe enjoys the moſt ſerene air and fruitful ſoil of all the quarters, and produces the moſt delicious fruits, odoriferous ſhrubs, ſpices, and valuable drugs, gums, &c.

Idolatry and Mahometiſm are almoſt univerſal in this quarter of the globe, except in a few European ſettlements. The languages in uſe in this quarter are chiefly the Arabic, Perſian, Malayan, Chineſe, Japaneſe, Tartarian, Ruſſian, and Turkiſh.

Aſia is bounded on the weſt by the Red Sea, the Mediterranean, the Archipelago, the Black Sea, and Europe; on the north by the Frozen Ocean; on the eaſt and ſouth by the Pacific and Indian Oceans. It is ſituated between the equator and the frigid zone, and between 25 and 180 degrees of eaſt longitude; it is about 4800 miles in length, and 4300 in breadth, and contains the following countries.

TURKEY in Aſia, being the other part of the Turkiſh empire, is about 1000 miles in length from eaſt to weſt, and 800 in breadth from the northern parts to the deſerts of Arabia. It is bounded on the north by the Black Sea and Circaſſia; on the eaſt by Perſia; on the ſouth by Arabia and the Levant ſea; and on the weſt by the Archipelago and the Helleſpont.

This part of Turkey was the principal ſcene of all the tranſactions recorded in ancient writ, ſacred and profane.

TARTARY is an extenſive country taken in its full extent, and ſtretches from Muſcovy on the weſt, to the Pacific Ocean on the eaſt; and from the nations of China, India, Perſia, and Turkey, on the ſouth, to the impenetrable regions of the north. It extends from the thirtieth degree of north latitude to the frozen regions of the north pole; and from 50 to 190 degrees eaſt longitude; and contains Ruſſian, Chineſe, Mogulean,

gulean, and Independent Tartary, which are its four grand diviſions, 4000 miles in length, and 2400 in breadth.

Through ſuch an extenſive tract of country the ſoil and climate muſt neceſſarily partake of a great variety.

Their manners, language, &c. muſt alſo be as various.

CHINA lies on the eaſtern borders of the continent of Aſia, and is divided from Chineſe Tartary on the north, by a prodigious wall, and, in ſome places, by inacceſſible mountains; on the eaſt it is bounded by the Yellow Sea and Pacific Ocean, which ſeparates it from America; on the ſouth by the Chineſe ſea, and the kingdom of Tonquin; and on the weſt by Tibet. It extends from 21 to 44 degrees north latitude, and from 94 to 133 degrees eaſt longitude.

In ſuch an extenſive country there muſt no doubt be a variety of climates. The ſouthern parts are very hot, and have violent rains, while the northern parts are very cold, and their rivers frozen for ſome months during the winter; but the middle parts are temperate and pleaſant. The ſoil alſo partakes of a great variety, though there is no part of this extenſive country but is fruitful, either from nature or art; for ſuch is the induſtry of the Chineſe, that they ſuffer very little, if any land, to lie uncultivated.

The Chineſe have a conſiderable trade with every European nation, and with North America, exporting ſilks, cotton, gold and ſilver ſtuffs, painted gauzes, teas, china-ware, paper, and Indian ink, for which they receive ready money; deſpiſing the manufactures of every other country but their own.

There are a great number of natural and artificial curioſities in China. Among the latter are reckoned the famous wall which divides China from Tartary, extending over mountains and vallies, of 1500 miles in length, and from 20 to 25 feet in height, and broad enough for ſix horſemen to travel abreaſt. It has ſtood near 1800 years, and is now almoſt entire. 2. Their canals are works of great magnitude, infinitely exceeding thoſe in Europe. 3. The bridge over the river Saffrany, which conſiſts of a ſingle arch, whoſe ſpan is 400 cubits, and its height 500. 4. The Cientao, or road of pillars, which is a road broad enough for four horſes to travel abreaſt, and near four miles in length, defended by an iron railing; and unites the ſummits of ſeveral mountains, in order to avoid the winding of the roads. It reſts upon ſtrong ſtone pillars for the moſt part. 5. The bridge of chains, which is a bridge built upon a number of ſtrong iron chains, and hangs over a very deep valley, in the neighbourhood of King-Tung. 6. The triumphal arches of China, of which there are above 1100; 200 of them are very magnificent; they were erected in memory of their great princes, legiſlators, &c. 7. The tower of Nan-King, called the Porcelain Tower, being wholly covered with the moſt beautiful china; upwards of three hundred feet in height, nine ſtories high; each ſtory decreaſing gradually to the top. The whole forms the moſt correct and grand piece of architecture to be met with in the Eaſt.

Among the natural curioſities may be reckoned their water-falls and volcanoes.

Their religion is that of Paganiſm; the deities are men that have been eminent in arts and ſciences. They alſo worſhip inanimate beings, as mountains, woods, and rivers; but they acknowledge only one Supreme Being.

INDIA, or HINDOSTAN, is an extenſive country taken in its full extent. Bounded on the north by Tibet and Uſbeck Tartary; on the ſouth by the Indian Ocean; on the eaſt by China and the Pacific; and on the weſt by Perſia and the Indian Ocean. It extends from 1 degree to 40 degrees north latitude, and from 66 to 109 degrees eaſt longitude; and is principally divided into three parts:—1. The peninſula of India beyond the Ganges, on the eaſt; 2. the main land, or empire of the Great Mogul, on the north; 3. the peninſula within the Ganges, or on this ſide of it, on the weſt.

A great part of the ſea-coaſt of India, as well as conſiderable diſtricts in the interior, belong to the Engliſh Eaſt India Company, where there are many large and rich ſettlements, from which we receive great quantities of Eaſt India commodities.

As the country extends through ſo many degrees of latitude, there is a great difference in the climates of the different parts. In the northern parts the air is very dry and healthy; but in the ſouthern parts near the ſea, in low lands, the air is very hot and moiſt: they divide the year into the dry and wet ſeaſons.

The ſoil, in general, throughout the whole country, is very fruitful, producing all the variety of plants, drugs, and fruits, to be met with in the other tropical climates. There are alſo mines of gold, diamonds, rubies, topazes, and other precious ſtones.

In the European ſettlements the religion is Chriſtianity; but in the northern and inland parts they are either Mahometans or Pagans; and divided into ſeveral kingdoms, each of which is governed by one or more abſolute monarchs.

Persia extends from 25 to 45 degrees north latitude, and from 45 to 67 degrees eaſt longitude. It is bounded on the eaſt by the Mogul's dominions; on the north by Uſbeck Tartary, the Caſpian Sea, and Circaſſia; on the ſouth by the Indian Ocean and Gulf of Perſia; and on the weſt by Arabia and the Turkiſh empire.

The climates of this country are very various. In the northern parts, and near the mountains, which are covered with ſnow, the air is very cold; in the midland parts it is ſerene, pure, and healthy; but towards the ſouthern parts there are ſometimes hot ſuffocating winds, which blow over a ſandy deſert from ſouth and eaſt; a blaſt of which has ſometimes ſtruck the unwary traveller with death in an inſtant. The ſoil is various, being in ſome parts very barren, but where it is well watered it is very fruitful.

The principal commodities of traffic are ſilks, camlets, carpets, leather, embroidery, gold and ſilver threads, mohair, &c.

The national religion of Perſia is that of Mahometiſm, and the ſect of Ali.

ARABIA extends from 35 to 60 degrees eaſt longitude, and from 12 degrees 30 minutes to 30 degrees north latitude. It is bounded on the north by Aſiatic Turkey; on the ſouth by the Indian Ocean; on the eaſt by the Euphrates and Gulf of Baſſora; and on the weſt by the Red Sea.

Arabia is divided into three parts, viz. Arabia Petræa, or the Stony; Arabia Deſerta, or the Deſert; and Arabia Felix, or the Happy.

Arabia the Stony is the wilderneſs in which the children of Iſrael ſojourned 40 years: and in it may be ſeen the mountains of Horeb and Sinai, mentioned in Sacred Writ.

Arabia the Deſert principally conſiſts of a large ſandy deſert; it has, however, a few ſpots of fruitful land, covered with verdure, which are interſperſed in different parts of the deſert. It is over this deſert that ſome of the eaſtern nations bring their commodities of traffic from the Eaſt, travelling in large caravans.

Arabia the Happy is, in general, barren; but ſome of the vallies between the mountains, and thoſe plains which are well ſupplied with water, are very fruitful. From this part great quantities of drugs are exported to Europe, and alſo Turkey coffee.

The Arabs are, in general, a wandering people: many of their tribes live wholly in tents, and ſubſiſt partly by robbing the caravans which travel through the deſert, and partly by the produce of their country, and the fleſh of their cattle; raiſing no grain of any kind for domeſtic uſe.

Their religion is that of Mahometiſm; but many of the tribes are ſtill Pagans. Their language is ſaid to exceed even the Greek itſelf in copiouſneſs. The Arabians have never yet been ſubdued by any military force, though ſeveral attempts have been made for that purpoſe.

SECT. VIII.

OF AFRICA.

The continent of Africa is in the form of a peninſula, ſurrounded on each ſide by water, except where it joins to Aſia by the Iſthmus of Suez. Several countries, famous in antiquity for the arts and ſciences, were ſituated in the northern parts of this quarter. And in the early days of Chriſtianity ſeveral Chriſtian churches were founded here; but at the preſent period Mahometiſm and idolatry degrade this moſt fertile quarter of the globe. That moſt inhuman commerce, trafficking in men, alſo is carried on here by the European nations.

The ancients believed the greater part of this quarter of the globe to be uninhabited, as alſo the greater part of Aſia, and, indeed, all that part of the globe lying between the tropics; but modern travellers have diſcovered, that the tropical countries are in general the moſt fertile and beſt populated; and of theſe the ſouthern and interior parts of Africa are found the moſt eligible, both for vegetation and population. Its ſea-coaſts are the only parts with which we are particularly acquainted; but travellers are now buſily employed in making diſcoveries in the internal parts.

Africa is bounded on the weſt by the Atlantic Ocean; on the north by the Mediterranean; on the eaſt by the Red Sea; and on the ſouth by the Southern Ocean. It lies between 37 degrees north, and 36 degrees ſouth latitude, the equator running nearly through the middle thereof; and between 17 degrees weſt, and 51 degrees eaſt longitude. In length, from north to ſouth, it is about 4600 miles; and in breadth, from eaſt to weſt, 3500 miles.

Egypt is bounded on the north by the Iſthmus of Suez; on the eaſt by the Red Sea; on the ſouth by Nubia; and on the weſt by the interior parts of Africa. It lies between 30 and 36 degrees eaſt longitude; and between 20 and 32 degrees north latitude; and is divided into Upper and Lower Egypt.

The climate, during the ſummer ſeaſon, is exceſſively hot; when the ſouth winds often raiſe ſuch a cloud of ſand as to obſcure the light of the ſun, and cauſe epidemical diſeaſes.

The ſoil is exceedingly fruitful, owing to the annual overflowing of the Nile. This river, ſo famous in ancient hiſtory, has its riſe in Abyſſinia, at between 11 and 12 degrees of north latitude, and purſues a northern courſe for above 1500 miles; when it divides into two branches, about ſix miles below Grand Cairo; one branch extending eaſtward, and the other weſtward. It begins to riſe in the beginning of ſummer, and increaſes three or four inches in height each day, for the firſt week: the next fortnight it increaſes in a ſtill greater proportion: and it is near four months before it is reduced into its channel again. The principal cities and towns are built on eminences on the banks of the Nile, and, during the inundation, correſpond with each other by means of boats. When the Nile riſes to the height of 49 feet, it produces a plentiful ſeaſon, but if it exceed that height it is productive of great miſchief, ſweeping away both houſes and cattle.

In Egypt they generally have three crops in a year: the firſt, of lettuces and cucumbers; the ſecond, of corn; the third, of melons, and all the fruits common to hot climates.

Their paſtures are the richeſt in the world, the graſs being uſually as high as the cattle.

Their trade conſiſts of great quantities of flax and cotton, both prepared and unmanufactured; leather of different kinds; alſo a great variety of drugs, and roots for dying.

The common language ſpoken here is the vulgar Arabic, as it is under the dominion of the Turks.

BARBARY extends from Egypt to the Atlantic Ocean, and from the Mediterranean Sea to the Libyan Deſerts, being 750 miles in breadth, and near 2000 in length: containing the countries of Morocco and Fez, which form one diſtinct empire; and the ſtates of Algiers, Tunis, Tripoli, and Barca, compoſing ſeveral diſtinct ſtates, united together in confederacy, under the Turkiſh government.

Its ſoil is exceedingly fruitful, producing excellent corn, cattle, and paſture, and all the variety of tropical fruits; and vaſt quantities of fiſh and fowl; alſo a great variety of tame and wild animals.

The commerce of this country is chiefly carried on by caravans: their exports conſiſt of leather, mats, handkerchiefs, carpets, elephant's teeth, oſtrich feathers, copper, tin, wool, fruits, gum, drugs, &c. for which they receive timber, artillery, gunpowder, &c.

Their religion is that of Mahometiſm. Their language varies according to the different parts of the country. That ſpoken in the inland parts, is either an African language, or a corrupt Arabic. The latter is alſo ſpoken in moſt of the ſea-port towns: but in ſome parts they uſe a mixed language, ſuch as is ſpoken in moſt of the Mediterranean ports.

Moſt of the Barbary ſtates ſubſiſt by piracy: and their ſailors fight deſperately when they meet a veſſel belonging to any power with whom they are at war.

The government is that of an abſolute monarchy. The emperor is in general both judge and executioner; and he acknowledges the Grand Seignior of Turkey to be his ſuperior. When there is a vacancy in the government, every ſoldier in the army has a vote in chooſing a new emperor, which is often attended with great bloodſhed.

The parts of Africa, from the tropic of Cancer to the Cape of Good Hope, are very little known, except the ſea-coaſt thereof. The natives in general are black, except thoſe of Abyſſinia, who are of a tawney complexion, and are a mixture of Jews, Chriſtians, and Pagans. The religion of the other countries in this part is generally that of Paganiſm, and the form of government every where monarchical, except in a few ſettlements formed by the Europeans, on the ſea-coaſt. Few of their princes, however, poſſeſs an extenſive degree of territory. As the natives are ignorant of all the arts of utility and refinement, the different kingdoms are therefore unconnected with each other, and are generally at war.

The ſoil of Africa is in general very fruitful: though in ſome parts it is perfectly barren, particularly where there is very little water; the heat of the ſun reducing the ſoil to a perfect ſand; ſuch are the countries of Anian and Zaara; but the countries of Mandingo, Ethiopia, Congo, Angola, Batua, Truticui, Monomotapa, Caſati, and Melunenrugi, are extremely fruitful, and very rich in gold and ſilver.

On the weſtern coaſt the Engliſh trade is carried on at James's Fort, and other ſettlements, near and up the river Gambia, where woollen and linen cloths, hardware, and ſpirituous liquors are exchanged for the perſons of the natives. Many of the negroes will ſell their own families for thoſe ſuperfluities. The natives are often trepanned by foreigners, or their own countrymen, and then ſold to the Europeans: and many more are ſold by the princes of the different ſtates, being captives taken in war. Gold and ivory form the principal branches of commerce, next to that of the ſlaves.

The Portugueſe poſſeſs the greater part of the eaſt and weſt coaſt of Africa, from the tropic of Capricorn to the equator. The Dutch have ſome ſettlements towards the ſouthern parts of the continent; and Cape Town, at the Cape of Good Hope, belongs to them, and is well fortified, and where the ſhips bound for India uſually put in, and trade with the natives, or Hottentots, for their cattle and other proviſions, for which they give them ſpirituous liquors. There are ſeveral iſlands near the coaſt of Africa, lying in the Eaſtern or Indian Ocean, or in the Weſtern or Atlantic Ocean, of which the chief are:

1. Zocotra, ſituated in 53 degrees eaſt longitude, and 12 degrees north latitude; 30 leagues eaſt of Cape Guardafui, on the continent of Africa; being 80 miles in length, and 54 in breadth, and has two good harbours. It is a populous, plentiful country, governed by a prince who is tributary to the Porte.

2. Babelmandel, ſituated in the ſtrait of the ſame name, at the entrance of the Red Sea, in 44 degrees 30 minutes eaſt longitude, and 12 degrees north latitude, being a ſmall ſandy iſland, not five miles round.

3. The iſlands of Joanna, Mayotta, Mohilla, Angezeia, and Comora, ſituated between 41 and 46 degrees eaſt longitude, and between 10 and 14 degrees ſouth latitude: the chief of theſe is Joanna, to which the others are tributary, being 30 miles long, and 15 broad: affording excellent fruits and proviſions. The natives are a friendly ſet of people, and profeſs the Mahometan religion.

4. Madagascar, the largeſt of the African iſlands, ſituated between 43 and 51 degrees eaſt longitude, and between 10 and 26 degrees ſouth latitude; three hundred miles ſouth-eaſt of the continent of Africa, being near 1000 miles in length, from north to ſouth; and between 2 and 300 miles in breadth. Between this iſland and the Cape of Good Hope, or the continent of Africa, the ſea rolls with great force, and is exceedingly rough. In this channel, all European ſhips paſs in their voyage to and from India, except the water be too rough. Madagaſcar is a fertile country, abounding in all the variety of fruits and vegetables to be met with in the ſame climate. The air is temperate alſo, and healthy. It is inhabited by both blacks and whites, profeſſing different religions; but principally Mahometiſm and Paganiſm; and governed by ſeveral petty princes.

5. **MAURITIUS**, or Maurice, ſituated in 56 degrees eaſt longitude, and 20 degrees ſouth latitude; about 400 miles eaſt of Madagaſcar. It is of an oval form, and about 150 miles in circumference, with a large fine harbour. The climate is healthy and pleaſant; and the iſland is well watered with ſeveral rivers: though the ſoil is not ſo fruitful as that of the former, it nevertheleſs feeds a great number of cattle, ſheep, deer, and goats.

6. **BOURBON**, ſituated in 54 degrees eaſt longitude, and 21 degrees ſouth latitude; about 300 miles eaſt of Madagaſcar, and about 90 miles in circumference. Surrounded for the moſt part with blind rocks, a few feet under water. The climate is in general healthy, though hot. It affords very good paſture and cattle.

There are ſeveral other ſmall iſlands about Madagaſcar, and on the eaſtern coaſt of Africa.

7. **ST. HELENA**, ſituated in 6 degrees weſt longitude, and 16 degrees ſouth latitude; being 1200 miles weſt from the continent of Africa, and 1800 eaſt from South America. The whole iſland is ſituated on a rock, and is about 21 miles in circumference. There is but one landing-place in the iſland, which is at the eaſt ſide thereof. It is very fertile, diverſified by hills and vallies, and abounds in all the conveniences and comforts of life. There are about 200 families, moſtly deſcended from Engliſh parents.

8. **ASCENSION**, ſituated in 7 degrees 40 minutes ſouth latitude, and 600 miles north-weſt of St. Helena. It is a mountainous barren iſland, and uninhabited; about 20 miles round.

9. **ST. MATTHEW**, ſituated in 6 degrees 1 minute weſt longitude, and 1 degree 30 minutes ſouth latitude; and uninhabited.

10. **CAPE VERD ISLANDS** are ſituated between 23 and 26 degrees weſt longitude, and between 14 and 18 degrees north latitude. They are about 20 in number; but the principal are St. Jago, Bravo, Fogo, Mayo, Bonaviſta, Sal,

St. Nicholas, St. Vincent, Santa Cruz, and St. Antonio. They moſtly belong to the Portugueſe and Spaniards. The air in general is very hot, and in ſome unwholeſome. They are inhabited by Europeans and their deſcendants.

11. GOREE, ſituated in 14 degrees 43 minutes north latitude, and 17 degrees 20 minutes weſt longitude. It is a ſmall ſpot not exceeding two miles in circumference; but an important ſituation for trade.

12. The CANARIES, or FORTUNATE ISLANDS, are ſeven in number, and ſituated between 12 and 19 degrees weſt longitude, and between 27 and 29 degrees north latitude. Theſe iſlands have a pure temperate air, and abound in moſt delicious fruits, from whence they have thoſe rich wines called Canary, of which they export 10,000 hogſheads annually.

13. The MADEIRAS are three iſlands, ſituated in 32 degrees 27 minutes north latitude, and between 18 degrees 30 minutes and 19 degrees weſt longitude. Theſe iſlands are moſtly famous for producing the Madeira wine, of which no leſs than 20,000 hogſheads are annually exported.

14. The AZORES, or WESTERN ISLANDS, are ſituated between 25 and 32 degrees weſt longitude, and between 37 degrees and 40 degrees north latitude. Being 900 miles weſt of Portugal; and lying in the midway between Europe and America. Of theſe St. Michael is the largeſt, being near 100 miles in circumference, and containing 50,000 inhabitants. Tercera is the moſt important of theſe iſlands, on account of its harbour, which is very ſpacious, and affords good anchorage. There are ſeven other of theſe iſlands: their names are Santa Maria, St. George, Gracioſa, Fayal, Pica, Flores, and Corvo.

SECT. IX.

OF AMERICA *.

America, or the Great Weſtern Continent, frequently called the New World, extends from the 80th degree of north latitude, to the 56th degree of ſouth latitude; and where the breadth is known, from the 35th degree, to the 136th degree weſt longitude; extending near 9000 miles in length, and 3690 in breadth. As it extends into both hemiſpheres, it has two ſummers and two winters. It is waſhed by the two great oceans, the Atlantic and Pacific; having the former on the eaſt, and the latter on the weſt: by theſe ſeas it has a direct communication with the other three quarters of the world. It is compoſed of two great continents, North and South America, connected together by the kingdom of Mexico, which is an iſthmus of 1500 miles long, and in one part only 60 miles broad.

America is the beſt watered of any part of the globe; even thoſe vaſt tracts of country ſituated beyond the Apalachian Mountains, at an immenſe diſtance from the ocean, are watered by inland ſeas, as the Lakes of Canada, which give riſe to ſeveral large rivers, as the Miſſiſſippi, the Miſſouri, the Ohio, and on the north, the river St. Lawrence, all of them being navigable to their heads, which is a great advantage for commerce.

South America is better watered, if poſſible, than North America, having the two largeſt rivers in the world: viz.—

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* To ſuch perſons as wiſh to make themſelves thoroughly acquainted with this part of the world, I would recommend a peruſal of the Rev. W. Winterbotham's Hiſtorical, Geographical, Commercial, and Philoſophical View of the United States of America, in four large volumes 8vo. illuſtrated by a complete Atlas and other plates, price 1*l.* 16*s.*

the river of Amazons, and the river of La Plata; the former having a courſe of about 3000 miles.

A country of ſuch vaſt extent on both ſides of the equator muſt neceſſarily have all the varieties of ſoils and climates to be met with in every other part of the globe. It alſo produces moſt of the metals, minerals, plants, fruits, trees, and wood, to be met with in the other parts of the world, and many of them in greater quantities and higher perfection.

This country likewiſe produces diamonds, pearls, emeralds, amethyſts, and other precious ſtones; alſo cochineal, indigo, anatto, logwood, brazil, fuſtic, pimento, lignum vitæ, rice, ginger, cocoa or chocolate, ſugar, cotton, tobacco, the balſams of Peru, Tolu, and Chili, Jeſuit's bark, mechoacan, ſaſſafras, ſarſaparilla, caſſia, tamarinds, and a great variety of other woods, roots, and plants, many of which were not known before the diſcovery of America; beſides hides, furs, and ambergris.

Though the Indians ſtill live in quiet poſſeſſion of many large tracts of country, in the inland parts, yet America, ſo far as is known, is generally claimed by four powers: viz.—the Spaniards, Engliſh, Portugueſe, and American ſettlers, being the deſcendants of Europeans, and who have the largeſt ſhare of country, except the Spaniards, who poſſeſs the largeſt and moſt extenſive portion of all, extending from New Mexico and Louiſiana, in North America, to the Straits of Magellan, in South America; except the large province of Brazil, which belongs to Portugal, Surinam, claimed by the Dutch, and Cayenne, the property of the French, all in South America. The United States of America poſſeſs all that tract of country which is bounded by the Miſſiſſippi, the river St. Lawrence, and the Lakes of Canada, on the north and weſt; and waſhed by the Atlantic Ocean on the eaſt; and on the ſouth by the Gulf of Mexico.

The AMERICAN ISLANDS, commonly called the Weſt

Indies, was the firſt of America diſcovered by the Europeans, and are ſituated in the gulf called the Caribbean Sea, between the continents of North and South America, extending from the coaſt of Florida, to the river Oroonoko: they are divided between five European nations: viz.—the Engliſh, French, Spaniards, Dutch, and Danes.

As all theſe iſlands lie between the tropics, their climates and ſoil are pretty much alike: the heat would be intolerable, if it were not for the trade-winds which blow during the fore part of the day, and the ſea and land breezes. Their ſeaſons are divided into the wet and dry: in the wet ſeaſons, the rain pours down with ſuch impetuoſity as to overflow the rivers, and lay the low country under water.

The principal trade of the Weſt Indies conſiſts of ſugar and rum: they alſo export cotton, indigo, chocolate, coffee, and dying and phyſical drugs, ſpices, and hard woods; for which they receive from Europe, manufactures; from the African iſlands, wine; and from the neighbouring continent, lumber and proviſions.

The BAHAMA ISLANDS, which are ſaid to be 500 in number, lie to the ſouth of Carolina, between 21 and 27 degrees north latitude, and between 73 and 81 weſt longitude. There are, however, not above twelve of them of any magnitude, the reſt being little better than rocks or banks, and almoſt uninhabited, except Providence Iſland.

The BERMUDAS, or SUMMER ISLANDS, lie in the Atlantic Ocean, about 500 leagues eaſt from Carolina, in 32 degrees north latitude, and in 65 degrees weſt longitude: theſe are ſaid to be about 400 in number; but containing not more than 20,000 acres.

The iſlands of NEWFOUNDLAND, CAPE BRETON, and ST. JOHN, lie at the mouth of the river St. Lawrence; and are celebrated for the quantity of fiſh found on their coaſts, which is ſuppoſed to increaſe the national ſtock upwards of 300,000*l.* annually: in this branch of commerce 3000 ſmall craft are employed, and 10,000 hands.

British America, or the territories on the continent belonging to the Engliſh, are New Britain, Canada, or the province of Quebec, and Nova Scotia, or Acadia: bounded on the eaſt and ſouth by the Atlantic Ocean and the American States; on the north and weſt, their boundaries have never been defined, but are blended with the lands of the Indian nations. New Britain contains Labrador, and New North and South Wales. Canada contains the towns of Quebec, Trois Rivieres, and Montreal, all ſituated on the river St. Lawrence.

Nova Scotia contains the towns of Halifax, Annapolis, and St. John's.

The United States of America are bounded on the weſt by the Indian nations; on the north, by Britiſh America; on the weſt, by the Atlantic; and on the ſouth, by Spaniſh America; containing the following ſtates or colonies: New Hampſhire, Maſſachuſetts, Rhode Iſland, Connecticut, New York, New Jerſey, Pennſylvania, Delaware, Maryland, Virginia, North Carolina, South Carolina, Georgia, Vermont, Weſtern Territory, and Kentucky.

The United States, in the year 1776, were only 13 in number: Vermont, Kentucky, and the Weſtern Territory, have been added ſince. The Weſtern Territory is of ſuch extent, that the Congreſs have determined to divide it into ten new ſtates.

From the lateſt accounts, it appears, that the population of the United States amounts to upwards of 3,083,600 perſons, who are compoſed of almoſt all nations, languages, characters, and religions: the greater part, however, have deſcended from the Engliſh.

The language generally ſpoken through all theſe ſtates is the Engliſh, in which all their civil and eccleſiaſtical matters are performed, and their records kept.

There are, however, great numbers of Dutch, French, Germans, Spaniards, Jews, and Swedes, who retain in a great degree each their native language, and have their reſpective places of worſhip; and in general live comfortably and unmoleſted, as to principles of conſcience.

New England is bounded on the north by Canada; on the eaſt by Nova Scotia and the Atlantic Ocean; on the ſouth by the Atlantic; and on the weſt by New York; and is divided into five ſtates: viz. New Hampſhire, Maſſachuſetts, Rhode Iſland, Connecticut, and Vermont, which are ſubdivided into counties, and thoſe counties again ſubdivided into townſhips.

New England is a fine country for paſtures; the valleys are generally interſected with brooks of water, the banks of which are covered with a tract of rich meadow land.

The ſtate of New York is bounded on the ſouth-eaſt by the Atlantic Ocean; on the eaſt by Connecticut, Maſſachuſetts, and Vermont; on the north by Canada; on the ſouth and ſouth-weſt by Pennſylvania and New Jerſey: being 350 miles in length, and 300 in breadth; and containing about 44,000 ſquare miles, equal to 28,160,000 acres. The river St. Lawrence divides this ſtate from Canada. The ſettlements formed in this ſtate are chiefly upon two oblongs, extending from the city of New York eaſt and north. The eaſt is Long Iſland, which is 140 miles in length; the other, extending north, is about 40 miles in breadth. This ſtate exports to the Weſt Indies, biſcuits, peas, Indian corn, apples, onions, boards, ſtaves, horſes, ſheep, butter, cheeſe, pickled oyſters, beef, and pork: but the principal part of their trade is wheat, of which, in the year 1775, they exported 677,700 buſhels, and 2555 tons of bread, beſides 2828 tons of flour, for which they receive in exchange the commodities of the Weſt India iſlands.

New Jersey is 160 miles in length, and 52 in breadth. Bounded on the eaſt by Hudſon's river and the ſea; on the ſouth by the ſea; on the weſt by Delaware bay and river, which divides it from Pennſylvania; and on the north by a line drawn from the mouth of Mahakkamak river, in latitude 41 degrees 24 minutes, to a point in Hudſon's river, in latitude 41 degrees; containing about 8320 ſquare miles.

This ſtate has a great variety of ſoil, from the worſt to the beſt. But it contains a greater portion of barren land than any other ſtate, there being nearly one fourth of this ſtate unfit for cultivation. But thoſe parts which are fruitful are equal in fertility to any part of the United States.

The ſtate of PENNSYLVANIA is bounded by Delaware river on the eaſt; by the ſtate of New York on the north; by the parallel of latitude, 39 degrees 43 minutes 18 ſeconds, on the ſouth; and by a meridian line drawn from the ſaid parallel at five degrees of longitude, from a point on Delaware river weſt. This ſtate lies in the form of a parallelogram. The north ſide of Pennſylvania is the beſt ſoil, and moſt populated, owing to the great number of new roads which have lately been made.

The ſtate of DELAWARE is bounded on the north by the territorial line (which is a circle), deſcribed with a radius of 12 Engliſh miles, and whoſe centre is in the middle of the town of Newcaſtle, which divides it from Pennſylvania; on the eaſt by Delaware river and bay; on the ſouth by a line drawn due eaſt and weſt from Cape Henlopen in 38 degrees 30 minutes north latitude, to the middle of the peninſula, which line divides the ſtate from Worceſter county, in Maryland; and on the weſt by Maryland: containing about 1400 ſquare miles; being 92 miles in length, and 16 in breadth.

The ſtate of MARYLAND is bounded on the north by Pennſylvania; on the eaſt by the Delaware ſtate; on the ſouth and ſouth-eaſt by a line drawn from the ocean over the peninſula (dividing it from Accomac county in Virginia), to the mouth of Patomac river, and from thence up the river to its firſt ſource; from thence, by a due north line, till it interſects the ſouthern boundary at Pennſylvania: being 134 miles in length, and 110 in breadth. The ſoil of Maryland, where it is good, will produce from 12 to 16 buſhels of wheat, or from 20 to 30 buſhels of Indian corn, per acre. Their trade is chiefly with the other ſtates, the Weſt Indies, and ſome parts of Europe: to which places they annually export 30,000 hogſheads of tobacco, beſides great quantities of pig-iron, lumber, flax-ſeed, and proviſions.

The ſtate of VIRGINIA is bounded on the eaſt by the Atlantic; on the north by Pennſylvania and the river Ohio; on the weſt by the Miſſiſſippi; and on the ſouth by North Carolina: being 758 miles in length, and 224 in breadth.

The ſtate of KENTUCKY is bounded on the north-weſt by the river Ohio; on the weſt by Cumberland river; on the ſouth by North Carolina; and on the eaſt by Sandy river, and a line drawn full ſouth from its ſource, to the northern boundary of North Carolina: being 250 miles in length, and 200 in breadth. The fertility of the ſoil is ſuch, that the land, in common, will produce 30 buſhels of wheat, or rye, an acre. The beſt lands are too rich for wheat, and will produce from 50 to 80 buſhels of good corn per acre; and few ſoils yield more and better tobacco.

The ſtate of NORTH CAROLINA is bounded on the north by Virginia; on the eaſt by the Atlantic; on the ſouth by South Carolina and Georgia; and on the weſt by the Miſſiſſippi: being 758 miles in length, and 110 in breadth.

The ſtate of SOUTH CAROLINA is bounded on the eaſt by the Atlantic; on the north by North Carolina; on the ſouth and ſouth-weſt by the Savannah river. The weſtern boundary is not aſcertained. It is reckoned 200 miles in length, and 125 in breadth.

The ſtate of GEORGIA is bounded on the eaſt by the Atlantic; on the ſouth by Florida; on the weſt by the river Miſſiſſippi; on the north and north-eaſt by South Carolina: being 600 miles in length, and 250 in breadth.

The WESTERN TERRITORY includes all that part of the United States on the north-weſt of the river Ohio; bounded on the weſt by the Miſſiſſippi river; on the north by the lakes;

on the eaſt by Pennſylvania; and on the ſouth and ſouth-eaſt by the river Ohio; containing 411,000 ſquare miles, equal to 263,040,000 acres, from which deducting 43,040,000 acres occupied by the water, there remain 220,000,000 acres, which are to be ſold by Congreſs, for the diſcharge of the national debt.

Territories of Spain in North America.

The dominions of Spain, in North America, extend from 81 degrees to 120 degrees weſt longitude, and from 8 to 43 degrees north latitude. Bounded on the north by the United States and the Indian nations; on the weſt by the Pacific Ocean; on the eaſt by the Gulf of Mexico and the Atlantic; and on the ſouth terminating in the Iſthmus of Darien. They contain the following countries, viz. Eaſt Florida, Weſt Florida, New Mexico, California, and Old Mexico.

The ſoil of this extenſive tract of country is very various, but in general fertile: and it is in the moſt mountainous parts that the mines of gold and ſilver are found. The air is in general warm and pleaſant; but the northern parts have very cold winds; and the ſouthern parts, lying within the torrid zone, are exceedingly hot.

South America.

South America, from the northern coaſt of Terra Firma, and the Iſthmus of Darien, to the Straits of Magellan, belongs to the Spaniards, except the province of Brazil, which belongs to the Portugueſe; and the ſettlements of the Dutch in Surinam, and thoſe of the French in Cayenne.

Brazil, belonging to the Portugueſe, extends from the equator to 35 degrees ſouth latitude, and from 35 to 60 degrees weſt longitude. Bounded on the north by the mouth of the river Amazons and the Atlantic; on the eaſt by the Atlantic; on the ſouth by the mouth of the river Plata; and on the weſt by the unknown country of the Amazons.

Cayenne is the only ſettlement in the ſouthern continent of America retained by the French, and is ſituated between the equator and 5 degrees north latitude; and between 52 degrees 15 minutes, and 55 degrees 30 minutes weſt longitude. It extends 240 miles along the coaſt of Guiana, and near 300 miles inland. Bounded by Surinam on the north; by the Atlantic on the eaſt; by Amazonia on the ſouth; and by the territories of the Indians on the weſt.

Surinam, or Dutch America, lies between 5 and 7 degrees north latitude; and is bounded on the north by Cayenne; on the weſt by Terra Firma; on the ſouth by the Indian nations; and on the eaſt by the river Oroonoko.

The dominions of Spain, in South America, contain the following extenſive countries, viz. Terra Firma, Peru, Chili, Paraguay, Amazonia, and Patagonia; extending, as before obſerved, through the whole continent of South America.

The climate and ſoil of Spaniſh America vary greatly, from the hot burning ſand and ſmoking ſwamp in the northern parts in the torrid zone, to the cold region of the ſouthern part, near Cape Horn.

The iſlands of South America are Terra del Fuego, the Falkland Iſlands, and the iſland of Juan Fernandes; the latter of which gave riſe to the famous ſtory of Robinſon Cruſoe, from one Alexander Selkirk, mariner, native of Scotland, who was put aſhore on this iſland by his captain, in the year 1697, and diſcovered by Woodes Rogers in the year 1709, who took him on board, and brought him to Europe; after having been on this uninhabited iſland for twelve years.

The number of inhabitants in the known parts of the world, computed at a medium from the beſt calculations, are about nine hundred and fifty-three millions; viz.

Europe contains	153	millions
Aſia - - -	500	
Africa - - -	150	
America - -	150	
	953	

CHAP. XIV.

OF ASTRONOMY.

SECT. I.

OF THE PRIMARY PLANETS.

BEFORE I proceed to the deſcription of the Primary Planets, it will be neceſſary to take a view of the ſolar ſyſtem, with the order and economy of the motions and courſes of thoſe planets.

The ſyſtems which have been moſt generally received in aſtronomy, are the *Ptolomaic*, the *Copernican*, *Pythagorean*, and the *Tychonic*.

The *Ptolomaic* ſyſtem, ſo called from its inventor Ptolomy, ſuppoſes the earth to be placed at reſt in the centre of the univerſe, and the heavens revolving about it from eaſt to weſt, in the ſpace of twenty-four hours, and by this motion carrying the ſun, ſtars, and planets, completely round the earth in that ſpace of time.

It was this ſyſtem which Ariſtotle, Hipparchus, and moſt of the philoſophers of antiquity defended ſo ſtrenuouſly, and was followed by the whole world for many ages, and longer retained in many learned univerſities. But latter improvements and more evident demonſtrations have now utterly exploded it.

In the *Tychonic* ſyſtem, invented by Tycho, the Dane, the earth is alſo ſuppoſed to be fixed in the centre of the univerſe, and all the heavenly bodies performing a revolution round it in the ſpace of twenty-four hours, as in the Ptolomaic ſyſtem; but with this difference, that it allows a monthly motion to the moon round the earth, and alſo the proper motions of the ſatellites about Jupiter and Saturn. It alſo ſuppoſes the ſun to be the centre of all the primary planets: the primary planets being carried round the ſun in their reſpective periods, while the ſun, with all the aforeſaid planets, revolve round the earth every twenty-four hours. But this ſyſtem was ſo inconſiſtent with obſervations, that it had but few followers. It was therefore altered by Longomontanus and others, who allowed the diurnal motion of the earth on its own axis, but denied its annual motion round the ſun. This improved hypotheſis is called the *ſemi-tychonic* ſyſtem.

But theſe ſyſtems have now given place to that called the *Copernican* ſyſtem, which undoubtedly is the moſt ancient in the world. It was firſt introduced into Greece and Italy by Pythagoras, and from him called the Pythagorean ſyſtem. It was adopted by Philolaus, Plato, Archimedes, and all the moſt ancient philoſophers, but was at length loſt under the Peripatetic philoſophy, and reſtored again about the year 1500, by Nic. Copernicus.

This ſyſtem has been proved by the moſt evident demonſtrations to be the only true one. I ſhall therefore confine myſelf to the deſcription of this alone, and the phenomena that ariſe from it.

Fig. 1. Plate 15, is a repreſentation of this ſyſtem, where the ſeven concentric circles, marked Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and Georgian Sidus, repreſent the orbits of theſe ſeven primary planets, performing each its annual rotation round the ſun, which is placed in the centre. The next two circles repreſent the twelve ſigns of the zodiac, with all its diviſions into thirty degrees in each ſign. And, laſtly, the two next outer circles ſhow the twelve calendar months of the year, with their diviſions into days. Of each of which in their order.

A planet, in the literal ſenſe of the word, ſignifies a wanderer, or wandering ſtar; and is therefore uſed in contradiſtinction to the ſtars which we call fixed. It is a celeſtial body, globular and opake, and revolving around the ſun, or ſome other planet, as a centre, at leaſt as a focus to its orbit, which always has a moderate degree of eccentricity, ſo that it is never much further from the ſun, or centre of motion, at one time than at another, in proportion to the diameter of its orbit.

The planets are either primary or ſecondary.

Primary planets, ſometimes called planets, by way of eminence, are thoſe ſeven above deſcribed, which move round the ſun, as their common centre, or focus of their orbits.

Secondary planets, or ſatellites, are ſuch as move round ſome primary planet, as their focus, in the ſame manner as the primary planets move round the ſun. Such are the moon, which moves round the earth; and the four moons of Jupiter, the ſeven moons of Saturn, and the ſix of the Georgian Sidus.

The primary planets are divided into ſuperior and inferior: the ſuperior planets are thoſe that perform their revolutions round the ſun at a greater diſtance from the ſun, than the

Earth is, as Mars, Jupiter, Saturn, and the Georgian. The inferior planets are thoſe included within the orbit of the earth, as Venus and Mercury; as may be ſeen in the figure.

The planets were formerly repreſented by the ſame characters which the chemiſts made uſe of, to repreſent their metals by, on account of a ſuppoſed analogy between the planets and thoſe metals; and theſe characters are ſtill uſed, to avoid confuſion.

Thus, *Mercury*, anciently called the meſſenger of the Gods, was ſignified by the character ☿; which ſtood for the metal mercury, and alſo bore a rude reſemblance to the heathen deity of that name, being a man with wings on his head and feet.

Venus, the next planet in order, ſo named from the Goddeſs of Love, was characterized by ♀, for the figure of a woman; and denoted the metal copper.

Tellus, or the Earth, was characterized by ⊖; and is the third planet diſtant from the Sun.

Mars, or the God of War, was denoted by ♂, and repreſented iron; and ſuppoſed to bear a reſemblance to a man holding out a ſpear.

Jupiter, the chief of the heathen gods, was marked ♃, to repreſent thunderbolts; and ſignified the metal tin.

Saturn, the father of the gods, was repreſented by ♄, to reſemble an old man, ſupporting himſelf with a ſtaff; the ſame character being uſed for the metal lead.

The *Georgian*, or Herſchel, is denoted by ♅, the initial of Dr. Herſchel's name, with a croſs for the Chriſtian planet.

The *orbit* of a *planet* is the path it deſcribes in going round the Sun; they are repreſented in the figure by concentric circles. The Earth's orbit is called the ecliptic.

Kepler was the firſt aſtronomer who diſcovered that the orbits of the planets were not circular, but of an elliptic form, in the form of the figure (fig. 2), having the Sun in one of the foci thereof: and he further diſcovered theſe two primary laws, from hence called *Kepler's Laws*: viz. that a radius drawn from the centre of the Sun to the centre of the planet, always deſcribes equal areas in equal times; or, which is the ſame thing, in unequal terms, it deſcribes areas proportional to thoſe times: and the ſquares of the periodical times of the planets are as the cubes of the mean diſtances of the planets from their centres.

Thus, let *d e f* (fig. 2) be the orbit of a planet, or an ellipſe, in which *c* is the centre, and *a b* the two foci of the ellipſe, in one of which foci the ſun is placed, as ſuppoſe *b*: then if the area *h b g* be equal to the area *b g f*, the planet will be as long in deſcribing that part of its orbit from *h* to *g*, as from *g* to *f*. A ſtraight line *e f* drawn through the two foci of the orbit is called the line of the apſides, and the point *e*, being the fartheſt from the Sun, or centre of motion, is called the higher apſis, or *aphelion*; and the point *f*, being that point of the line next the Sun, or centre of motion, is called the lower apſis, or *perihelion*; therefore, when the planet is in its aphelion, its motion is the ſloweſt of all, and when in the perihelion the quickeſt.

The axis of a planet is an imaginary line, ſuppoſed to be drawn through the centre, about which the planet performs a diurnal rotation.

The method of deſcribing an ellipſe is this: having fixed two pins, or points, *a b*, with a piece of thread doubled, and a pen, or pencil, *d*, in the double of the thread, deſcribe the ellipſe *d e f*, keeping the thread extended to the full length by the pencil, *d*. And note: the two fixed points *a* and *b*, which hold the thread, are always the two foci of the ellipſe; and the nigher theſe two foci are together, the nearer the ellipſe will approach to the form of a circle; and the farther they are diſtant from each other, the farther the ellipſe departs from a circular form. The diſtance *a c*, or *c b*, that is, the diſtance from the centre to either focus, is called the eccentricity.

The orbits of the planets are not all in the ſame plane; neither are any two of them in the ſame plane: but the plane of the ecliptic, or Earth's orbit, interſects the plane of the orbit of every other planet in a right line, which paſſes through the Sun, called the line of the nodes; and the points of interſection of the orbits are called the nodes.

The different inclinations of the axis of a planet to the plane of its orbit are the cauſe of the different ſeaſons of the year in that planet; and the diurnal rotation of the planet round its axis produces the ſucceſſive changes of the day and night. Thus, in the Earth's orbit (fig. 3), let A repreſent the Earth in the ſummer ſeaſon, on the 21ſt of June. N S is the axis of the Earth, N being the north, and S the ſouth pole, about which the Earth performs her revolution every twenty-four hours. Let *c a* repreſent the path deſcribed by the city of London, in the rotation of the globe round its axis. Now the reaſon why the days are longer than the nights at this ſeaſon of the year in London, will be evident, from a bare inſpection of the figure. For the axis of the Earth being inclined to a perpendicular drawn to the ecliptic, in an angle of twenty-three degrees and a half, and the city of London being ſituated in the fifty-ſecond degree of north latitude, which is nigher the north pole than to the equinoctial, it will not paſs through near ſo great a ſpace of the dark ſide of the Earth, or that ſide oppoſite to the Sun, as it will through the illuminated part of the Earth, or part next to the Sun. The nights, in this ſeaſon of the year, only continue while London deſcribes that part of its track, from *c* to *b*, and from *b* to *c*, on the other ſide: but as ſoon as it arrives at *b* it will be daybreak, and the day continues while London deſcribes that part of its track from *b* to *a*, and from *a* to *b* on the other ſide of the globe; and when it arrives at *a*, the Sun will be on the meridian, or it will be then noon-day. The length of the day, therefore, will be to that of the night, as the diſtance *c b* to that of *b a*.

But when the Earth arrives at B, which is on September the 23d, the axis of the Earth always keeping in the ſame poſition, the days will be equal to the nights; for the illuminated parts of the Earth will extend exactly from the north to the ſouth poles; and therefore, in the paſſage of the Earth from A to B, the length of the days will gradually decreaſe in the northern latitudes, but increaſe in the ſouthern latitudes.

Fig. 1. *Mercury* is the ſmalleſt of all the primary planets, and neareſt the Sun; performing his revolution in a leſs ſpace of time than any of the reſt, round the Sun, and with a very rapid motion. This occaſioned the Greeks to give it its preſent name and character, calling it, as before obſerved, the meſſenger of the gods.

The mean diſtance of this planet from the Sun, compared with that of the Earth from the Sun, is as 387 to 1000; therefore, his diſtance from the Sun is about 37,000,000 miles, or little more than one third of the Earth's diſtance from the Sun. Hence, the diameter of the Sun, ſeen from Mercury, will appear near three times as large as when ſeen from the Earth; conſequently, that planet receives about ſeven times as much heat and light from the Sun, as this Earth does.—This is a degree of heat ſufficient to make water boil.

The diameter of Mercury is not one third of that of the Earth, or about 2600 miles; therefore, its ſurface is nearly one ninth, and its ſolidity one twenty-ſeventh of that of the Earth.

The orbit of Mercury is inclined to the plane of the ecliptic, or Earth's orbit, in an angle of 6 degrees 54 minutes. He performs one entire revolution round the Sun in the ſpace of 87 days 23 hours 15½ minutes; therefore, the ſummers and winters in that planet cannot be more than 44 days each. His greateſt elongation from the Sun, that is, the greateſt diſtance that he is ſeen by us to depart from the Sun, is 28 degrees 20 minutes: the eccentricity of his orbit is one fifth of his proportional mean diſtance from the Sun, which is far greater than that of any of the other planets; and the pace with which he moves in his orbit is at the amazing rate of about 95,000 miles in an hour.

The place of his aphelion is 14 degrees 13 minutes of Sagittary; the place of his aſcending node is 15 degrees 47 minutes of Taurus, and conſequently that of the deſcending node, 15 degrees and 47 minutes of Scorpio.

The length of his days, or rotation on his own axis, inclination of his axis to his orbit, gravity on his ſurface, denſity and quantity of matter, are all unknown.

Mercury is obſerved to appear with various phaſes, like the Moon; varying according to his various poſitions with regard to the Earth and Sun; therefore he never appears with a full face towards us, except when he is too near the Sun to be diſtinctly ſeen; for his enlightened face is always towards the Sun. From theſe obſervations it is plain, that Mercury, like all the other planets, is a dark opake body, having no light of his own, but what he receives from the Sun; for, if he had, he would always appear completely round,

The beſt time to make an obſervation on this planet, is when it is ſeen on the Sun's diſk, called its tranſit; at which time it paſſes before the Sun, and appears like a little round black ſpot on the Sun's ſurface, eclipſing a ſmall part thereof, and is only viſible through a teleſcope. And as theſe tranſits can happen only when the Earth and Mercury are both in the ſame node of Mercury, (that is, on the 6th of November, and the 4th of May;) and at which time Mercury muſt alſo be in an inferior conjunction, it will follow, that at theſe times, when Mercury is in the inferior conjunction with the Sun, he will appear to paſs over the Sun's diſk.

But in all the other parts of his orbit, he will never make a tranſit over the Sun, though he may be in an inferior conjunction, becauſe he goes either above or below the face of the Sun. Gaſſendi, in November 1631, firſt took an obſervation of this kind; and Mr. Whiſton has given a liſt of ſeveral tranſits of Mercury: viz.—November 12th, at 3 hours 44 minutes, afternoon, in 1782; May 4th, at 6 hours 57 minutes, forenoon, 1786; November 5th, 3 hours 55 minutes, afternoon, 1789; and the 7th, 2 hours 34 minutes, afternoon, 1799.

Venus, the brighteſt, and, to appearance, the largeſt of all the planets, is the next inferior planet, and is diſtinguiſhed from all the reſt by her brightneſs, and white appearance: her light is ſo conſiderable, that in a duſky place ſhe will often cauſe an object to project a ſenſible ſhadow; and her brightneſs is ſo great, as often to render her viſible in the day-time.

As Venus moves round the Sun in an orbit within the Earth's orbit, like Mercury, ſhe can never be ſeen in oppoſition to the Sun, though ſhe departs farther from him than Mercury does; her greateſt diſtance from the Sun being about 47 degrees 48 minutes, as ſeen from the Earth.

When ſhe is on the weſt of the Sun, which happens from her inferior to her ſuperior conjunction with him, ſhe riſes in the morning before the Sun, and is then called the *morning ſtar;* when ſhe is to the eaſt of the Sun, which is from her ſuperior to her inferior conjunction, ſhe ſets after the Sun in the evening, and is then called the *evening ſtar.*

The diameter of Venus is nearly equal to that of the Earth, being about 7687 miles; her apparent mean diameter, ſeen from the Earth, is 58 ſeconds; but her apparent diameter, ſeen from the Sun, or her horizontal parallax, is only 30 ſeconds: her diſtance from the Sun 68,000,000 miles; her eccentricity $\frac{7}{1000}$ of her diſtance, or near 476,000 miles, The inclination of her orbit to the plane of the ecliptic is 3 degrees 23 minutes; the points of their interſections, or nodes, are in 14 degrees of Gemini and Sagittary. The place of her aphelion is 9 degrees 38 minutes of Aquarius. Her axis is inclined to her orbit in an angle of 75 degrees: her periodical courſe round the Sun is performed in 224 days 16 hours 49 minutes. The diurnal rotation round her axis is not certainly known: Caſſini makes it 23 hours, but others make it much more.

When Venus is obſerved through a good teleſcope, ſhe is perceived to have various phaſes and changes, like thoſe of the Moon; her illuminated parts being conſtantly turned towards the Sun.

Dr. Herſchel made many obſervations on this planet, between the years 1777 and 1793. The reſults of his obſervations are, that this planet has a revolution about its own axis, but the periodical time of which he was not able to aſcertain; that the poſition of its axis cannot certainly be known; that the planet's atmoſphere is very conſiderable: the ſurface of the planet he alſo found to be diverſified with hills and vallies, and other inequalities. But the atmoſphere of the planet appeared ſo denſe, or ſome other obſtruction in the region of the planet prevented him, as he ſays, from having a particular view of the mountains, though aſſiſted by the beſt inſtruments.

The tranſits of Venus happen but ſeldom. One of theſe tranſits was ſeen in England, in the year 1639, by Mr. Horrox and Mr. Crabtree: two more were ſeen in the laſt century; the one, June the 6th, 1761; and the other in June 1769. There will not happen another till the year 1874.

In all other reſpects Venus has the ſame appearance to us regularly every eight years; that is, her conjunctions, elongations, times of riſing and ſetting, being very nearly the ſame, and on the ſame days.

Some aſtronomers have diſcovered, or imagined they have diſcovered, a ſatellite belonging to Venus: of this number was Caſſini, who, with a teleſcope of 34 feet, in the years 1672 and 1686, thought he ſaw a ſatellite move round this planet, at the diſtance of about three fifths of Venus's diameters. And Mr. Short, in 1740, with a reflector of 16½ inches focus, perceived a ſmall ſtar near Venus: and with another teleſcope of the ſame focus, and magnifying to the 60

60th power, he found its diſtance from Venus about 10 minutes; and with a glaſs magnifying to the 240th power, he obſerved the different phaſes of this ſatellite; and its diameter appeared to be near one third of that of Venus. And ſeveral other acute obſervers have imagined they diſcovered the ſame thing.

The *Earth*, the next primary planet in order from the Sun, is our habitation. It performs its revolution round the Sun, at the diſtance of 95,000,000 miles. Like all the other primary planets, it has both a diurnal and annual motion:—its diurnal motion is that by which it turns round its own axis in the ſpace of 24 hours nearly, from weſt to eaſt, and thereby cauſing the continual ſucceſſion of day and night. Its annual motion is that by which it is carried round the Sun in its own orbit, and between the orbits of Venus and Mars, having the orbits of Venus and Mercury within its own orbit, or between it and the Sun, which is in the centre; and thoſe of Mars, Jupiter, Saturn, the Georgian, &c. without, or above it, which are therefore called the ſuperior planets, and Mercury and Venus the inferior ones. This annual motion is accompliſhed in the ſpace of a year, or 365 days 6 hours, or rather 365 days, 5 hours, 48 minutes, 48 ſeconds:—this is called the tropical year. But the time the Earth takes to perform its annual revolution, from any fixed ſtar to the ſame again, as ſeen from the Sun, is 365 days, 6 hours, 9 minutes, 17 ſeconds, which is the ſidereal year. The figure of the Earth's orbit, as that of all the other planets, is elliptical; the eccentricity of the orbit, or diſtance of the Sun in the focus from the centre of the orbit, is about one ſixtieth part of the mean diſtance of the Earth from the centre. The Earth, as well as all the other planets, performs its annual revolution according to the natural order of the ſigns.

By the diurnal rotation of the Earth on its axis the ſame appearance is produced as if it were fixed, and the ſun and ſtars moved round them every 24 hours. For their turning round from weſt to eaſt, cauſes the Sun, and all the heavenly bodies, to appear to move the contrary way, or from eaſt to weſt, as is imagined to be the caſe by the vulgar and illiterate. Thus, when, by the rotation of the earth, the obſerver is brought to that part where he ſees the Sun or a ſtar juſt riſing above the horizon, in the eaſt, they are ſaid to be riſing; and as the Earth continues to move, other ſtars will appear to riſe, and advance weſtward; and when, by the motion of the Earth, the obſerver is brought directly under the Sun or ſtar, they are then ſaid to be on the meridian; after which, by a continuance of the ſame motion, the obſerver is brought to the eaſtern ſide of the Earth, when the Sun or ſtar will appear to ſet on the weſtern ſide thereof.

The diameter of the Earth is 7957¾ miles; though ſome make it 7964 miles.

The Earth, throughout its annual orbit, always keeps its axis parallel to itſelf in every part of its orbit; thereby occaſioning all the varieties of different ſeaſons of the year, and the different length of the days and nights, as ſeen in *fig.* 3; the Sun enlightening more of the north polar parts, at one ſeaſon of the year, and more of the ſouthern parts at the oppoſite ſeaſon of the year; thus producing the different degrees of heat in the different ſeaſons; for in the ſummer ſeaſon the heat of the Sun is increaſed by two cauſes, viz. 1. By the Sun's rays being more vertical to thoſe ſituated in the north parts of the globe; and, conſequently, the heat is not diminiſhed in paſſing through ſo great a portion of the atmoſphere, as when the rays come more oblique. And, 2dly, having the light, and conſequently the heat of the Sun for a longer ſpace of time in the ſummer than in the winter. All which will be evident from an inſpection of the figure.

What is here aſſerted, concerning the Earth, holds good with

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The primary planets are divided into ſuperior and inferior: the ſuperior planets are thoſe that perform their revolutions round the ſun at a greater diſtance from the ſun, than the

Earth is, as Mars, Jupiter, Saturn, and the Georgian. The inferior planets are thoſe included within the orbit of the earth, as Venus and Mercury; as may be ſeen in the figure.

The planets were formerly repreſented by the ſame characters which the chemiſts made uſe of, to repreſent their metals by, on account of a ſuppoſed analogy between the planets and thoſe metals; and theſe characters are ſtill uſed, to avoid confuſion.

Thus, *Mercury*, anciently called the meſſenger of the Gods, was ſignified by the character ☿; which ſtood for the metal mercury, and alſo bore a rude reſemblance to the heathen deity of that name, being a man with wings on his head and feet.

Venus, the next planet in order, ſo named from the Goddeſs of Love, was characterized by ♀, for the figure of a woman; and denoted the metal copper.

Tellus, or the Earth, was characterized by ⊖; and is the third planet diſtant from the Sun.

Mars, or the God of War, was denoted by ♂, and repreſented iron; and ſuppoſed to bear a reſemblance to a man holding out a ſpear.

Jupiter, the chief of the heathen gods, was marked ♃, to repreſent thunderbolts; and ſignified the metal tin.

Saturn, the father of the gods, was repreſented by ♄, to reſemble an old man, ſupporting himſelf with a ſtaff; the ſame character being uſed for the metal lead.

The *Georgian*, or Herſchel, is denoted by ♅, the initial of Dr. Herſchel's name, with a croſs for the Chriſtian planet.

The *orbit* of a *planet* is the path it deſcribes in going round the Sun; they are repreſented in the figure by concentric circles. The Earth's orbit is called the ecliptic.

Kepler was the firſt aſtronomer who diſcovered that the orbits of the planets were not circular, but of an elliptic form, in the form of the figure (fig. 2), having the Sun in one of the foci thereof: and he further diſcovered theſe two primary laws, from hence called *Kepler's Laws:* viz. that a radius drawn from the centre of the Sun to the centre of the planet, always deſcribes equal areas in equal times; or, which is the ſame thing, in unequal terms, it deſcribes areas proportional to thoſe times: and the ſquares of the periodical times of the planets are as the cubes of the mean diſtances of the planets from their centres.

Thus, let *d e f* (fig. 2) be the orbit of a planet, or an ellipſe, in which *c* is the centre, and *a b* the two foci of the ellipſe, in one of which foci the ſun is placed, as ſuppoſe *b*: then if the area *b b g* be equal to the area *b g f*, the planet will be as long in deſcribing that part of its orbit from *b* to *g*, as from *g* to *f*. A ſtraight line *e f* drawn through the two foci of the orbit is called the line of the apſides, and the point *e*, being the fartheſt from the Sun, or centre of motion, is called the higher apſis, or *aphelion;* and the point *f*, being that point of the line next the Sun, or centre of motion, is called the lower apſis, or *perihelion;* therefore, when the planet is in its aphelion, its motion is the ſloweſt of all, and when in the perihelion the quickeſt.

The axis of a planet is an imaginary line, ſuppoſed to be drawn through the centre, about which the planet performs a diurnal rotation.

The method of deſcribing an ellipſe is this: having fixed two pins, or points, *a b*, with a piece of thread doubled, and a pen, or pencil, *d*, in the double of the thread, deſcribe the ellipſe *d e f*, keeping the thread extended to the full length by the pencil, *d*. And note: the two fixed points *a* and *b*, which hold the thread, are always the two foci of the ellipſe; and the nigher theſe two foci are together, the nearer the ellipſe will approach to the form of a circle; and the farther they are diſtant from each other, the farther the ellipſe departs from a circular form. The diſtance *a c*, or *c b*, that is, the diſtance from the centre to either focus, is called the eccentricity.

The orbits of the planets are not all in the ſame plane; neither are any two of them in the ſame plane: but the plane of the ecliptic, or Earth's orbit, interſects the plane of the orbit of every other planet in a right line, which paſſes through the Sun, called the line of the nodes; and the points of interſection of the orbits are called the nodes.

The different inclinations of the axis of a planet to the plane of its orbit are the cauſe of the different ſeaſons of the year in that planet; and the diurnal rotation of the planet round its axis produces the ſucceſſive changes of the day and night. Thus, in the Earth's orbit (fig. 3), let A repreſent the Earth in the ſummer ſeaſon, on the 21ſt of June. N S is the axis of the Earth, N being the north, and S the ſouth pole, about which the Earth performs her revolution every twenty-four hours. Let *c a* repreſent the path deſcribed by the city of London, in the rotation of the globe round its axis. Now the reaſon why the days are longer than the nights at this ſeaſon of the year in London, will be evident, from a bare inſpection of the figure. For the axis of the Earth being inclined to a perpendicular drawn to the ecliptic, in an angle of twenty-three degrees and a half, and the city of London being ſituated in the fifty-ſecond degree of north latitude, which is nigher the north pole than to the equinoctial, it will not paſs through near ſo great a ſpace of the dark ſide of the Earth, or that ſide oppoſite to the Sun, as it will through the illuminated part of the Earth, or part next to the Sun. The nights, in this ſeaſon of the year, only continue while London deſcribes that part of its track, from *c* to *b*, and from *b* to *c*, on the other ſide: but as ſoon as it arrives at *b* it will be daybreak, and the day continues while London deſcribes that part of its track from *b* to *a*, and from *a* to *b* on the other ſide of the globe; and when it arrives at *a*, the Sun will be on the meridian, or it will be then noon-day. The length of the day, therefore, will be to that of the night, as the diſtance *c b* to that of *b a*.

But when the Earth arrives at B, which is on September the 23d, the axis of the Earth always keeping in the ſame poſition, the days will be equal to the nights; for the illuminated parts of the Earth will extend exactly from the north to the ſouth poles; and therefore, in the paſſage of the Earth from A to B, the length of the days will gradually decreaſe in the northern latitudes, but increaſe in the ſouthern latitudes.

Fig. 1. *Mercury* is the ſmalleſt of all the primary planets, and neareſt the Sun; performing his revolution in a leſs ſpace of time than any of the reſt, round the Sun, and with a very rapid motion. This occaſioned the Greeks to give it its preſent name and character, calling it, as before obſerved, the meſſenger of the gods.

The mean diſtance of this planet from the Sun, compared with that of the Earth from the Sun, is as 387 to 1000; therefore, his diſtance from the Sun is about 37,000,000 miles, or little more than one third of the Earth's diſtance from the Sun. Hence, the diameter of the Sun, ſeen from Mercury, will appear near three times as large as when ſeen from the Earth; conſequently, that planet receives about ſeven times as much heat and light from the Sun, as this Earth does.—This is a degree of heat ſufficient to make water boil.

The diameter of Mercury is not one third of that of the Earth, or about 2600 miles; therefore, its ſurface is nearly one ninth, and its ſolidity one twenty-ſeventh of that of the Earth.

The orbit of Mercury is inclined to the plane of the ecliptic, or Earth's orbit, in an angle of 6 degrees 54 minutes. He performs one entire revolution round the Sun in the ſpace of 87 days 23 hours 15½ minutes; therefore, the ſummers and winters in that planet cannot be more than 44 days each. His greateſt elongation from the Sun, that is, the greateſt diſtance that he is ſeen by us to depart from the Sun, is 28 degrees 20 minutes: the eccentricity of his orbit is one fifth of his proportional mean diſtance from the Sun, which is far greater than that of any of the other planets; and the pace with which he moves in his orbit is at the amazing rate of about 95,000 miles in an hour.

The place of his aphelion is 14 degrees 13 minutes of Sagittary; the place of his aſcending node is 15 degrees 47 minutes of Taurus, and conſequently that of the deſcending node, 15 degrees and 47 minutes of Scorpio.

The length of his days, or rotation on his own axis, inclination of his axis to his orbit, gravity on his ſurface, denſity and quantity of matter, are all unknown.

Mercury is obſerved to appear with various phaſes, like the Moon; varying according to his various poſitions with regard to the Earth and Sun; therefore he never appears with a full face towards us, except when he is too near the Sun to be diſtinctly ſeen; for his enlightened face is always towards the Sun. From theſe obſervations it is plain, that Mercury, like all the other planets, is a dark opake body, having no light of his own, but what he receives from the Sun; for, if he had, he would always appear completely round,

The beſt time to make an obſervation on this planet, is when it is ſeen on the Sun's diſk, called its tranſit; at which time it paſſes before the Sun, and appears like a little round black ſpot on the Sun's ſurface, eclipſing a ſmall part thereof, and is only viſible through a teleſcope. And as theſe tranſits can happen only when the Earth and Mercury are both in the ſame node of Mercury, (that is, on the 6th of November, and the 4th of May;) and at which time Mercury muſt alſo be in an inferior conjunction, it will follow, that at theſe times, when Mercury is in the inferior conjunction with the Sun, he will appear to paſs over the Sun's diſk.

But in all the other parts of his orbit, he will never make a tranſit over the Sun, though he may be in an inferior conjunction, becauſe he goes either above or below the face of the Sun. Gaſſendi, in November 1631, firſt took an obſervation of this kind; and Mr. Whiſton has given a liſt of ſeveral tranſits of Mercury: viz.—November 12th, at 3 hours 44 minutes, afternoon, in 1782; May 4th, at 6 hours 57 minutes, forenoon, 1786; November 5th, 3 hours 55 minutes, afternoon, 1789; and the 7th, 2 hours 34 minutes, afternoon, 1799.

Venus, the brighteſt, and, to appearance, the largeſt of all the planets, is the next inferior planet, and is diſtinguiſhed from all the reſt by her brightneſs, and white appearance: her light is ſo conſiderable, that in a duſky place ſhe will often cauſe an object to project a ſenſible ſhadow; and her brightneſs is ſo great, as often to render her viſible in the day-time.

As Venus moves round the Sun in an orbit within the Earth's orbit, like Mercury, ſhe can never be ſeen in oppoſition to the Sun, though ſhe departs farther from him than Mercury does; her greateſt diſtance from the Sun being about 47 degrees 48 minutes, as ſeen from the Earth.

When ſhe is on the weſt of the Sun, which happens from her inferior to her ſuperior conjunction with him, ſhe riſes in the morning before the Sun, and is then called the *morning ſtar;* when ſhe is to the eaſt of the Sun, which is from her ſuperior to her inferior conjunction, ſhe ſets after the Sun in the evening, and is then called the *evening ſtar.*

The diameter of Venus is nearly equal to that of the Earth, being about 7687 miles; her apparent mean diameter, ſeen from the Earth, is 58 ſeconds; but her apparent diameter, ſeen from the Sun, or her horizontal parallax, is only 30 ſeconds: her diſtance from the Sun 68,000,000 miles; her eccentricity $\frac{7}{1000}$ of her diſtance, or near 476,000 miles, The inclination of her orbit to the plane of the ecliptic is 3 degrees 23 minutes; the points of their interſections, or nodes, are in 14 degrees of Gemini and Sagittary. The place of her aphelion is 9 degrees 38 minutes of Aquarius. Her axis is inclined to her orbit in an angle of 75 degrees: her periodical courſe round the Sun is performed in 224 days 16 hours 49 minutes. The diurnal rotation round her axis is not certainly known: Caſſini makes it 23 hours, but others make it much more.

When Venus is obſerved through a good teleſcope, ſhe is perceived to have various phaſes and changes, like thoſe of the Moon; her illuminated parts being conſtantly turned towards the Sun.

Dr. Herſchel made many obſervations on this planet, between the years 1777 and 1793. The reſults of his obſervations are, that this planet has a revolution about its own axis, but the periodical time of which he was not able to aſcertain; that the poſition of its axis cannot certainly be known; that the planet's atmoſphere is very conſiderable: the ſurface of the planet he alſo found to be diverſified with hills and vallies, and other inequalities. But the atmoſphere of the planet appeared ſo denſe, or ſome other obſtruction in the region of the planet prevented him, as he ſays, from having a particular view of the mountains, though aſſiſted by the beſt inſtruments.

The tranſits of Venus happen but ſeldom. One of theſe tranſits was ſeen in England, in the year 1639, by Mr. Horrox and Mr. Crabtree: two more were ſeen in the laſt century; the one, June the 6th, 1761; and the other in June 1769. There will not happen another till the year 1874.

In all other reſpects Venus has the ſame appearance to us regularly every eight years; that is, her conjunctions, elongations, times of riſing and ſetting, being very nearly the ſame, and on the ſame days.

Some aſtronomers have diſcovered, or imagined they have diſcovered, a ſatellite belonging to Venus: of this number was Caſſini, who, with a teleſcope of 34 feet, in the years 1672 and 1686, thought he ſaw a ſatellite move round this planet, at the diſtance of about three fifths of Venus's diameters. And Mr. Short, in 1740, with a reflector of 16½ inches focus, perceived a ſmall ſtar near Venus: and with another teleſcope of the ſame focus, and magnifying to the

60th power, he found its diſtance from Venus about 10 minutes; and with a glaſs magnifying to the 240th power, he obſerved the different phaſes of this ſatellite; and its diameter appeared to be near one third of that of Venus. And ſeveral other acute obſervers have imagined they diſcovered the ſame thing.

The *Earth*, the next primary planet in order from the Sun, is our habitation. It performs its revolution round the Sun, at the diſtance of 95,000,000 miles. Like all the other primary planets, it has both a diurnal and annual motion:—its diurnal motion is that by which it turns round its own axis in the ſpace of 24 hours nearly, from weſt to eaſt, and thereby cauſing the continual ſucceſſion of day and night. Its annual motion is that by which it is carried round the Sun in its own orbit, and between the orbits of Venus and Mars, having the orbits of Venus and Mercury within its own orbit, or between it and the Sun, which is in the centre; and thoſe of Mars, Jupiter, Saturn, the Georgian, &c. without, or above it, which are therefore called the ſuperior planets, and Mercury and Venus the inferior ones. This annual motion is accompliſhed in the ſpace of a year, or 365 days 6 hours, or rather 365 days, 5 hours, 48 minutes, 48 ſeconds:—this is called the tropical year. But the time the Earth takes to perform its annual revolution, from any fixed ſtar to the ſame again, as ſeen from the Sun, is 365 days, 6 hours, 9 minutes, 17 ſeconds, which is the ſidereal year. The figure of the Earth's orbit, as that of all the other planets, is elliptical; the eccentricity of the orbit, or diſtance of the Sun in the focus from the centre of the orbit, is about one ſixtieth part of the mean diſtance of the Earth from the centre. The Earth, as well as all the other planets, performs its annual revolution according to the natural order of the ſigns.

By the diurnal rotation of the Earth on its axis the ſame appearance is produced as if it were fixed, and the ſun and ſtars moved round them every 24 hours. For their turning round from weſt to eaſt, cauſes the Sun, and all the heavenly bodies, to appear to move the contrary way, or from eaſt to weſt, as is imagined to be the caſe by the vulgar and illiterate. Thus, when, by the rotation of the earth, the obſerver is brought to that part where he ſees the Sun or a ſtar juſt riſing above the horizon, in the eaſt, they are ſaid to be riſing; and as the Earth continues to move, other ſtars will appear to riſe, and advance weſtward; and when, by the motion of the Earth, the obſerver is brought directly under the Sun or ſtar, they are then ſaid to be on the meridian; after which, by a continuance of the ſame motion, the obſerver is brought to the eaſtern ſide of the Earth, when the Sun or ſtar will appear to ſet on the weſtern ſide thereof.

The diameter of the Earth is 7957¾ miles; though ſome make it 7964 miles.

The Earth, throughout its annual orbit, always keeps its axis parallel to itſelf in every part of its orbit; thereby occaſioning all the varieties of different ſeaſons of the year, and the different length of the days and nights, as ſeen in *fig.* 3; the Sun enlightening more of the north polar parts, at one ſeaſon of the year, and more of the ſouthern parts at the oppoſite ſeaſon of the year; thus producing the different degrees of heat in the different ſeaſons; for in the ſummer ſeaſon the heat of the Sun is increaſed by two cauſes, viz. 1. By the Sun's rays being more vertical to thoſe ſituated in the north parts of the globe; and, conſequently, the heat is not diminiſhed in paſſing through ſo great a portion of the atmoſphere, as when the rays come more oblique. And, 2dly, having the light, and conſequently the heat of the Sun for a longer ſpace of time in the ſummer than in the winter. All which will be evident from an inſpection of the figure.

What is here aſſerted, concerning the Earth, holds good with regard to all the other planets, as each of them revolves about an axis, which is not perpendicular to the plane of its orbit, but inclined thereto in a greater or leſs angle; and which axis is always parallel to itſelf in every part of the planet's orbit.

The figure of the Earth, like that of all the other planets, is that of an oblate ſpheroid, which Sir Iſaac Newton demonſtrated to ariſe from the rotation of the Earth about its axis; and by the obſervation and experiments of later aſtronomers, the polar diameter is to the equatorial diameter, as 178 to 179, as affirmed by that great genius.

The abſolute gravity or denſity of the whole maſs of the Earth is to that of the water as 9 is to 2, and to common ſtone as 9 is to 5. Thus, we diſcover the very conſiderable mean denſity of the Earth, which is almoſt double to that of common ſtone; from whence it may be preſumed, that the internal parts of the Earth contain great quantities of metals.

Having the denſity of the Earth, its quantity of matter is eaſily found; being always equal to the product of its denſity multiplied by its magnitude.

The Earth is, moreover, every where ſurrounded by an atmoſphere, which is that large quantity of fluid matter extending over the whole ſurface of the Earth, conſiſting of air, aqueous and other vapours, electric fluid, &c. which ſurround the Earth to a conſiderable height, and partake of all its motions, both annual and diurnal.

The atmoſphere ſerves for innumerable purpoſes, and is even eſſential to both animal and vegetable life; it is this, inſinuating itſelf into all the vacuities of bodies, which cauſes thoſe mutations of generation, corruption, diſſolution, &c.

This atmoſphere, like all other matter, has the properties of weight and preſſure, the quantity of which is now pretty well known, and is found by the barometer to be equal to an equal column of quickſilver, of 30 inches high; therefore, becauſe a cubical inch of quickſilver is found to weigh near half a pound avoirdupois, a column of quickſilver of 30 inches in height, whoſe baſe is one ſquare inch, will weigh near 15 pounds; from whence it follows, that the weight of the atmoſphere on every ſquare inch of ſurface on the face of the Earth is alſo 15 pounds. Thus it appears, that the preſſure upon the human body muſt be very conſiderable; for, as every ſquare inch of ſurface ſuſtains a preſſure of 15 pounds, every ſquare foot will ſuſtain 144 times as much, or 2160 pounds.

The atmoſphere has been proved, by late experiments, to become more rare, and of leſs denſity, the farther we remove from the Earth, and that in the following proportion: —At the height of three miles and a half from the Earth, the denſity of the atmoſphere is nearly two times rarer than at the ſurface of the Earth: at the height of ſeven miles, four times rarer; at the height of 14 miles, 16 times rarer; at the height of 21 miles, 64 times rarer; at the height of 28 miles, 256 times rarer; and at the height of 35 miles, 1024 times rarer, &c.

Mars is the firſt of the four ſuperior planets, and placed immediately next above the Earth, including the orbits of the Earth, Venus, and Mercury, within that of his.

The mean diſtance of Mars from the Sun is 1523 of thoſe parts of which the diſtance of the Earth from the Sun is 1000, or upwards of 144,000,000 miles: his eccentricity 142 of thoſe parts of which the Earth is diſtant from the Sun 1000: his mean diameter is 4189 miles. The length of his year, or the period of his completing one revolution round the Sun, is $686\frac{23}{24}$ days. His revolution round his own axis is performed in 24 hours 39 minutes and 22 ſeconds. His mean diameter, ſeen from the Sun, is 11 ſeconds. The inclination of his axis to his orbit is nothing, his axis being perpendicular. The inclination of his orbit to the ecliptic is 1 degree 51 minutes; the place of his aphelion 2 degrees 6 minutes 15 ſeconds of Virgo; his aſcending node is 17 degrees 59 minutes of Taurus.

Dr. Herſchel has made many obſervations on the rotation of this planet about its axis, from which he inferred that the mean diurnal rotation was between 24 hours 39 minutes 5 ſeconds, and 24 hours 39 minutes 22 ſeconds. He alſo obſerved ſeveral ſmall remarkably bright ſpots, near both the poles, which had a ſort of motion. He alſo concludes that the inclination of his axis to the ecliptic is 59 degrees 22 minutes; and the node of the axis to be in 17 degrees 47 minutes of Piſces; the obliquity of the planet's ecliptic 28 degrees 42 minutes, and the point Aries on Mars' ecliptic to anſwer to our 19 degrees 28 minutes of Sagittary.

The figure of Mars is that of an oblate ſpheroid, like the Earth, having his equatorial diameter to the polar one, as 1355 to 1272, or nearly as 16 to 15.

This planet alſo has a conſiderable, but moderate atmoſphere: ſo that its inhabitants probably are, in their nature, ſimilar to the inhabitants of this earth.

Mars always appears with a ruddy, diſturbed light, occaſioned by the nature of his atmoſphere.

When he is in oppoſition to the Sun, that is, when the Earth comes between the Sun and him, he is nearly five times nearer to us than when in conjunction with the Sun, or when the Sun is between him and us; and conſequently, in the former caſe, he appears near five times more large and bright than in the latter caſe.

As he receives his light from the Sun, like the other planets, he muſt neceſſarily have an increaſe and decreaſe apparently, like the Moon; he may alſo be obſerved ſometimes almoſt biſected, when in the quadratures; but he is never ſeen cornicular, as the inferior planets are; which ſhows that his orbit includes that of the Earth, and that he ſhines with a borrowed light.

Between the orbits of Mars and Jupiter there have lately been diſcovered two new planets: viz. one named the *Pallas*, diſcovered March 23, 1802: its eccentricity is greater than that of Mercury: the inclination of its orbit to the ecliptic

33 degrees 39 minutes: its periodical revolution four years five months, and real diameter 95 miles. And the *Ceres de Ferdinand*, diſcovered by Mr. Piazzi, at Palermo, in Sicily, on January 1ſt, 1801: it is not apparently larger than a fixed ſtar of the eighth magnitude; its elements, according to M. Gauſs, are all follow:—inclination of its orbit to the ecliptic 10 degrees 36 minutes 57 ſeconds, aſcending node $2^s\ 21^\circ$, aphelion $10^s\ 26^\circ\ 27'\ 38''$, eccentricity of the orbit .082 parts of its mean diſtance, periodical revolution 1652 days 5 hours nearly; and real diameter, according to Dr. Herſchel, 162 miles.

Jupiter is the next ſuperior planet, and the largeſt of all the planets in the ſolar ſyſtem; in brightneſs, he is next to the planet Venus: his orbit is ſituated between thoſe of Mars and Saturn. His diameter is above 11 times that of the Earth; conſequently his magnitude exceeds the magnitude of the Earth above 1300 times. His annual revolution round the Sun is performed in 4332 days 8 hours 51 minutes 30 ſeconds, or nearly 12 years. His diurnal revolution about his own axis, he performs in the ſhort ſpace of 9 hours 56 minutes, by which motion every part on his equator is carried round at the rate of 26,000 miles in an hour, being about 25 times faſter than the equatorial parts of the Earth move.

The axis of Jupiter is nearly perpendicular to the plane of his orbit; therefore he has no ſenſible change of ſeaſons, except very near the poles. This is wiſely ordered by Providence. For if his axis made any conſiderable angle with the perpendicular of his orbit, juſt ſo many degrees as it was inclined thereto would be near ſix years in darkneſs round each of their poles, in their turn.

The orbit of Jupiter is inclined to the ecliptic in an angle of 1 degree 19 minutes 15 ſeconds: the place of his aphelion is 10 degrees 57 minutes 30 ſeconds of Libra: the place of his aſcending node is 8 degrees 50 minutes of Cancer, and of his deſcending node 8 degrees 50 minutes of Capricorn.

The eccentricity of his orbit is one twentieth part of his mean diſtance from the Sun.

There are ſeveral faint ſhining ſubſtances which ſurround Jupiter, of different dimenſions, called his zones or belts, which are conſtantly changing their ſize and ſituations, and are, therefore, generally believed to be clouds. They have ſometimes appeared of different breadths, at other times of the ſame breadth. Large ſpots have alſo been obſerved in theſe belts; and whenever a belt vaniſhes, as is often the caſe, the ſpots contiguous to it have alſo vaniſhed. Theſe belts have ſometimes been interrupted and broken, and the broken ends of ſuch belts have often been obſerved to revolve round the planet in the ſame time with the ſpots. The ſpots have alſo aſſumed different appearances; ſome of them changing their ſhape from a circular to an oblong form, others uniting together in one, and ſometimes one large one dividing into two or three.

The difference between the equatorial and polar diameters of Jupiter is upwards of 6000 miles, the former being to the latter as 13 to 12.

Jupiter is attended by four moons, or ſatellites, ſome of them larger than our earth, which perform their reſpective revolutions round Jupiter in different periods of time; ſo that there is ſcarcely any part of this great planet but what is enlightened by one or more of theſe moons during the whole night. The periods, diſtances in ſemidiameters of Jupiter, and the angles of the orbits of theſe moons, ſeen from the Earth, are as follow:

Moons.	Periods round Jupiter.			Diſtances in Semidiameters of Jupiter.	Angles of the Orbits.	
	D.	H.	M.			
1	1	18	27	$5\frac{2}{3}$	3′	55″
2	3	13	13	$9\frac{1}{30}$	6	14
3	7	3	42	$14\frac{5}{13}$	9	58
4	16	18	32	$25\frac{3}{10}$	17	30

The three neareſt moons of Jupiter fall within his ſhadow, and are eclipſed once in every revolution; but the orbit of the fourth ſatellite is ſo much inclined, that it paſſes the ſhadow of Jupiter without falling into it two years in every ſix.

Saturn is the next primary planet, and the outermoſt from the Sun, except the Georgian. He ſhines with a feeble light, on account of his great diſtance from the Sun, which is alſo apparently increaſed by his great diſtance from us. This planet has attracted the moſt attention of all the primary planets, on account of his wonderful ring. This ring, or rather a double ring, one within the other, ſurrounds the body of Saturn, at a diſtance equal to the diameter of the planet. Beyond the ring, ſeven moons perform their reſpective revolutions round Saturn. The rings and the moons are all in the ſame plane, and are all dark denſe bodies, and therefore caſt their ſhadows upon each other.

The phenomena of the rings have engaged the attention of all the aſtronomers, ſince their diſcovery; ſome contending it was one entire ring, others dividing it into two or more; but the obſervations of Dr. Herſchel have been more ſatiſfactory on this head than thoſe of his predeceſſors: he divides them into two rings, one within the other: their dimenſions and ſpaces, he ſtates in the following proportion to each other:—

	Miles.
Inſide diameter of ſmaller ring	146,345
Outſide diameter of ditto	184,393
Inner diameter of larger ring	190,248
Outſide diameter of ditto	204,883
Breadth of the inner ring	20,000
Breadth of the outer ring	7,200
Breadth of the vacant ſpace	2,839

This ring revolves in its own plane in 10 hours 32 minutes 15 ſeconds.

From the above ſtatement it appears, that the outſide diameter of the larger ring is almoſt 26 times the diameter of the Earth. This ring is inclined to the plane of the ecliptic in an angle of 30 degrees. When we ſee the ring moſt open, its ſhadow upon the planet is broadeſt; and from that time the ſhadow grows narrower as the ring appears to do to us, until, by the annual motion of Saturn, the Sun comes to the plane of the ring, or even with its edge; which, being then directed towards us, becomes inviſible.

Saturn is found to have certain zones or belts, ſomewhat like thoſe of Jupiter. Dr. Herſchel has diſcovered and demonſtrated that Saturn has a denſe atmoſphere; that he has a revolution about his axis; that his axis is perpendicular to the plane of his rings; that his figure, like that of the other planets, is that of an oblate ſpheroid, the polar diameter being to the equatorial as 10 to 11; that his rings have a revolution in their own plane, their axis being the ſame as that of Saturn.

The annual period of Saturn about the Sun is near 30 years, or 10,761 days 14 hours 36 minutes 45 ſeconds; his diameter is about 79,042 miles, being near ten times that of the Earth; his diſtance from the Sun is about 9½ times that of the Earth.

From his great diſtance from the Sun, ſome have imagined that the portion of light and heat derived from the Sun is not ſufficient for animal life. But that they have a greater portion of light, and conſequently heat, than is at firſt imagined, is evident from the brightneſs of this planet and its ſatellites, in the night-time. Alſo, as the Sun's light to us is 45,000 times as great as that of the full moon, the Sun will afford 500 times as much light to Saturn, as the full moon does to us; and 1600 times as much to Jupiter. Thus, theſe two planets, without any moons to enlighten them, would receive more light from the Sun than might be at firſt imagined; their number of ſatellites, the rings of Saturn, and the nature of their atmoſpheres, may alſo have a conſiderable effect in increaſing their light and heat. For we find that, in our earth, the different degrees of heat do not entirely depend on the rays of the Sun. The inhabitants alſo of thoſe planets are, no doubt, adapted to their ſituations.

Saturn has ſeven ſatellites, or moons, performing their revolutions round him in their reſpective periods, as follows:

Satellites.	Periods.				Diſtances in Semidiamet. of Saturn.	Diſtances in Miles.	Diameters of the Orbits.	
	D.	H.	M.	S.				
1	1	21	18	27	$4\frac{3}{5}$	170,000	1′	27″
2	2	17	41	22	$5\frac{1}{2}$	217,000	1	52
3	4	12	25	12	8	303,000	2	36
4	15	22	41	13	18	704,000	6	18
5	79	7	48	0	54	2,050,000	17	4
6	1	8	53	9	$3\frac{2}{5}$	120,000	1	14
7	0	22	40	46	$2\frac{5}{6}$	91,000	0	57

The four firſt of theſe ſatellites deſcribe ellipſes, like thoſe of the rings, and are alſo in the ſame plane. Their inclination to the ecliptic is from 30 to 31 degrees. The fifth ſatellite deſcribes an orbit which is inclined from 17 to 18 degrees to the orbit of Saturn; the plane of the orbit of this ſatellite lies between the plane of the ecliptic and the planes of the orbits of the other ſatellites. Dr. Herſchel diſcovered that this ſatellite turns once round its own axis in the time that it makes one revolution about the planet Saturn; in which reſpect it reſembles our moon.

The *Georgium Sidus*, or *Herſchel*, is the moſt diſtant of all the primary planets from the Sun; its apparent diameter is about four ſeconds. It is but ſeldom he can be ſeen by the naked eye; but, having his ſituation, he may be plainly ſeen with a good teleſcope, in a clear night.

This planet is twice the diſtance from the Sun that Saturn is, and is nearly 83 years performing his annual courſe. It is 90 times as large as the Earth. The degree of cold in this planet is ſuppoſed to be extreme, as it is computed that the light of the Sun is not above the 300th part of what we enjoy on our earth.

This planet is attended by ſix ſatellites, which perform their revolutions round him as follows:

Satellites.	Periods.			Diſtances in Semi-diameters of the Georgian.
	D.	H.	M.	
1	5	21	25	12½
2	8	17	1	16½
3	10	23	4	19
4	13	11	5	22
5	38	1	49	44
6	107	16	40	88

The orbits of theſe ſatellites are nearly perpendicular to the plane of the ecliptic; and by the beſt obſervations that have been made, it is probable that their magnitudes are equal to, if not greater than, that of Jupiter. The motion of theſe ſix ſatellites is retrograde, or contrary to the order of the ſigns.

Theſe are the primary planets which conſtitute the ſolar ſyſtem, each of them moving, in his own proper orbit, round the Sun, as the common centre. The Sun, therefore, can hardly be conſidered as a planet, though reckoned as ſuch by the ancient aſtronomers; but it may rather be ranked as a fixed ſtar. He has, in every reſpect, the ſame properties, being a fixed luminous body, imparting light and heat to all the planets, both primary and ſecondary, found in his ſyſtem. The reaſon he appears brighter and larger than any of the fixed ſtars, is his nearneſs to the Earth, in compariſon with the great diſtance of the former. For a ſpectator, placed as near to any fixed ſtar as we are to the Earth, would ſee that ſtar, in every reſpect, as large and as bright as the Sun appears to us; and an obſerver, as far diſtant from the Sun as we are from the neareſt fixed ſtar, would ſee the Sun as ſmall as the ſtar appears to us, and would reckon it as one of the ſtars.

Though the Sun is ſaid to be a fixed body, yet he has a revolution round his own axis, which he performs in the courſe of 27 days 12 hours and 20 minutes, which is found, by an obſervation of the ſeveral ſpots to be ſeen on the Sun's diſk, which paſs from the weſtern edge of his diſk to the eaſtern edge thereof, in the ſpace of leſs than 14 days. And theſe ſpots are found to perform one entire revolution round the Sun in the ſpace of 27 days 12 hours and 20 minutes; therefore we reaſonably ſuppoſe, that this is the Sun's proper motion from weſt to eaſt, like that of all the planets.

Philoſophers have been greatly divided in opinion concerning the matter of the Sun; ſome contending it was a ball of fire, from the property of the Sun's rays acting like fire, when collected by concave mirrors or convex lenſes; others, as Boerhaave, maintain the contrary; the particulars of which arguments I have not room to inſert; but the following properties of the Sun are demonſtrated by Sir Iſaac Newton:—1. That the denſity of the Sun's heat and light is ſeven times as great on the planet Mercury as it is with us.—2. That the quantity of matter in the Sun is to that in Jupiter nearly as 1100 is to 1; and that the diſtance of that planet from the Sun is in the ſame ratio to the Sun's ſemidiameter.—3. That the quantity of matter in the Sun is to that in Saturn as 2360 to 1; and the diſtance of Saturn from the

the Sun is in a ratio but little leſs than that of the Sun's ſemidiameter. And hence the common centre of gravity of the Sun and Jupiter is nearly in the ſuperficies of the Sun; and that of the Sun and Saturn a little within it.—4. Hence the common centre of gravity of all the planets cannot be more than the length of the ſolar diameter diſtant from the centre of the Sun. This common centre of gravity is always at reſt; and though the Sun, by the various poſitions of the planets, may be moved every way; yet it cannot recede far from this common centre.—5. The axis of the Sun is inclined to the ecliptic in an angle of 87 degrees 30 minutes nearly. The Sun's apparent diameter being ſenſibly larger in December than in June, the Sun muſt be proportionably nearer to the Earth in winter than in ſummer. In winter, the Earth will be in the perihelion; in the ſummer, in the aphelion. This is demonſtrated by the Earth's motion being quicker in December than in June, by about a fifteenth part: for the Earth and every planet deſcribe equal areas in equal times; thus, when it moves ſwifter, it muſt be nearer the Sun. From this we find that we have about eight days more in real time from the Sun's vernal equinox to the autumnal, than from the autumnal to the vernal.—6. The Sun's diameter is equal to 100 diameters of the Earth; therefore, the body of the Sun is 1,000,000 times greater than that of the Earth.—7. The apparent mean diameter of the Sun is 32 minutes 12 ſeconds. The Sun's horizontal parallax is now fixed at eight ſeconds, five twentieths of a ſecond.—8. If 360 degrees (the whole ecliptic) be divided by the quantity of time in the ſolar year, it will give 59 minutes 8 ſeconds, &c. which is the mean quantity of the Sun's diurnal motion. And if theſe 59 minutes 8 ſeconds be divided by 24, the number of hours in a day, the quotient is 2 minutes 28 ſeconds, which is the Sun's motion in one hour; and which, divided by 60, will give his motion in one minute, &c. By this method, the tables of the Sun's mean motion are conſtructed.

Though the planets above deſcribed perform their periods round the Sun, or rather round the centre of gravity, yet many of the planets ſeen from the Earth will appear to move in a contrary motion to the order of the ſigns; particularly the inferior planets; and ſometimes they may appear ſtationary, or not to move at all, for ſeveral nights together. But theſe appearances are nothing but optical deceptions, ariſing partly from the motions of the planets, and partly from the motion of the Earth on which we are placed; for we always judge a planet to be in that part of the ecliptic which is on the oppoſite ſide of the planet to us; this is called its Geocentric Longitude: but the part of the ecliptic in which the planet is ſeen by an obſerver ſuppoſed to be placed in the Sun, is called the Heliocentric Longitude. And the longitude o any planet or ſtar is an arch of the ecliptic, counted from the beginning of Aries to the place where the ecliptic is cut by a circle perpendicular to the ecliptic, and paſſing through the ſtar or planet.

SECT. II.

OF THE SECONDARY PLANETS.

The ſecondary planets, or ſatellites, are certain planets which perform a revolution round any other planet, as the Moon does round the Earth. They are called ſatellites, becauſe they are always found attending their primary planets, and making the tour about the Sun together with them.

There are but four primary planets that are certainly known to have ſatellites: viz. the Earth, Jupiter, Saturn, and the Georgium Sidus; though ſome have imagined they have diſcovered ſatellites attending ſome of the other planets, as hath been hinted in the laſt ſection; but theſe obſervations have not been ſufficiently confirmed.

The Earth is attended by one ſatellite, called the Moon, and marked ☾. She performs her revolution round the Earth in an elliptic orbit, the mean eccentricity of which is one eighteenth part nearly of her mean diſtance from the Earth, or about 13,000 miles; her mean diſtance from the Earth being 60⅓ ſemidiameters of the Earth; or about 240,000 miles.

The mean time of one revolution of the Moon about the Earth, or from one New Moon to another, when ſhe overtakes the Sun again, is 29 days 12 hours 44 minutes 3 ſeconds 11 thirds. But the mean time in which ſhe moves once round her whole orbit is 27 days 7 hours 43 minutes 8 ſeconds, which is at the rate of about 2290 miles in an hour. For the Moon has completed one revolution about the Earth before ſhe comes again in conjunction with the Sun; becauſe, while the Moon is performing her revolution, the Earth has advanced about a 13th part of the ecliptic forward.

The Moon turns once round her own axis exactly in the time that ſhe goes round the Earth. This is the reaſon the ſame ſide of the Moon is always turned towards the Earth; and day and night in the Moon, taken together, are juſt as long as a lunar month.

The diameter of the Moon is to that of the Earth, as 20 to 73; therefore it is equal to 2180 miles. The ſurface of the Moon is to that of the Earth as 3 is to 40, or as 1 to 13⅓ nearly; therefore, the Earth reflects 13 times as much light upon the Moon, as ſhe does upon the Earth, when ſhe is at her full. The ſolid content of the Moon is to that of the Earth as 3 is to 146: the denſity of the Moon's body is to that of the Earth as 5 is to 4; therefore, her quantity of matter is to that of the Earth as 1 is to 39 nearly. The force of gravity on her ſurface is to that on the Earth as 100 is to 293. The axis of the Moon is almoſt perpendicular to the plane of the ecliptic; therefore, ſhe has little or no difference of ſeaſons. The mean apparent diameter of the Moon is 31 minutes 16½ ſeconds.

The various phaſes and appearances of the Moon have puzzled all the aſtronomers of antiquity. Her wanings and increaſings, her various poſitions with regard to the Earth, and her frequent eclipſes, were matters of conſtant admiration. The moon being a dark ſpherical body, and ſhining only with the borrowed light of the Sun, can only have one half of her body illuminated at the ſame time, the oppoſite half remaining in its native darkneſs; therefore, as the Moon performs a revolution round the Earth, ſhe will ſometimes turn the whole of her illuminated face towards the Earth; at which time ſhe appears perfectly round, and is a *full moon:* at other times only a certain portion of her illuminated face will be turned towards the Earth; ſhe will then appear either horned, half round, or gibbous, according to the quantity of her illuminated part which is ſeen by us.

To illuſtrate this, let A B C D E F G H repreſent the orbit of the Moon, (*fig.* 9, *plate* 17.) Now, when the Moon is at A, in conjunction with the Sun, her dark ſide will be turned towards the Earth, and therefore ſhe will be inviſible, as at *a*, which is then called the New Moon. When ſhe arrives at B, or has run through one eighth part of her orbit, one quarter of her illuminated face will be turned towards the Earth; ſhe will then appear horned, as at *b*. When ſhe arrives at C, one half of her illuminated face is turned towards the Earth, as at *c*, when ſhe is ſaid to be in her quadrature. When ſhe arrives at D, which is called her ſecond octant, three parts of her illuminated face will be turned towards the Earth, and ſhe will appear gibbous, as at *d*. When ſhe arrives at E, the whole of her illuminated face is turned towards the Earth, and ſhe appears quite round, as at *e*, when ſhe is ſaid to be a full Moon. As ſhe proceeds through the other half of her orbit ſhe decreaſes again from *e* to *a*, and nearly in the ſame ratio as ſhe increaſed in the former half of her orbit. And the Earth has all the ſame appearances to an obſerver in the Moon, as the Moon has to us, but in a contrary order: viz. the Earth being at the full to them, when the Moon changes to us, and *vice verſa*; as is evident from a view of the figure.

The motions of the Moon are all very irregular; the only equable motion ſhe has, is the rotation on her own axis in the ſpace of a month, being the time in which ſhe moves round the Earth; which is the reaſon that ſhe always expoſes the ſame face towards the Earth.

The orbit of the Moon is very changeable, and does not long preſerve the ſame figure; for though the orbit of the Moon be an ellipſe, having the Earth in one of her foci thereof; yet the eccentricity is ſometimes greater than at other times.

The plane of the Moon's orbit is inclined to that of the ecliptic, in an angle of five degrees.

The face of the Moon has the appearance, when viewed through a teleſcope, of being diverſified with hills and vallies; this is alſo proved to be the caſe, from the edge or border of the Moon appearing jagged, eſpecially about the line which ſeparates the illuminated part of the Moon from the dark ſide thereof. The ſpots alſo of the Moon, which are taken for mountains, are found to caſt a triangular ſhadow in the direction oppoſite to the Sun; and thoſe parts which are taken for vallies or cavities are always dark on that ſide next the Sun, and illuminated on the oppoſite ſide; which is agreeable to experience. Sometimes the tops of the mountains are ſeen illuminated by the Sun, while their baſes are in the dark ſide of the Moon; and by theſe means we have a good method of taking the height of the lunar mountains.

Thus, let E D (*fig.* 14) be the Moon's diameter, E C D the line dividing the dark from the illuminated part of the Moon; and A the top of a hill in the dark part, juſt beginning to be illuminated: with a teleſcope, take the proportion of A E to the diameter E D, then there are given the two ſides A E, E C, of the right-angled triangle A E C; the ſquares of the two ſides of the right-angled triangle being added together give the ſquare of the hypothenuſe A C, from the ſquare root of which, ſubtracting B C, the radius, there remains A B, the height of the mountain.

From late obſervations, Dr. Herſchel has diſcovered that very few of the lunar mountains exceed half a mile in perpendicular height. The ſame gentleman has alſo obſerved three volcanoes in the Moon, which he thus deſcribes: "I perceived (April 19th, 10 hours 36 minutes, ſidereal time) three volcanoes in different parts of the dark part of the New Moon: two of them are either already nearly extinct, or otherwiſe in a ſtate of going to break out; which, perhaps, may be decided next lunation: the third ſhows an actual eruption of fire, or luminous matter; its light is much brighter than the nucleus of the comet which Mr. Mechain diſcovered at Paris the 10th of this month." The following night he diſcovered it burn more violently; and by meaſuring, he found the ſhining or burning matter to be more than three miles in diameter. The actual fire or eruption of a volcano exactly reſembled a ſmall piece of burning charcoal, when it is covered by a very thin coat of white aſhes; and it had a degree of brightneſs, about as ſtrong as that with which a coal would be ſeen to glow in faint daylight.

Theſe are the chief phenomena obſervable in the Moon. All the other ſatellites are of a ſimilar nature to this; but from their great diſtance from the Earth, we are unable to be ſo particular in our deſcription of them.

Though it be aſſerted, that the Moon and the other ſecondary planets revolve round the primary planets as their centres, and the primary planets revolve round the Sun for their centre, yet it muſt be remembered, that this aſſertion is not the real mathematical truth. For the primary planets do not regard the Sun as their exact centre; but each primary planet, and the Sun, revolve round their common centre of gravity: which common centre of gravity is that point where the two bodies, or the Sun and planet, will equiponderate each other. Thus, the centre of gravity in a common balance-beam, or ſteelyard, is the point of ſuſpenſion.

To diſcover the common centre of gravity of two bodies, is to find that point, whoſe diſtance from the greater body is leſs than its diſtance from the leaſt body, in the ſame proportion as the gravity of the leſs body is leſs than that of the greater; and in two bodies of equal gravities, their common centre of gravity is equally diſtant from their two centres.

Thus, if B be a body four times as great in magnitude as the body l, and both be ſuppoſed to be connected by an inflexible wire B l, the common centre of gravity of the two bodies will be at the point g, which is four times nearer to B than to l; or as B g is to g l, ſo is l to B.

Therefore, the common centre of gravity of the Earth and Sun is nearer to the centre of the Sun than to that of the Earth, by as great a ratio as the quantity of matter in the Sun exceeds that in the Earth, which centre of gravity is in the body of the Sun. The common centre of gravity of Jupiter and the Sun is alſo within the body of the Sun, though very near its ſuperficies.

The Sun is not acted upon by our Earth only, ſo as to turn round the common centre of gravity of that and the Earth, without having regard to the other planets; but there is a common centre of gravity between the Sun and each of the primary planets; and each of theſe planets has its effect in cauſing the Sun to turn round their centres of gravity.

As the centre of gravity of Jupiter and the Sun is the fartheſt diſtant from the Sun's centre, owing to the great ſize of Jupiter, and its diſtance from the Sun, and as this centre is within the body of the Sun, it follows that the Sun is never removed above one of its own diameters out of its place.

Each of the ſecondary planets, and its primary planet alſo, turn round their common centre of gravity.

The figures 1, 2, 3, 4, 5, 6, 7, and 8, ſhow the true proportions of the planets Mercury, Venus, Mars, the Earth, Jupiter, Saturn, Georgium Sidus, and the Moon.

Jupiter has four ſatellites; the times of whoſe periods and diſtances have been noticed in the laſt ſection. Their periods were found from their conjunction with Jupiter, after the ſame manner as the periods of the primary planets were diſcovered. Their diſtances from the body of Jupiter are meaſured by a micrometer, and computed in ſemidiameters of Jupiter, and then reduced into miles.

The ſatellites of Jupiter are of very great uſe in aſtronomy; for, by an obſervation of the eclipſes of theſe ſatellites, we derive three great advantages:—1ſt, in determining the diſtance of Jupiter from the Earth; 2dly, we find the progreſſive motion of light: for by theſe eclipſes, it is evident that light does not come to us from Jupiter in an inſtant; for if the motion of light were inſtantaneous, it is evident we ſhould ſee the commencement of the eclipſes of the ſatellites at the ſame moment they really happen, whatever diſtance they might be from us; but, on the contrary, if light have a progreſſive motion, it is evident, the farther we are from a planet, the later we ſhould be in ſeeing the beginning of its eclipſe; and ſo it is found to happen: the ſatellites of Jupiter appear to be eclipſed later than the true computed time, and always proportionally later, as the Earth is removed farther from the planet. When the Earth and Jupiter are neareſt to each other, that is, when they are both in conjunction, on the ſame ſide of the Sun, then the eclipſes of Jupiter's ſatellites are ſeen to happen ſooner than when the Sun is directly between Jupiter and the Earth; in which laſt caſe, the diſtance of Jupiter from the Earth is greater than it is in the former caſe, by the whole diameter of the Earth's annual orbit, or by double the Earth's diſtance from the Sun: in this laſt caſe, we cannot obſerve an eclipſe of Jupiter's ſatellites, till near a quarter of an hour after the time we could have diſcovered it in the former caſe, that is, when Jupiter was at his leaſt diſtance from the Earth. From hence it follows, that the light is near a quarter of an hour in paſſing through a ſpace equal to the diameter of the Earth's orbit, or near eight minutes in paſſing from the Sun to the Earth; which is at the rate of about twelve million of miles in a minute.

But the third and greateſt advantage derived from the obſervation of theſe eclipſes, is the diſcovery of the longitude of different places on the Earth: for having the difference of time between two obſervations of the ſame eclipſe, taken in two different places, we have the difference of longitude between the two places. For example, ſuppoſe there be two obſervers of an eclipſe, the one at London, the other at Barbadoes, the eclipſe will appear at the ſame moment of *real time* to each perſon; but being under different meridians, the hour of the day will be different at each place; thus, if it be 12 o'clock at noon at London, it will be 8 o'clock in the morning at Barbadoes, by which the obſervers find the difference of longitude between the two places to be ſixty degrees, or four hours in time.

The planet Saturn has ſeven ſatellites, the ſixth and ſeventh of which were diſcovered by Dr. Herſchel, in the years 1787 and 1788. Their periods, diſtances, &c. have been deſcribed in the laſt ſection.

The Georgian planet, or Herſchel, is alſo found to have ſix ſatellites revolving round him, like thoſe of Jupiter and Saturn. These ſatellites were diſcovered by Dr. Herſchel, for which ſee the laſt ſection.

Theſe are the only primary planets which we are certain are attended by ſatellites. Some aſtronomers have imagined they diſcovered a ſatellite belonging to Venus; but the many repeated obſervations which have been made by others to obſerve it, and without effect, leave us room to ſuſpect they were deceived.

SECT. III.

OF THE FIXED STARS.

THE fixed ſtars, generally denominated *ſtars* by way of eminence, are thoſe heavenly bodies which uſually keep the ſame diſtance with regard to each other. All the heavenly bodies, except the primary and ſecondary planets, and comets, are of this claſs.

The diſtances of the fixed ſtars are ſo great, that we have no diſtance in the planetary ſyſtem with which we can compare them; for the diameter of the Earth's orbit, which is nearly 190,000,000 miles, bears no ſenſible proportion to the diſtance of the neareſt fixed ſtar from the Earth.

The diſtances of the fixed ſtars have been the ſubject of inveſtigation to ſeveral aſtronomers. Various methods have been deviſed for the purpoſe of diſcovering the diſtances of theſe heavenly bodies, but without ſucceſs, on account of their almoſt infinite diſtance; the moſt accurate obſervations only gave us a diſtant approximation; but by the beſt obſervations, however, we can ſafely conclude that the neareſt fixed ſtar is upwards of forty thouſand diameters of the Earth's orbit diſtant from us, or eighty thouſand times farther diſtant from us than the Sun is.

The magnitudes of the fixed ſtars appear very different in different ſtars, owing, in ſome ſort, to their real magnitudes, which are different, but principally owing to their different diſtances from us.

The ſtars are generally divided, according to their apparent magnitudes, into ſix, and by ſome into eight claſſes. The firſt claſs, called ſtars of the firſt magnitude, are thoſe that appear largeſt: ſtars of the ſecond magnitude appear ſomewhat leſs; and thus every following claſs comprehends thoſe ſtars next in ſize to the former claſs: the ſtars of the ſixth magnitude containing the ſmalleſt ſtars viſible to the naked eye. All others that cannot be perceived, but by the help of teleſcopes, are called teleſcopic ſtars. It is not to be inferred from hence that Nature has divided the ſtars into thoſe claſſes; for there are almoſt as many claſſes as there are ſtars, ſo great is their variety of magnitude and brightneſs!

The number of ſtars is alſo very great, and appears to be almoſt infinite; but aſtronomers have deduced all that are viſible to the naked eye into catalogues. Mr. Flamſtead reduced 3000 ſtars into a catalogue, which contains all that are viſible to the naked eye at any time of the year, and a great number that are only viſible through a teleſcope. The number of ſtars that are viſible at one time, in the cleareſt heaven, ſeldom exceeds 1000; their appearing ſo much more numerous ariſes from their twinkling, and from viewing them confuſedly, without reducing them into any order. But a good teleſcope will, nevertheleſs, bring great numbers to our view: and the more the magnifying power of the teleſcope is increaſed, the greater will be the number of ſtars diſcovered; till the number becomes ſo great as to baffle our computation.

From the great diſtance of the ſtars, we are at a loſs to diſcover many of their properties; but from their phenomena we can with certainty deduce the following theorems concerning them:—1. That they are much greater than our Earth; for, if that were not the caſe, they could not be viſible at ſuch a diſtance.—2. They are farther diſtant than the moſt diſtant of the planets; for we often find them hidden behind the body of the planets.—And, 3. They ſhine with their own natural light; for though they be much farther from the Sun than Saturn is, and appear much ſmaller to us, yet they ſhine much brighter than that planet. And it is known, that the more the teleſcope magnifies, the leſs is the angle under which the ſtar is ſeen; becauſe the teleſcope deſtroys all the adventitious rays. Thus a teleſcope magnifying 200 times will ſhow a ſtar leſs in magnitude than it appears to the naked eye, inſomuch, that it will appear to be only an indiviſible point.

From hence we conclude, that the fixed ſtars are ſo many Suns; and that, in all probability, they are not much ſmaller than our Sun, but perhaps larger.

Therefore, it is generally believed that every ſtar is the centre of a ſyſtem, and has planets revolving round it in the ſame manner as the Earth and the other primary planets revolve round the Sun; for our Sun, together with the orbits of all the planets, would be almoſt inviſible from the neareſt fixed ſtar.

To imagine that the ſtars are formed only to afford us a faint light, would be abſurd, as we have incomparably more light from the Moon than from all the fixed ſtars taken together.

The fixed ſtars have two apparent motions; one called the firſt, common, or diurnal motion; the other called the ſecond or proper motion. The former of theſe motions ariſes from the Earth's motion round its axis; by which the ſtars appear to be carried round the Earth, from eaſt to weſt, in the ſpace of 24 hours. The latter is that motion by which they appear to go backwards from weſt to eaſt round the poles of the ecliptic with a very ſlow motion, deſcribing only one degree of a circle in the ſpace of 71½ years. This apparent motion is owing to the preceſſion of the equinoxes; or, in other words, the axis of the Earth is directed to different parts of the heavens every year, deſcribing a circle, one degree of which it deſcribes in 71½ years.

The *Zodiac* is an imaginary ring or zone in the heavens, in the ſpace of which all the primary planets revolve in their orbits: its breadth is made different by different aſtronomers, but is from eight to ten degrees on each ſide the ecliptic; and is divided into twelve parts, called the Twelve Signs of the Zodiac, and each ſign is ſubdivided into 30 degrees; the degrees are each again divided into minutes, ſeconds, &c. But as the ſtars have a motion from weſt to eaſt, theſe conſtellations, or ſigns of the Zodiac, do not now correſpond to their proper ſigns; for the vernal equinox formerly happened when the Sun was in the firſt degree of Aries, and the Earth in the oppoſite degree of the Zodiac, or firſt degree of Libra; whereas now, the Sun has advanced a whole ſign from that point at the vernal equinox.

The twelve ſigns of the Zodiac are diſtinguiſhed by the following names and characters, viz. ♈ *Aries*, ♉ *Taurus*, ♊ *Gemini*, ♋ *Cancer*, ♌ *Leo*, ♍ *Virgo*, ♎ *Libra*, ♏ *Scorpio,*

pio, ♐ *Sagittarius*, ♑ *Capricornus*, ♒ *Aquarius*, ♓ *Piſces*: or, according to the Engliſh names, the Ram, the Bull, the Twins, the Crab, the Lion, the Virgin, the Balance, the Scorpion, the Archer, the Goat, the Water-bearer, and the Fiſhes.

Beſides theſe conſtellations in the Zodiac, the ſtars in every other part of the heavens are reduced into conſtellations of ſome certain figures, to which it is ſuppoſed each ſet of ſtars bears ſome reſemblance. In the northern hemiſphere are 21 conſtellations, of which the following are the names:—the Little Bear, Great Bear, the Dragon, Cepheus, Boötes, the Northern Crown, Hercules, the Harp, the Swan, Caſſiopeia, Perſeus, Auriga, Serpentary, the Serpent, the Arrow, the Eagle, the Dolphin, the Horſe, Pegaſus, Andromeda, and the Triangle. In the ſouthern hemiſphere are 15 conſtellations, viz. the Whale, Orion, the Eridanus, the Hare, the Great Dog, the Little Dog, the Ship, the Hydra, the Cup, the Raven, the Centaur, the Wolf, the Altar, the Southern Crown, and the Southern Fiſh.

This diviſion was introduced by Ptolemy, and to theſe Bayer added 12 more, about the ſouthern pole, viz. the Peacock, the Tucan, the Crane, the Phœnix, the Dorado, the Flying Fiſh, the Hydra, the Chamelion, the Bee, the Bird of Paradiſe, the Triangle, and the Indian.

To theſe, Mr. Royer has added 11 other conſtellations, viz. the Giraffe, the River Jordan, the River Tigris, the Sceptre, and the Flower-de-Luce, being on the north. The following ſix are on the ſouth part, viz. the Dove, the Unicorn, the Croſs, the Great Cloud, the Little Cloud, and the Rhomboid.

Hevelius alſo added the following new conſtellations, compoſed of ſome unformed ſtars, viz. the Unicorn, the Camelopardalis, the Sextant of Urania, the Dogs, the Little Lion, the Lynx, the Fox and Gooſe, Sobieſki's Crown, the Lizard, the Little Triangle, and the Cerberus, to which Gregory has added the Ring and the Armilla.

It

It muſt, however, be remarked, that ſome of theſe conſtellations introduced by Hevelius anſwer to ſome of thoſe of Royer; as the Camelopardalis to the Giraffe, the Dogs to the River Jordan, and the Fox to the River Tigris. The foregoing is the number of the conſtellations as they ſtand at preſent; but an attempt has lately been made by Dr. Hill, to introduce fourteen new ones; they are, however, not yet adopted by mathematicians.

Beſides the ſtars in the foregoing conſtellations, there are a great number of ſtars not included in any conſtellation, and therefore called unformed ſtars.

The Galaxy, or Milky Way, is that long, white, luminous tract which ſeems to encompaſs the heavens, and is eaſily ſeen in a clear night, when the Moon is not up. It is of a conſiderable breadth, and in ſome parts double. Its luminous appearance is owing to the great number of ſmall ſtars with which it is every where beſpangled, and which by a good teleſcope may be plainly diſcovered.

Of Comets.

A comet is a wandering body, appearing ſuddenly, and as ſuddenly diſappearing; and moves in its own proper orbit, like a planet.

It is uſually furniſhed with a long train of light, called its tail, which is always oppoſite to the Sun. Comets are divided into three kinds; viz. *bearded*, *tailed*, and *hairy* comets: which diviſion ariſes from the different ſituation of the comet. Thus, when the comet is eaſtward of the Sun, and moves from him, it is ſaid to be a bearded comet, becauſe the light precedes it in the manner of a beard: when the comet is weſtward of the Sun, and ſets after him, it is ſaid to be a tailed comet, becauſe the train of light follows it in the manner of a tail: and when the Sun and comet are in oppoſition to each other, the Earth being between them, the train of the comet is hid behind its body; except the extremities of the train, which being broader than the body of the comet, appear, as it were, round the edges of it like a border of hair, from which it is called a hairy comet.

The comets make a part of the ſolar ſyſtem, and move in elliptical orbits, having the Sun in one of their foci, and deſcribe areas proportional to the times of their motions, like the planets. The reaſon why they ſometimes appear viſible, and ſometimes not, is the great eccentricity of their orbits, which is very conſiderable, for when they are in that part of the orbit moſt remote from the Sun, they are much beyond the orbit of Jupiter; and in their perihelion they frequently deſcend within the orbit of Mars, and ſometimes within thoſe of the inferior planets.

SECT. IV.

OF ECLIPSES.

AN eclipſe is the privation of the light of one of the luminaries by the interpoſition of ſome opake body, either between the luminary and the eye, or between it and the Sun.

The *duration* of an eclipſe is the time of its continuance.

The *immerſion*, or incidence of an eclipſe, is the moment when the eclipſe begins; or when part of the luminary firſt begins to be obſcured.

The *emerſion*, or expurgation of an eclipſe, is the time when the eclipſed luminary begins to reappear, or emerge out of the ſhadow.

The *quantity* of an eclipſe, is the part of the luminary eclipſed. To determine this quantity, the diameter of the eclipſed body is divided into 12 equal parts, called digits; and the eclipſe is ſaid to be of ſo many digits as are contained in that part of the diameter which is eclipſed.

Eclipſes are either thoſe of the Sun, the Moon, or of ſome of the ſatellites, and are either *total*, *partial*, *annular*, *central*, &c.

A *total* eclipſe, is when the whole body of the luminary is darkened.

A *partial* eclipſe, is when only a part of the luminary is eclipſed.

A *central* eclipſe, is when the centres of the two luminaries and the Earth come in a ſtraight line, and is always total.

An *annular* eclipſe, is when the whole body is eclipſed, except a ring or annulus, which appears round the border or edge.

An eclipſe of the Moon, is a privation of the light of the Moon, and occaſioned by the body of the Earth being directly between the Sun and the Moon, and ſo intercepting the Sun's rays, that they cannot arrive at the Moon; conſequently the Moon paſſes through a part of the conical ſhadow of the Earth, as ſeen in *fig.* 12, *plate* 17, where D E C repreſents the Earth, and D G F C the conical ſhadow thereof, in which is the Moon in an eclipſe. The dotted ſpaces D G *s*, and F C *r*, ſhow thoſe parts of the ſhadow called the *penumbra*, in which the Moon is deprived only of part of the Sun's light.

An eclipſe of the Sun, is an obſcuration of the Sun's body, occaſioned by the Moon's coming between the Earth and the Sun, and thus intercepting the light of the Sun from us, on which account ſome have conſidered it an eclipſe of the Earth.

The ſolar eclipſe is repreſented *fig.* 11, where *m* repreſents the Moon, C D the Earth, and *r m s o* the Moon's conical ſhadow, travelling over that part of the Earth C *o* D, and cauſing a complete eclipſe of the Sun to all the inhabitants who reſide in the tract C D. The ſpaces C *r o* and D *s o* include the penumbra, and all the inhabitants within thoſe ſpaces will perceive a faint ſhadow of the eclipſe.

Hence, an eclipſe of the Moon can happen only at the time of the full Moon, or when ſhe is oppoſite to the Sun; and an eclipſe of the Sun will take place only at the time of a new Moon, or when the Moon is between the Sun and Earth.

From hence ſome may imagine that there may be two eclipſes, viz. one of the Sun and another of the Moon, in every lunation, which would really be the caſe, if the Moon moved in the ſame plane with the ecliptic; but the orbit of the Moon not being in the plane of the ecliptic, but inclined thereto in an angle of 5 degrees 35 minutes, and paſſing through the plane of the ecliptic, it muſt neceſſarily follow, that an eclipſe can only take place when the Moon is near that part of its orbit which paſſes through the plane of the ecliptic. Theſe two oppoſite points where the Moon's orbit intercepts the ecliptic, are called its Nodes.

That point where the Moon aſcends from the ſouth to the north ſide of the ecliptic, is called the *aſcending node*, or dragon's head, and marked ☊; and the oppoſite point, where the Moon deſcends from the north to the ſouth ſide of the ecliptic, is called the *deſcending node*, or dragon's tail, and marked ☋; and a line drawn from one node to the other, is called the line of the nodes. Thus, if (*fig.* 13) *a b c d* be the orbit of the Moon, and *e g* the ecliptic, the points *a c*, where the orbit cuts the ecliptic, are the two nodes, and the dotted line *a c* the line of the nodes. From a view of the figure, it is plain, when the full or new Moon happens when the Moon is at the points *b* or *d*, there can be no eclipſe, the ſhadow of the Moon or Earth falling either above or below the other luminary; but when the full or new Moon is at the points *a* or *c*, or within 17 degrees of theſe points, there will be an eclipſe of one of the luminaries.

In order to calculate an eclipſe, it is neceſſary to know how to take the parallax of the Sun, or any heavenly body; as alſo to take the parallactic angle.

The parallactic angle, called alſo the *parallax*, is the angle E S T (*fig.* 1, *plate* 18), made at the centre of a ſtar, or other bodies, by two lines, one drawn from the centre of the Earth T, and the other from its ſurface E; or, which is the ſame thing, it is the difference of the two angles C E A and B T A.

Parallax is an arch of the heavens intercepted between the true and apparent place of any ſtar, or heavenly body,

The true place of a ſtar, S, is that point of the heavens, B, where it would be ſeen by an obſerver placed in the centre of the Earth T: and the apparent place of the ſame ſtar is the point C in the heavens, where it would appear to an obſerver on the ſurface of the Earth, at E. This difference of the two places of the ſame ſtar is the parallax, ſometimes called, for diſtinction ſake, the parallax of altitude; and is an angle formed by two viſual rays, the one drawn from the centre, and the other from the circumference, of the Earth, and traverſing the body of the ſtar; the meaſure of it being an arch of a great circle, intercepted between the points of the true and apparent places, B and C.

The parallax B C is alſo the difference between the true diſtance of the ſtar from the zenith A, and the apparent diſtance A C. Hence the parallax diminiſhes the altitude of a ſtar, or increaſes its diſtance from the zenith.

The parallax is greateſt in the horizon, which is therefore called the horizontal parallax, as E F T. From the horizon the parallax decreaſes all the way to the zenith A, where the true and apparent places of the ſtar coincide.

The *parallax of the annual orbit of the Earth*, is the angle under which the ſemidiameter of the Earth's orbit is ſeen.

To find the parallax of a celeſtial body, obſerve when the body is in the ſame vertical line with a fixed ſtar which is near it; and while it is in that poſition, meaſure its apparent diſtance from the ſtar; then obſerve when the ſtar and body are at equal altitudes from the horizon, and there meaſure their diſtances again, and the difference of theſe diſtances will be the parallax.

The Aſtronomy of Eclipſes.

To calculate a lunar eclipſe it is neceſſary, firſt, to find the length of the Earth's conical ſhadow, which may be found by finding the diſtance between the Earth and Sun, and the proportion of their diameters. Thus, ſuppoſe the ſemi-axis of the Earth's orbit to be 95,000,000 miles, and the eccentricity of the orbit 1,377,000, which, added together, make 96,377,000 miles, or 24,194 ſemidiameters of the Earth; and the Sun's ſemidiameter being to that of the Earth as 112 to 1; then, as A D is to B E, ſo is D B to E C (*fig.* 2), that is, as 111 is to 1, ſo is 24,194 to 218 ſemidiameters of the Earth, equal to E C, the length of the Earth's ſhadow.

To find the apparent ſemidiameter of the Earth's ſhadow, in the place where the Moon paſſes through it, add together the parallaxes of the Sun and Moon, and from the ſum ſubtract the apparent ſemidiameter of the Sun, and the remainder will be the apparent ſemidiameter of the ſhadow at the place where the Moon paſſes through it.

Note. The Sun's parallax may very well be omitted in this calculation; and the apparent ſemidiameter of the ſhadow increaſed by adding one whole minute.

It is alſo neceſſary to have the true diſtances of the Moon from the node at the mean oppoſition; alſo the true time of the oppoſition, with the true place of the Sun and Moon reduced to the ecliptic, and the Moon's true latitude at the time of the true oppoſition; likewiſe the angles of the Moon's way with the ecliptic, and the true horary motions of the Sun and Moon; from which every particular concerning the eclipſe may be computed by common arithmetic and trigonometry.

The method of conſtructing an eclipſe of the Moon is as follows:—Let E W. (fig. 3) repreſent a part of the ecliptic, C the centre of the tranſverſe ſection of the Earth's ſhadow; Draw the line C N perpendicular to the ecliptic, and towards the north, if the Moon have north latitude; but if ſhe have ſouth latitude, draw a line C S. Make the angle N C D equal to 5 degrees 35 minutes, which is the angle the Moon's orbit makes with the ecliptic. Biſect this angle by the right line C F, and in this line the true equal time of oppoſition between the Sun and Moon falls by the tables.

Take the Moon's latitude at the true time of full Moon from a ſcale of equal parts, which is ſuppoſed to repreſent minutes of a degree, and ſet this diſtance from C to G on the line C F. Through the point G draw a line H I at right angles to C D, which line repreſents a portion of the Moon's orbit. Then L is the point where the Moon's centre is at the middle of the eclipſe; G the place of her centre at the tabular time of her being full; and K the point of her centre at the inſtant of her ecliptic oppoſition; I is the Moon's centre at the moment of her immerſion, and H her centre at the end of the eclipſe, or emerſion.

From the ſame ſcale take the Moon's ſemidiameter, and deſcribe three circles on the points I G H, which repreſent the Moon in the beginning, middle, and end of the eclipſe.

Then, to find the length of time or the duration of the eclipſe, meaſure the line I H on the ſame ſcale, and ſay, as the Moon's horary motion from the Sun is to H I, ſo is one hour, or 60 minutes, to the whole duration of the eclipſe.

From the above figure the eclipſes may be computed. For, firſt, in the right-angled triangle C G L, right-angled at L, there is given the hypothenuſe C G, which is the Moon's latitude at the time of the full Moon: alſo we have the angle G C L equal to the half of 5 degrees 35 minutes, wherefore the legs C L and G L may be found. Secondly, in the right-angled triangle C L H, or C L I, are given the legs C L, and C H or C I, which latter is the ſum of the ſemidiameter of the Moon and the Earth's ſhadow; therefore L H or L I may be found, which is equal to half the difference of the motions of the Sun and Moon during the eclipſe. Thirdly, as the difference of the horary motions of the luminaries is to one hour, or ſixty minutes, ſo is H L to the ſemiduration of the eclipſe; and ſo is G L to the difference between the oppoſition and middle of the eclipſe. This laſt, therefore, taken from the time of full Moon, gives the time of the middle of the eclipſe; from which, ſubtracting the time in L I, before found, gives the beginning of the eclipſe; and, added to the ſame, gives the end of the eclipſe.

Laſtly, from C O, the ſemidiameter of the Earth's ſhadow, ſubtract C L, and the remainder L O, added to L P, gives O P, the quantity eclipſed.

A ſolar eclipſe, or an eclipſe of the Sun, would be diſcovered in the ſame manner as a lunar eclipſe, if the Moon had no ſenſible parallax; but as the Moon has a parallax, the method is ſomewhat different.

1. Add the apparent ſemidiameters of both Sun and Moon together, both in the aphelion and perihelion, which gives 33 minutes 6 ſeconds for the greateſt ſum, and 30 minutes 31 ſeconds for the leaſt ſum.

2. As the parallax diminiſhes the northern latitude and increaſes the ſouthern, let the greateſt parallax in the latitude be added to the former ſums, and alſo ſubtracted from them: thus may be had, in each caſe, the true latitude, beyond which there can be no eclipſe. Having this latitude, the

Moon's diſtance from the nodes is found in the ſame manner as for the lunar eclipſe.

To find the digits eclipſed, add the apparent ſemidiameters of the luminaries together, from which ſubtract the Moon's apparent latitude; the remainder is the part of the diameter eclipſed. Then ſay, as the ſemidiameter of the Sun is to the ſcruples eclipſed, ſo are ſix digits reduced into ſcruples (that is, 360 ſcruples or minutes) to the digits eclipſed.

To determine the duration of a ſolar eclipſe, find the horary motion of the Moon from the Sun for an hour before the conjunction, and for one hour after it. Then ſay, as the former horary motion is to the ſeconds in an hour, ſo are the ſcruples of half the duration to the time of immerſion; and as the latter horary motion is to the ſame number of ſeconds, ſo are the ſcruples of half the duration, to the time of emerſion. Then adding the times of immerſion and emerſion together, the ſum is the whole duration.

To find the beginning, middle, and end of a ſolar eclipſe, find the arch G L from the Moon's latitude, for the time of conjunction. Then ſay, as the horary motion of the Moon from the Sun before the conjunction is to one hour, ſo is the diſtance of the greateſt darkneſs to the interval of time between the greateſt darkneſs and the conjunction. Subtract this interval in the firſt and third quarter of the anomaly from the time of the conjunction, but in the other quarters add it to the ſame, and the reſult is the time of the greateſt darkneſs. Laſtly, from the time of the greateſt darkneſs, ſubtract the time of incidence, to which is to be added the time of emerſion; the difference in the firſt caſe will be the beginning, and the ſum in the latter caſe will be the end of the eclipſe.

To calculate eclipſes of the Sun, it is neceſſary, firſt, to find the mean new Moon, and from thence the true one, with the place of the luminaries for the apparent time of the true new Moon.—2. For the apparent time of the true new Moon, compute the apparent time of the new Moon obſerved.—3. For the apparent time of the new Moon ſeen, compute the latitude ſeen.—4. Thence determine the number of digits eclipſed.—5. Find the times of the greateſt darkneſs, emerſion, and immerſion.—6. And from thence determine the beginning and ending of the eclipſe.

From this it is evident that the trouble in the calculation of eclipſes ariſes from the parallaxes of longitude and latitude, without which the calculation of ſolar eclipſes would be the ſame as that of lunar ones.

SECT. V.

OF TIME.

Time is a mode of duration marked by certain periods, meaſures, and motions; and the chief method we have of meaſuring time is by the revolution of the two luminaries, the Sun and Moon, but particularly that of the Sun.

Mr. Locke obſerves, that the idea we have of time is acquired by conſidering any part of infinite duration, as ſet out by periodical meaſures. The idea of any particular time, or length of time, as a day, an hour, &c. is acquired by obſerving certain appearances of ſome bodies, moving with a regular motion, and at regular and ſeeming equidiſtant periods. Now, by being able to repeat theſe lengths or meaſures of time as often as we pleaſe, we can imagine duration where nothing really endures or exiſts, and thus we imagine to-morrow, or next year, &c.

Time is alſo the duration of a thing which has both a beginning and an end, and in this ſenſe it is diſtinguiſhed from eternity.

Thus, time is the duration of ſome motion; for without ſome regular and uniform motion we ſhould have no methods to compute time, or diſtinguiſh it from eternity.

Time may be divided into *abſolute* and *relative.*

Abſolute time is time conſidered in itſelf, and without any relation to motion.

Relative or apparent time is the ſenſible meaſure of any duration, by means of motion, as, by the motions of the luminaries, the hands of a clock, watch, &c.

Relative time is ſubdivided into *aſtronomical* and *civil.*

Aſtronomical time is that which is meaſured by the motions of the heavenly bodies.

Civil time is formed for civil purpoſes, and diſtinguiſhed into years, months, days, hours, &c.

The year, in the full extent of the word, is a ſyſtem of ſeveral months, or a ſpace of time meaſured by the revolution of ſome celeſtial body in its orbit. Thus, the time in which the fixed ſtars make one revolution is called the great year; and the times in which Jupiter, Saturn, &c. complete their revolutions, and return to the ſame point again, are reſpectively called the years of Jupiter, Saturn, &c. For a year originally denoted a revolution, and is not limited to that of the Sun; therefore we find ſome ancient nations at different times called the revolutions of the Moon, or the ſpace of a month, a year; which occaſions ſuch ſtrange accounts in the chronology of ſome very ancient nations, as in thoſe of the Egyptians, Babylonians, &c.

The ſolar year, called alſo *year* by way of eminence, is that ſpace of time in which the Sun moves through the twelve ſigns of the Zodiac. This year, by the beſt obſervations, is found to contain 365 days 5 hours 48 minutes 48 ſeconds; but the quantity, according to the authors of the Gregorian calendar, is 365 days 5 hours 49 minutes. In the civil account, this year is ſaid to contain only 365 days; and one day is added to every fourth year, to make up for the odd hours, which is therefore called leap-year.

The Julian year, ſo named from Julius Cæſar, who eſtabliſhed it, conſiſts of 365 days 6 hours, which exceeds the true ſolar year by upwards of 11 minutes, which exceſs amounts to a whole day in near 131 years. And one day is added to the end of February every fourth year, which is compoſed of the odd ſix hours every year. This year is, therefore, called Biſſextile, or leap year.

The Gregorian year, introduced by Pope Gregory XIII. in 1582, is the Julian year corrected by this rule, viz. That inſtead of every hundredth year being a leap year, as it would be in the Julian calendar, in this way, only one hundredth year out of four is a leap year, the other three being common years. By this omiſſion of three days in every 400 years, the civil year would nearly keep pace with the ſolar year for time to come.

Yet this year is not quite perfect, for, as in four centuries the Julian year gains 3 days 2 hours 40 minutes, and as there are only three days omitted in the Gregorian account, there is ſtill an exceſs of 2 hours 40 minutes in 400 years, which amounts to a whole day in 3600 years.

In the year 1752 this ſtyle was adopted in England, and the eleven days were thrown out after the 2d of September, by accounting the 3d the 14th of that month. This was called the New Style, in diſtinction from the former, which was called the Old.

The ſolar year is either aſtronomical or civil. The aſtronomical ſolar year is that which is determined preciſely by aſtronomical obſervations, and is of two kinds, *viz. tropical*, and *ſideral* or *aſtral*.

The tropical or natural year is the time the Sun takes to paſs through the twelve ſigns of the Zodiac, and is the only proper natural ſolar year, becauſe the ſeaſons always fall in the ſame months.—The ſideral or aſtral year is the ſpace of time the Sun takes in paſſing from any fixed ſtar till his return to the ſame again, and is 20 minutes 29 ſeconds longer than the true ſolar year.

The lunar year is the ſpace of twelve lunar months, and is either aſtronomical or civil.

The lunar aſtronomical year conſiſts of twelve lunar ſynodical months; and is, therefore, 354 days 8 hours 48 minutes 38 ſeconds, being 10 days 21 hours 16 ſeconds ſhorter than the ſolar year.

The lunar civil year is either common or emboliſmic. The common lunar year conſiſts of twelve lunar civil months, and contains 354 days. The emboliſmic lunar year conſiſts of thirteen lunar civil months, and contains 384 days.

The civil or legal year, in England, formerly began on the 25th of March, or the day of the Annunciation of the Virgin Mary: but the hiſtorical year began on the 1ſt of January. The part of the year between theſe two terms was uſually expreſſed thus, 1735-6, or 173$\frac{5}{6}$. But according to the new ſtyle, the civil year now begins on the 1ſt of January.

The ancient Roman year, as firſt ſettled by the Romans, contained only ten months, and in all 304 days.

The Egyptian year, called alſo the year of Nabonaſſar, from the epoch of that name, contains only 365 days, divided into twelve months of 30 days each, with five intercalary days added at the end. Thus the Egyptian year loſes a whole day of the Julian year every four years, and after the ſpace of 1460 years it begins with the Julian year, which length of time is called the Sothic Period.

The ancient Greek year conſiſted of twelve months, which at firſt were divided into 30 days each, but afterwards each month contained 29 and 30 days alternately; and this year was computed from the firſt appearance of the new Moon, with the addition of an emboliſmic month of 30 days every 3d, 5th, 8th, 11th, 14th, 16th, and 19th year, in order to keep the new and full Moons to the ſame ſeaſons of the year.

The ancient Jewiſh year conſiſted of twelve months, containing 29 and 30 days alternately. To which were added eleven or twelve days to make it agree with the ſolar year.

The Syrian year was the ſame in quantity as the Julian year, but commenced in the beginning of October, according to the Julian year.

The Perſian year contained twelve months, of 30 days each, with five intercalary days added.

The Arabic, Mahometan, or Turkiſh year, called alſo the year of the Hegira, conſiſts of 354 days 8 hours 48 minutes, divided into twelve months, containing 29 and 30 days alternately: though ſometimes it contains 13 months; and intercalary days alſo added every 2d, 5th, 7th, 10th, 13th, 15th, 18th, 21ſt, 24th, 26th, and 29th year. The months commence with the firſt appearance of the new Moon.

The year is divided into twelve parts, called months, from the Moon, by whoſe motions it was regulated: the month is properly the time in which the Moon paſſes through the Zodiac, and is of ſeveral kinds; as,

1. The *illuminative month*, which is the interval between the appearance of one new Moon and that of the next; and always varies in quantity. This month is uſed by the Turks and Arabs.

2 The *lunar periodical month*, or the exact time in which the Moon runs through the Zodiac, and conſiſts of 27 days 7 hours 43 minutes and 8 ſeconds.

3. The *lunar ſynodical month*, called a lunation, is the time between two new Moons, as ſeen from the Earth, and conſiſts of 29 days 12 hours 44 minutes 3 ſeconds and 11 thirds.

4. The *ſolar month* is the time the Sun runs through one ſign of the ecliptic, and conſiſts at a mean rate of 30 days 10 hours 29 minutes 5 ſeconds.

5. The *civil* or *common month* is an interval of a certain number of whole days—ſuch are the calendar months.

6. The *civil lunar month* conſiſts alternately of 29 and 30 days. Thus two of theſe months are equal to two aſtronomical months, and the new Moon will be kept to the firſt day of ſuch civil months for a long time together. This month was in moſt common uſe till the time of Julius Cæſar.

7. The *civil ſolar month*, which conſiſted alternately of 30 and 31 days, excepting one month which had 29 days, introduced by Julius Cæſar. But under Auguſtus, the ſixth month, till then called Sextilis, received the name of Auguſtus, from thence called Auguſt; and one day more was added to it, which was taken from February. This is the regular civil month in uſe in England.

A week is a ſpace of time contained in ſeven days, and originated from the diviſion of the lunar month into four parts.

The diviſion of the month into weeks was uſed by the Syrians, Egyptians, and moſt of the Oriental nations. The Roman week conſiſted of nine days, and the ancient Greeks uſed decades, or a ſyſtem of ten days.

But the Jews uſed the week of ſeven days. The days of the week they denominated, the firſt, ſecond, third, fourth, fifth; and the ſixth day they called the preparation of the Sabbath, the Sabbath being the ſeventh day: and this diviſion is ſtill obſerved by the Chriſtians, Arabs, Perſians, Ethiopians, &c.

The ancient heathens denominated the days of the week from the ſeven planets, calling each day after that planet which they ſuppoſed governed the firſt hour of the day. Thus, the firſt day was called Dies Solis, or Sunday, from the Sun; the ſecond, Dies Lunæ, or Monday, from the Moon, &c.

But our Saxon anceſtors, before their converſion to Chriſtianity, named the days of the week from the Sun and Moon, and alſo from ſome of their deified heroes, to whom they were peculiarly conſecrated, which names we ſtill retain. Thus, Sunday was dedicated to the Sun; Monday to the Moon; Tueſday to Tuiſco, or Mars; Wedneſday to Woden, or Mercury; Thurſday to Thor, the Thunderer, or Jupiter; Friday to Friga, or Friya, the wife of Thor, or Venus; and Saturday to Seater, or Saturn. And the days of the week are often expreſſed by modern aſtronomers by the characters of the planets, as, ☉ for Sunday, and ☽ for Monday, &c.

A *day* is that ſpace of time which ariſes from the appearance or diſappearance of the Sun, and is either natural or artificial.

The *natural* day is the portion of time in which the Sun apparently performs one revolution round the Earth; that is, the time in which the Earth makes a rotation on its own axis.

The *artificial* day is the time from Sun riſing to Sun ſetting.

The *natural* day is either aſtronomical or civil.

The *aſtronomical* day begins at noon, or when the Sun's centre is on the meridian, and contains 24 hours to the following noon.

The *civil* day is the time allotted for the ſpace of a day in civil purpoſes, and includes one entire rotation of the Earth on its axis. This day begins at different times in different nations: at Sun riſing among the ancient Babylonians, Perſians, Syrians, and moſt other Eaſtern

nations,

nations, and the preſent inhabitants of the Balearic Iſlands, the Greeks, &c. It began at Sun ſetting among the ancient Athenians and Jews; it is alſo uſed by the Auſtrians, Bohemians, Marcomanni, Sileſians, modern Italians, and Chineſe. With all modern aſtronomers, and the ancient Umbri and Arabians, the day is begun at noon; and at midnight among the ancient Egyptians, Romans, and with the modern Engliſh, French, Dutch, Germans, Spaniards, and Portugueſe.

An *hour* is the twenty-fourth part, but ſometimes only the twelfth part of a day.

There are various kinds of hours, as, 1. Equal hours, which are the twenty-fourth part of a natural day. They are called equinoctial hours, becauſe they are meaſured on the equinoctial; and aſtronomical, becauſe uſed by aſtronomers. 2. Babyloniſh hours, of which there are 24 equal ones in the day, and reckoned from Sun riſing. 3. European hours, uſed in civil computations, and are reckoned from midnight; 12 hours from thence till noon, and 12 more from noon to midnight. 4. Jewiſh, or planetary, or ancient hours, which are the twelfth parts of the artificial day, and the ſame parts of the artificial night. They are called ancient or Jewiſh hours, becauſe uſed by the ancients, and ſtill uſed by the Jews: they are called planetary hours, becauſe ancient aſtrologers pretended that a new planet preſided over every hour. 5. Italian hours, of which there are 24 equal ones to a day, reckoning from ſunſet.

The hour is divided into 60 minutes; and each minute into 60 ſeconds; each ſecond into 60 thirds, &c.

As time, for the purpoſes of chronology, is calculated by years, it is neceſſary to have ſome certain fixed point of time from which calculations can be made with certainty, which fixed point of time is called an *epocha*, or epoch.

Different nations uſe different epochs or æras; the Chriſtians chiefly uſe that of the nativity of Jeſus Chriſt; the Mahometans, that of the Hegira, or flight of Mahomet;

the Jews, that of the creation of the world, or that of the Deluge; the ancient Greeks, that of the Olympiads; the Romans, that of the building of Rome; the ancient Perſians and Aſſyrians, that of Nabonaſſar, &c.

The doctrine and uſe of epochs is of great importance in chronology. And to find what year of one epoch correſponds with that of another, a period of years has been invented, which commenced before all the epochs, and is a common ſtandard of them all, and called the Julian period. To this period all the epochs are reduced; that is, the year of this period when each epoch commences is determined. Thus, adding the given year of one epoch to the year of the period correſponding with its beginning, and from the ſum ſubtracting the year of the ſame period correſponding to the other epoch, the remainder is the year of that other epoch.

A TABLE

OF THE

Years of the moſt remarkable Epochs.

N. B. *The years before Chriſt, are thoſe before the reputed year of his birth, and not reckoned back from the firſt year of his age, as is uſually done.*	*Julian Period*	*Years of the World.*	*Years before Chriſt.*
Creation of the World — — —	706	0	4007
The Deluge, or Noah's Flood — —	2362	1656	2351
Aſſyrian Monarchy, founded by Nimrod —	2537	1831	2176
Kingdom of Athens, founded by Cecrops —	3157	2451	1556
Entrance of the Iſraelites into Canaan — —	3262	2556	1451
The Deſtruction of Troy — — —	3529	2823	1184
Solomon's Temple founded — — —	3701	2995	1012
The Argonautic Expedition — — —	3776	3070	937
Lycurgus formed his Laws — — —	3829	3123	884
Arbaces, firſt King of the Medes — —	3838	3132	875
Olympiads of the Greeks began — —	3938	3232	775
The Building of Rome — — —	3961	3255	752
Æra of Nabonaſſar — — —	3967	3261	746
Firſt Babyloniſh Captivity, by Nebuchadnezzar	4107	3401	606
The Second Babyloniſh Captivity, and Birth of Cyrus	4114	3408	599
Solomon's Temple deſtroyed — —	4125	3419	588
Cyrus began to reign in Babylon — —	4177	3471	536
Peloponneſian War began — — —	4282	3576	431
Alexander the Great died — —	4390	3684	323
Captivity of 100,000 Jews, by Ptolomy —	4393	3687	320
Archimedes killed at Syracuſe — —	4506	3800	207
Julius Cæſar invaded Britain — —	4659	3953	54
He corrected the Calendar — — —	4667	3961	46
The true Year of Chriſt's Birth — —	4709	4003	4

The Chriſtian Æra.	*Julian Period*	*Years of the World.*	*Years ſince Chriſt.*
Dionyſian, or vulgar Æra of Chriſt's Birth —	4713	4007	0
Chriſt crucified, Friday, April 3d — —	4746	4040	33
Jeruſalem deſtroyed — — —	4783	4077	70
Adrian's Wall built in Britain — —	4833	4127	120
Diocleſian Epoch of Martyrs — —	4997	4291	284
The Council of Nice — — —	5038	4332	325
Conſtantine the Great died — —	5050	4344	337
The Saxons invited into Britain — —	5158	4452	445
Hegira, or Flight of Mahomet — —	5335	4629	622
The Death of Mahomet — — —	5343	4637	630
The Perſian Yeſdegird — — —	5344	4638	631
The Moon, and all the Primary Planets, ſeen in the Sign Libra, from the Earth — —	5899	5193	1186
Art of Printing diſcovered — — —	6153	5447	1440
The Reformation begun by Martin Luther —	6230	5524	1517
The Calendar corrected by Pope Gregory —	6295	5589	1582
Sir Iſaac Newton born, December 25th —	6355	5649	1642
Oliver Cromwell died — — —	6371	5665	1658
Sir I. Newton made Preſident of the Royal Society	6416	5710	1703
Died March 20th — — —	6440	5734	1727
The New Planet diſcovered by Dr. Herſchel —	6494	5788	1781
The Ceres de Ferdinand diſcovered by M. Piazzi	6514	5808	1801

SECT.

SECT. VI.

OF ASTRONOMICAL PROBLEMS, AND THE USE OF THE GLOBES.

The celeſtial globe differs from the terreſtrial one in having the images of the conſtellations and figures of the ſtars upon it, inſtead of the ſeveral parts of the Earth. The meridian circle drawn through the poles and the point Cancer repreſents the ſolſtitial colure; and that meridian, drawn through the point Aries, repreſents the equinoctial colure.

PROBLEM I.

To exhibit a true Repreſentation of the Face of the Heavens for any given Time and Place.

Rectify the globe for the latitude of the place (as taught in Geography), placing the north pole of the globe towards the north pole of the world. Having found the Sun's place in the ecliptic, and brought it to the meridian, ſet the index to 12 o'clock at noon; then turn the globe on its axis till the index points to the given hour. In this poſition the globe exactly repreſents the face of the heavens as it appears at that time; every conſtellation and ſtar in the heavens correſponding in ſituation to thoſe on the globe.

PROBLEM II.

To find the Declination and right Aſcenſion of any Star.

Bring the ſtar to the brazen meridian; and the number of degrees on the meridian between the equator and the ſtar is its declination; and the degree of the equator, cut by the meridian, is the right aſcenſion of the ſtar. Thus the right aſcenſion of any ſtar is an arch of the equator, intercepted between the firſt degree of Aries, and that point where the meridian or circle paſſing through the ſtar cuts the equator.

PROBLEM. III.

To find the Latitude and Longitude of any Star.

Bring the ſolſtitial colure to the meridian, and then fix the quadrant of altitude over the pole of the ecliptic, in the ſame hemiſphere with the ſtar, and bring its graduated edge to the ſtar; then the degree on the quadrant, cut by the ſtar, is its latitude, counted from the ecliptic; and the degree of the ecliptic, cut by the quadrant, is the ſtar's longitude.

PROBLEM IV.

To find the Place of any Star or heavenly Body, having its Declination and right Aſcenſion.

Find the point of right aſcenſion on the equinoctial by Problem II. and bring it to the meridian; then count the degrees of declination upon the meridian from the equinoctial, and there make a mark upon the globe, which will be the place of the ſtar, &c.

PROBLEM V.

To find the Place of a Star, Planet, Comet, &c. having the Latitude and Longitude.

Bring the pole of the ecliptic to the meridian, and there fix the quadrant of altitude, which turn round till its edge cut the given longitude on the ecliptic; then count the given latitude from the ecliptic upon the quadrant of altitude, and there make a mark upon the globe, which will be the place of the ſtar, planet, &c. The place of any ſtar, planet,

net, &c. being found by this or the foregoing
riſing, ſetting, or any other circumſtance c
may be found by the proper Problems, as thoſ
are found.

PROBLEM VI.

To find the riſing, ſetting, and culminatin or any celeſtial Body, and conſequently nuance above the Horizon for any Plac alſo its oblique Aſcenſion and Deſcenſio eaſtern and weſtern Amplitude and Azim

Adjuſt the globe to the ſtate of the heavens a
at noon on the given day; bring the ſtar, &c. t
ſide of the horizon, which will give its eaſte
and azimuth, and the time of riſing; as for the
turn the globe till the ſame ſtar comes to the we
the horizon; ſo will the weſtern amplitude a
with the time of ſetting, be found. Then the
ſubtracted from that of ſetting, leaves the conti
ſtar above the horizon; which, ſubtracted fro
leaves the time it is below the horizon. Laſtl
ſtar to the meridian, and the hour to which th
points is the time of its culminating, or ſouthi

PROBLEM VII.

To find the Altitude of a Star, &c. for Hour.

Adjuſt the globe to the poſition of the heave
it till the index point to the given hour; then
drant of altitude at 90 degrees from the horizo
it to the place of the ſtar: then the degrees of
intercepted between the horizon and the ſtar
altitude ſought.

PROBLEM VIII.

Having the Altitude of a Star by Night, or the Altitude of the Sun by Day, to find the Hour of the Day or Night.

Rectify the globe as in the foregoing Problem, and turn the globe and quadrant till that degree of the ecliptic where the Sun is, or the ſtar itſelf, cut the quadrant in the given degree of altitude; then the index will point to the hour required.

PROBLEM IX.

Having the Azimuth of a Star, or the Sun, to find the Time of the Night or Day.

Rectify the globe as before, and bring the quadrant to the given azimuth in the horizon; then turn the globe till the ſtar or Sun come to the quadrant, and the index will then ſhow the hour of the night or day.

Motions, Diſtances, &c.	Mercury.	Venus.	Earth.
Greateſt elongation of inferior and parallax of ſuperior planets	28° 20′	47° 48′	*
Periodical revolutions round the Sun	*d. h. m.* 87 23 15½	*d. h. m.* 224 16 49½	*d. h. m.* 365 6 9¼
Diurnal rotation upon their axes	*	*h. m.* 23 22	*h. m. s.* 23 56 4
Inclinations of their orbits to the ecliptic	70° 0′	3° 23½′	*
Place of the aſcending node	1^{s} 15° 46¾′	2^{s} 14° 44′	*
Place of the aphelion, or point fartheſt from the Sun	8^{s} 14° 13′	10^{s} 9° 38′	9^{s} 9° 15¼′
Greateſt apparent diameters ſeen from the Earth	11″	58″	*
Diameters in Engliſh miles — that of the Sun being 883217	2600	7687	7964
Proportional mean diſtances from the Sun	38710	72333	100000
Mean diſtances from the Sun in ſemidiameters of the Earth	9210	17210	23799
Mean diſtances from the Sun in Engliſh miles	37 millions	68 millions	95 millions
Proportional eccentricities, or diſtances of the focus from the centre	7960	510	1680
Proportion of light and heat — that of the Earth being 100	668	191	100
Proportion of bulk — that of the Sun being 1380000	$\frac{1}{15}$	$\frac{8}{9}$	1
Proportion of denſity — that of the Sun being ¼	2	1¼	1

Mars.	Pallas.	Ceres de Ferdinand.	Jupiter.	Saturn.	Herſchel, or Georgian.
47° 24′	*	*	11° 51′	6° 29′	3° 4¼′
d. h. m. 686 23 30¾	*d.* 1594	*d. h.* 1652 5	*d. h. m.* 4332 8 51½	*d. h. m.* 10761 14 36¾	*d. h.* 30445 18
h. m. s. 24 39 22	*	*	*h. m.* 9 56	*	*
1° 51′	33° 39′	10° 36′ 57″	1° 19¼′	2° 30⅓′	46′ 26″
1^s 17° 59′	*	2^s 21°	3^s 8° 50′	3^s 21° 48¼′	3^s 13° 1′
5^s 2° 6¼′	*	*	6^s 10° 57½′	9^s 0° 45½′	11^s 23° 23′
25″	*	*	46″	20″	4″
4189	95	162	89170	79042	35109
152369	*	*	520098	953937	1903421
36262	*	*	123778	227028	453000
44 millions	*	*	490 millions	900 millions	1800 millions
14218	*	*	25277	53163	4759
43	*	*	3.7	1.1	0.276
$\frac{7}{24}$	*	*	$1\frac{2}{5}$	1000	90
.7	*	*	.23	.02	*

AN EXPLANATION

OF THE

Principal Terms uſed in Aſtronomy.

Æras, certain periods of time from whence chronologers and aſtronomers begin their computations.

Altitude, the height of the Sun, Moon, or ſtars above the horizon; and it is always reckoned upon a vertical circle.

Amplitude, an arc of the horizon contained between the eaſt or weſt point of the heavens, and the centre of the Sun, or a ſtar, at the time of its riſing or ſetting.

Anomaly (true), the diſtance of a planet in ſigns, degrees, &c. from that point of its orbit which is the fartheſt from the Sun.

Anomaly (mean), is that which would take place, if the planet moved uniformly in the circumference of a circle.

Antecedentia, the motion of any heavenly body when it is contrary to the order of the ſigns, as, through Aries, Taurus, Gemini, &c. towards Piſces, &c.

Aphelion, that point in the orbit of a planet which is the fartheſt diſtant from the Sun.

Apogeon, that point in the orbit of a planet, in which it is at its greateſt diſtance from the Earth.

Apſides, the two points in the orbit of a planet, which are its greateſt and leaſt diſtance from the Sun: the line joining theſe points is called the line of the apſides.

Armillary ſphere, an inſtrument having the principal circles which are uſually drawn upon the artificial globe.

Aſcenſional difference, an arc of the equinoctial contained between that point of it which riſes with the Sun, Moon, or ſtar, and that point which comes to the meridian with them; or it is the time the Sun riſes or ſets before or after ſix o'clock.

Atmoſphere, that collection of vapours, or body of air, that ſurrounds the Earth.

Axis, of the Earth or any planet, is an imaginary line paſſing through the centre, from one pole to the other.

Azimuths, great circles paſſing through the zenith and nadir, and they are perpendicular to the horizon. The azimuth of any celeſtial object is an arc of the horizon, contained between the eaſt or weſt point of the heavens, and a vertical circle paſſing through the centre of that object.

Biſſextile, or *leap year*, every fourth year, ſo called becauſe the Romans reckoned the ſixth day of the calends of March, in this year, twice over.

Cardinal points, the Eaſt, Weſt, North, and South points of the compaſs.

Cardinal points of the ecliptic, the firſt points of the ſigns Aries, Cancer, Libra, and Capricorn.

Centrifugal force, that force by which any body, revolving in a circular orbit, endeavours to fly off from the centre of motion in a right line, or tangent to the circle.

Centripetal force, that force which attracts any heavenly body towards the centre of its orbit, and which, together with the centrifugal force, preſerves the body in the proper path of its orbit.

Colures, two great circles or meridians, one of which paſſes through the ſolſtitial points Cancer and Capricorn, and is called the ſolſtitial colure; the other paſſes through the equinoctial points Aries and Libra, and is called the equinoctial colure.

Conjunction, is when two ſtars or planets, ſeen from the Sun or Earth, appear in the ſame point of the heavens.

Conſtellation, ſeveral ſtars lying near each other, which aſtronomers, for the ſake of remembering with more eaſe, ſuppoſed to be circumſcribed by the outlines of ſome animal, or other figure.

Coſmical riſing or ſetting of a ſtar, is when they riſe with the Sun in the morning, or ſet with him in the evening.

Conſequentia, the motion of the planets according to the order of the ſigns, as from Aries towards Taurus, &c.

Culminating of the Sun or a ſtar, is when they come to the meridian of any place.

Cycle of the Moon, a period of 19 years, in which time the changes and eclipſes of the Moon are nearly the ſame, and happen at the ſame time.

Day, that portion of time in which the Earth performs an entire revolution upon its axis; and is either natural, artificial, or aſtronomical.

Declination of any celeſtial body, is its diſtance north or ſouth from the equator, reckoned in degrees, minutes, &c. upon a circle perpendicular to the equator.

Degree, the 360th part of a circle.

Direct, a planet is ſaid to be direct when it moves according to the order of the ſigns.

Diſk of the Sun, or Moon, is its round face, which, on account of its diſtance from us, appears flat, like a plane ſurface.

Digit, is the twelfth part of the Sun or Moon's diameter.

Eccentricity, the diſtance between the centre of an ellipſe and either of its foci.

Ecliptic, that great circle in which the Sun appears to move.

Elevation of the pole, is an arc of the meridian contained between the pole and the horizon, and is always equal to the diſtance of the zenith from the equator, that is, the latitude of the place.

Elongation, the angular diſtance of a planet from the Sun, as it appears to a ſpectator upon the Earth.

Ellipſis, a figure formed by cutting a cone obliquely. The orbits of all the planets are of this form.

Emerſion, the time when any planet that is eclipſed begins to recover its light again.

Epact, the Moon's age at the end of the year, or the difference between the ſolar and lunar year.

Equations, certain quantities by which are eſtimated the inequalities in the motion of a planet: the Moon being ſubject to many irregularities, has a great number of equations.

Equation of Time, the difference between equal and apparent time, or that ſhown by a clock and a ſun-dial.

Equinoxes, the two points where the ecliptic cuts the equator.

Galaxy, or the Milky Way, a large irregular zone in the heavens, illuminated with a great number of ſtars.

Geocentric place of a planet, is that part where it is ſeen from the Earth.

Heliacal riſing of a ſtar, is when it appears above the horizon, before the Sun in the morning: and heliacal ſetting of a ſtar, is when it is not ſeen after the Sun in the evening.

Heliocentric place of a planet, is that part in which the planet is ſeen from the Sun.

Hemiſphere, the half of a globe or ſphere, and is either celeſtial or terreſtrial.

Horizon, is the circle which ſeparates the viſible from the inviſible hemiſphere, and is either ſenſible or rational. The former paſſing over the ſurface of the Earth, and the latter through the centre.

Hour circles, are great circles paſſing through the poles of the world.

Immerſion, the moment when an eclipſe begins on a planet.

Inclination, the angle which the orbit of one planet makes with that of another.

Latitude of a ſtar or planet, is its diſtance from the ecliptic, reckoned in degrees, minutes, &c. upon the arc of a great circle.

Longitude of a ſtar or planet, is its diſtance from the firſt point of Aries, in degrees, minutes, &c. upon the ecliptic.

Luminaries, the Sun and Moon, ſo called by way of eminence.

Lunation, the time between one new Moon and the next.

Magnitudes, the different claſſes of the ſtars, of which there are uſually reckoned ſix or eight.

Mean motion of a planet, is that which would take place if it moved in a perfect circle, and an equal ſpace every day.

Meridian, that great circle which paſſes through the poles and the zenith of any place.

Minute, the 60th part of an hour in time, or the ſame part of a degree of ſpace.

Nadir, that point in the heavens directly under our feet.

Nodes, the two points where the orbit of a planet interſects the ecliptic.

Northern ſigns of the ecliptic, are thoſe ſix on the north of the equinoctial, viz. Aries, Taurus, Gemini, Cancer, Leo, and Virgo.

Nucleus, the head of a comet, or the central part of a planet.

Oblique aſcenſion of the Sun, or a ſtar, is an arc of the equinoctial contained between the firſt degree of Aries, and that point of it which riſes with the Sun or ſtar.

Oblique Sphere, is that poſition of the globe when either pole is above the horizon leſs than 90 degrees.

Oppoſition, when two ſtars or planets are 180 degrees diſtant from each other.

Orbit, the path a planet deſcribes in its courſe round the Sun.

Orbis magnus, the orbit of the Earth.

Parallax, the difference between the places of any celeſtial body, as ſeen from the centre, and from the ſurface of the Earth.

Parallax of the Earth's annual orbit, is the angle at any planet which is ſubtended by the diſtance between the Sun and Earth.

Parallels of latitude, are ſmall circles of the ſphere, parallel to the equator.

Perigeon, that point of a planet's orbit in which it is neareſt the Earth.

Perihelion, that point of a planet's orbit neareſt the Sun.

Pole ſtar, a ſtar of the ſecond magnitude in the tail of the Greater Bear, ſo called from being ſituated near the North Pole of the world.

Poles of the world, the two points at the extremities of the Earth's axis.

Preceſſion of the equinoxes, a ſlow motion of theſe two points, whereby they are found to go backwards about 50 ſeconds in a year.

Quadrant, the fourth part of a circle; alſo an inſtrument for meaſuring angles.

Retrograde, is that motion by which ſome of the planets ſeem to go backwards, or contrary to the order of the ſigns.

Right aſcenſion, is that degree of the equator which comes to the meridian with any celeſtial body, reckoning from the firſt point of Aries.

Satellites, the ſecondary planets.

Second, the ſixtieth part of a minute, either of time, or ſpace.

Solſtitial points, are the two points in the ecliptic through which the ſolſtitial colure paſſes.

Stationary, a planet is ſaid to be ſtationary when it has no apparent motion.

Syſtem, a number of bodies revolving round a common centre; as the ſolar ſyſtem.

Syzigies, thoſe points of the Moon's orbit where ſhe is at the new and full.

Teleſcopic ſtars, are ſtars only diſcoverable by means of a teleſcope.

Tranſit, is the paſſing of celeſtial bodies before one another.

Twilight, that faint light we perceive before the riſing and after the ſetting of the Sun, occaſioned by the Earth's atmoſphere.

Vector radius, a line ſuppoſed to be drawn from any planet to the Sun, which moving with the planet, deſcribes equal areas, in equal times.

Zenith, that point of the heavens directly over our heads.

Zodiac, that zone ſurrounding the heavens on each ſide of the ecliptic, in which all the planets perform their motions.

CHAP. XV.

OF MECHANICS.

Definitions.

1. THE *mechanical powers* are certain ſimple machines uſed for raiſing greater weights, or overcoming greater reſiſtances, than the natural ſtrength of man can perform without them.

2. Theſe ſimple machines are reckoned ſix in number: viz. 1. The *lever*; 2. the *wheel*; 3. the *pulley*; 4. the *ſcrew*; 5. the *wedge*; 6. the *inclined plane*.

3. *Force* is a power exerted on a body to move it; if it act inſtantaneouſly, it is called *percuſſion*, or impulſe; if conſtantly, it is an *accelerative* force.

4. *Gravity* is that force wherewith the body endeavours to fall downwards: it is called *abſolute gravity* when in an empty ſpace, and *relative gravity* when immerſed in a fluid.

5. *Specific* gravity is the proportion which the weight of one body bears to that of another.

6. The *centre of gravity* is a certain point in a body, upon which the body, when ſuſpended, will reſt in any poſition.

7. The *centre of motion* is a fixed point round about which a body moves. And the axis of motion is that fixed line about which it moves.

8. *Power* and *weight*, when oppoſed to each other, ſignify the body that moves another, and the body that is moved;

the body which communicates the motion is the power, and that which receives the motion is the weight.

9. *Friction* is the reſiſtance which any machine ſuffers by the parts rubbing againſt each other.

In the practice of mechanics, though all bodies are rough in ſome degree, and all engines imperfect; yet it is neceſſary to conſider all planes as perfectly even, all bodies perfectly ſmooth, and all bodies and machines to move without friction or reſiſtance, all lines ſtraight and inflexible, all cords very pliable, &c.

SECT. I.

ON THE SIX MECHANICAL POWERS.

The whole principles of relative motion in mechanics depend upon this one ſingle rule;—*That the whole force of a moving body is the reſult of its quantity of matter multiplied by the velocity of its motion.* Thus, when the product ariſing from the multiplication of the particular quantities of matter in any two bodies by their reſpective velocities are equal, the entire forces are ſo too. For example:—ſuppoſe a body, A, which weighs 40 pounds, to move at the rate of two miles in a minute; and another body B, which weighs only four pounds, to move 20 miles in a minute: the entire forces with which theſe two bodies will ſtrike againſt any other would be equal to each other, and therefore it would require equal powers to ſtop them; for 40 multiplied by 2, gives 80, the force of the body A: and 80 is alſo the product of 4, multiplied by 20, the force of the body B. Thus, the heavier any body is, the greater is the power required, either to move or ſtop it. And again, the ſwifter it moves, the greater is its force; therefore, when two bodies are ſuſpended on any machine, ſo as to act contrary to each other, if the machine be put in motion, and the perpendicular aſcent of one bedy, multiplied into its weight, be equal to the perpendicular deſcent of the other body, multiplied into its weight; thoſe bodies, how unequal ſoever in their weights, will balance one another in all ſituations; for as the whole aſcent of one is performed in the ſame time with the whole deſcent of the other, their reſpective velocities muſt be directly as the ſpaces through which they move; and the exceſs of weight in one body is compenſated by the exceſs of velocity in the other. Upon this principle the power of any machine may be eaſily computed; for it is only finding how much ſwifter the power moves than the weight does (that is, how much farther in the ſame time), and juſt ſo much power is gained by the engine.

A *lever* is a bar, either of iron or of wood, one part of which is ſupported by a prop, as its centre of motion. And the velocity of every part or point in the lever is directly as its diſtance from the prop.

There are four kinds of levers:—1. The *common lever*, where the prop is placed between the weight and power, but much nearer the weight than the power. 2. Where the prop is at one end of the lever, the power at the other end, and the weight between them. 3. Where the prop is at one end, the weight at the other end, and the power applied between them. 4. The *bended lever*, which differs from a lever of the firſt ſort only in being bent. Levers of the firſt and ſecond kind are often uſed in mechanical engines; but the third kind are ſeldom uſed, as no power can be gained by them.

When the power is at the ſame diſtance from the prop as the weight is, and the power and weight are both alike, the machine will remain in equilibrium, and no power can be gained. This is the principle upon which the common balance is formed. Let C D (*fig.* 14, *plate* 19) be a beam or lever, E the middle point, or centre of motion, which may be conſidered as the prop; A B two weights hanging at the ends C and D; then, when the machine is ſuſpended at the point E, and put in motion, the points C and D, being equidiſtant from E, will deſcribe equal arches, therefore their velocities will be equal; and if the bodies A and B be alſo equal, then the motion of A will be equal to that of B, as the velocities and quantities of matter are equal; and conſequently, if the machine be at reſt, neither of the weights can move the other, but they will remain in equilibrium.

The uſe of the balance or common pair of ſcales, is to compare the weights of different bodies; for any body, whoſe weight is required, put into one ſcale, will be balanced by a body of the ſame weight, put into the other ſcale.

In order to have a pair of ſcales perfect, they ſhould poſſeſs the following properties:—1. The points of ſuſpenſion of the ſcales, and the centre of motion in the beam C, E, D, muſt be in a right line. 2. The arms C E and D E, muſt be of equal length. 3. The centre of gravity muſt be in the centre of motion E. 4. There ſhould be as little friction as poſſible. 5. The ſcales muſt be in equilibrium when empty.

If the centre of gravity of the beam be above the centre of motion, and one end of the balance be put lower than the other, that end will continually deſcend, till it be ſtopped at the handle H; but if the centre of gravity of the beam be below the centre of motion, the balance will preſerve an equilibrium.

Hence, to examine a pair of ſcales, let the weights in the two ſcales be in equilibrium, then change the weights to the contrary ſcales, and if they remain in equilibrium, the balance is true, otherwiſe it is falſe.

Let A B C (*fig.* 1, *plate* 19) repreſent a lever of the firſt kind, ſupported by the prop D; the parts A B and B C, on each ſide of the prop D, are called the arms of the lever; the end A of the ſhorter arm A B is applied to the weight to be raiſed, and the power is applied to the end C, of the other arm B C. The principal uſe of this lever is to looſen large ſtones which are fixed in the ground, or to raiſe great weights to a ſmall height, in order to place rollers under them, or ropes for raiſing them higher by other machines.

In this lever, the ſhorter arm A B ſhould be as much thicker than the longer arm B C, as will be ſufficient to balance it on the prop D. Thus, if P repreſent a power whoſe weight is equal to one ounce, and W a weight of twelve ounces, and if the power be twelve times farther from the prop than the weight is, they will exactly counterpoiſe each other; and a ſmall addition applied to the power P will raiſe the weight W; and the velocity with which the power deſcends will be to the velocity with which the weight riſes, as 12 is to 1; that is, directly as their diſtances from the prop; and conſequently, as the ſpaces through which they move. Thus, it is evident, that if a man, by his natural ſtrength, could lift an hundred weight, he will, by a lever of this ſort, be able to raiſe twelve hundred weight. If the weight be leſs, or the power greater, than in the foregoing caſe, the prop may be placed ſo much farther from the weight, and then it can be raiſed to a proportionably greater height by the ſame addition of force: but if the weight be greater, or the power leſs, the prop ſhould be placed ſo much nearer the weight. For, univerſally, if the gravity of the weight, multiplied by its diſtance from the prop, be equal to the gravity of the power, multiplied by its diſtance from the prop, the power and weight will exactly balance each other. Thus, if the weight W be twelve ounces, and its diſtance from the prop 1 inch; the product of 12 multiplied by 1 is 12; and if the power P be 1 ounce, and its diſtance from the prop 12 inches, the product of theſe two is alſo 12; therefore they counterpoiſe each other. If a power equal to 2 ounces be applied at 6 inches diſtance from the prop, it will alſo balance the weight W, for 6 multiplied by 2 is 12. And if the power be 3 ounces, and placed at 4 inches diſtance from the prop, it would alſo balance the weight W, for 3 times 4 is 12. And the like in any other proportion. A poker ſtirring a fire is a lever of this kind; the bar upon which it reſts is the prop, the hand applied to the end of it is the power, and the incumbent coals on the other end the weight. Several ſorts of inſtruments are formed of two levers of this kind, as, ſciſſars, ſnuffers, pincers, &c.; the prop, or centre of motion, is the pin which holds them together.

The *ſtatera*, or Roman ſteelyard, is a lever of this kind, and is uſed to find the weight of any body by one ſingle weight, placed at different diſtances from the prop. G X (*fig.* 13) is a ſteelyard, ſuſpended by the hook O, from the centre of motion D; the ſhorter arm D G is of ſuch a weight as exactly to counterpoiſe the longer arm D X; if this longer arm be divided into as many equal parts as it will contain, and each part equal to O D, the ſingle weight P will weigh any body as heavy as itſelf, or as many times heavier as there are diviſions in the arm D X. Thus, if the weight P be one pound, and placed at the firſt diviſion 1, in the arm D X, it will balance one pound in the ſcale W; if it be removed to the ſecond diviſion at 2, it will balance two pounds in the ſame ſcale; if to the third, three pounds, &c. And if each of theſe integral diviſions could be divided into as many equal parts, as a pound contains ounces, then the weight P, placed at any of theſe ſubdiviſions, would ſhow the odd ounces, over and above the number of pounds of the body in the ſcale.

The ſecond kind of levers have the weight between the prop and the power (*fig.* 2). In this, as well as the former, the advantage gained, is as the diſtance of the power from the prop, to the diſtance of the weight from the prop; and the rules for computing the force of this lever are the ſame with thoſe of the former. Thus, if W be a weight of ſix ounces, hanging at the diſtance of one inch from the prop G, and P a power or weight of one ounce, hanging at the end B, ſix inches diſtant from the prop (by the cord C D running over the fixed pulley E), the power will juſt ſupport the weight: and a ſmall addition to the power will raiſe the weight one inch for every ſix inches that the power deſcends. Thus the power acts with the ſame force upon the weight, as it would do, if the weight were at the ſame diſtance from the prop, and on the other ſide thereof, in which caſe it would be a lever of the firſt ſort.

Two men carrying a burden upon a ſtick exhibit a ſpecimen of a lever of this kind; and the portion of weight borne by each man, is in proportion to his diſtance from the weight. In yoking two horſes of an unequal ſtrength to draw any load, the point of taction is placed as much nearer to the ſtronger horſe than to the weaker, as the ſtrength of the former exceeds that of the latter.

Of this kind of levers are oars, rudders of ſhips, doors turning upon hinges, cutting knives fixed at the point, &c.

The third kind of lever has the power applied between the weight and the prop, in which, in order that the power may counterpoiſe the weight, the gravity of the power muſt exceed that of the weight, as much as the diſtance of the weight from the prop exceeds the diſtance of the power from the prop. Thus, if E (*fig.* 3) be the prop of the lever A B, and W a weight of one pound, which is placed three times as far from the prop as the power P, which acts at F by the cord C going over the pulley D; the power P muſt be three pounds, to counterpoiſe the weight of W of one pound;

and for every inch the power P deſcends, the weight W will aſcend three inches, &c.

Levers of this kind are very little uſed, becauſe they give no advantage in point of force, though they give an advantage in point of motion; but in ſome caſes they become neceſſary, as in raiſing a ladder againſt a wall, in which caſe the foot of the ladder, which is fixed againſt the ground, is the prop, the man's hand who raiſes it, the power, and the whole length of the ladder, from the hand to the upper end, is the weight.

The bones of a man's arm are likewiſe levers of this kind; for the muſcle which raiſes the arm, is fixed to the bone about a tenth part as far below the elbow as the hand is. Therefore the elbow may be conſidered as the prop upon which the lower part of the arm turns, and the muſcle muſt conſequently exert a force ten times as great as the weight which is raiſed in the hand.

The fourth kind of lever has all the properties of the firſt kind, and differs from it only in being bent, which is done for the ſake of convenience. A C B (*fig.* 10) is a lever of this ſort, bent at C, its prop or centre of motion; W the weight, and P the power acting at A, over a pulley by means of the cord D. As the mechanical power of this lever is the ſame with that of the firſt ſort, it need not be repeated. A hammer drawing a nail is a lever of this ſort.

The ſecond mechanical power is the *wheel* and *axle* (*fig.* 4), in which the power is generally applied to the circumference of the wheel, and the weight W to that of the axle; the weight being raiſed by a rope winding round the axle as the wheel turns round. In this inſtrument it is evident, that the velocity of the power muſt be, to that of the weight, as the circumference of the wheel is to the circumference of the axle: and the power gained is in proportion to the circumference of the wheel to that of the axle. Therefore, when the gravity of the power is to that of the weight, as the circumference of the axle is to the circumference of the wheel, the power and weight will balance each other. Again, let A B be a wheel, E D the axle, and the circumference of the wheel eight times as great as that of the axle; then a power P of one pound weight hanging by the cord I, which goes round the wheel, will balance the weight W of eight pounds hanging by the rope K, which goes round the axle; and a ſmall addition to the power will cauſe it to deſcend and raiſe the weight; but the weight will riſe with only an eighth part of the velocity wherewith the power deſcends; and conſequently will move through only an eighth part of an equal ſpace in the ſame time. If the wheel be pulled round by the handles S S, the power will be increaſed in proportion to their length.

In this mechanical power the radius of the wheel and the oppoſite radius of the axle may be conſidered as the longer and ſhorter arms of a lever of the firſt kind, the centre of the axle being the prop.

Sometimes the wheel, or the axle, is indented, or cut into teeth, which have another wheel working in them, as in jacks, clocks, mill-work, &c. by which means they give a much greater mechanical force. To compute the power of a combination of wheels, multiply the radii of all the axles continually together, as alſo the radii of all the wheels; then, as the former product is to the latter, ſo is a given power applied to the circumference of a wheel to the weight it can ſuſtain. For example: in a combination of five wheels and axles, to find the weight a man can ſuſtain or raiſe, whoſe force is equal to 150 pounds, the radii of the wheels being 30 inches, and the radii of the axles three inches. Here 3 multiplied four times into itſelf, produces 243; and 30 multiplied four times into itſelf, produces 24,300,000; therefore, as 243 is to 24,300,000, ſo is 150 to 15,000,000 pounds, the weight he can ſuſtain, which is more than 6696 tons, or above 100,000 times as great a weight as he could ſuſtain by his own natural force.

But here it muſt be obſerved, that though there is a prodigious gain of power in theſe combinations of wheels, yet there is a great loſs of time; that is, the weight in this caſe will move 100,000 times ſlower than the power; and this is true in all mechanical caſes whatever.

The third mechanical power is the *pulley*, or ſometimes a ſyſtem of pullies. Sometimes theſe are fixed in a block or caſe, which is alſo fixed; at other times they are in a block which is moveable, and riſes with the weight. The ſingle pulley A (*fig.* 9) gives no mechanical advantage, though it may ſerve to change the direction of the power; but is only as the beam of a balance, whoſe arms are of equal length and weight; and is, in fact, but another form of the balance.

The ſyſtem of pullies is repreſented *fig.* 12, where the four pullies are faſtened to an immoveable block above; three of them, A, B, C, by the three diſtinct cords running under them. The power of this ſyſtem of pullies is diſcovered by ſuppoſing W a weight of 16 pounds, ſuſpended from the pulley C, which is alſo ſuſpended by the cord C, one end of which is faſtened to the block above, and the other end ſupported by the pulley B; therefore the pulley B ſuſtains only half the weight of the weight W, or eight pounds; the other half being ſuſtained by the cord C, fixed to the block. Then the cord B, which goes under the ſecond pulley, ſuſtains the weight of eight pounds, which is alſo divided, four pounds being ſuſtained by the cord B, fixed in the block above, and the other four pounds by the next pulley A. This next pulley A alſo has its weight divided, one half being ſupported by the cord A fixed to the block, and the other half ſupported by the ſmall pulley, which ſmall pulley again divides the weight it ſupports: ſo that the power P is equal to only one pound, which will counterpoiſe the weight W of 16 pounds.

The velocity of the weight to that of the power is as the gravity of the power is to that of the weight. Thus if P deſcend eight inches, A will aſcend four inches, B two inches, C one inch, and W half an inch.

A, B, C, D, (*fig.* 11,) are four pullies, two of which, A and B, are in a fixed block X; the two others, C and D, in a moveable block. Here the weight W is raiſed by pulling the cord at P, which goes ſucceſſively over the four pullies, and is faſtened at the end to the fixed block at f. The purchaſe of this machine is ſeen to be as 4 to 1, for P is ſuſtained by the ſingle cord; but W by four folds of the cord, viz.—*e*, *ſ*, *u*, *k*, ſo that if P be one pound, W will be four.

The velocity of the power to that of the weight is alſo, as in the former caſe, as the gravity of the weight to that of the power, or as 4 to 1; for when P deſcends four inches, the parts of the cord at k will aſcend four inches towards e, and all the other parts of the cord will equally follow each other; and as there are four folds in the cord, viz. *e*, *s*, *u*, *k*, they will each of them be ſhortened one inch, and C or W will be ſo much raiſed.

In the ſame manner the purchaſe of any combination of pullies may be determined; for the momenta of the weight and power will always be equal; as in the other mechanical powers.

The fourth mechanical power is the *inclined plane*. In this machine the advantage gained is as great as its length exceeds its perpendicular height. Let A B (*fig.* 5) be a plane parallel to the horizon, and C D a plane inclined to it; if the length C D be three times as great as the perpendicular height G F, the cylinder E will be ſupported on the plane C D, by a power equal to a third part of the weight of the cylinder; or, it may be rolled up the plane with a third part of the power, which would be ſufficient to draw it up the ſide of an upright wall. If the plane were four times as long as the perpendicular height, it would require only the fourth part of the power; and ſo on in proportion. The uſe of this power is to raiſe a great weight to any eminence, which is uſually done by puſhing it up a ſtout plank, ſet ſloping to the place deſigned; and ſuch plank, or other contrivance ſimilar thereto, is called an inclined plane.—Now it is evident, the ſteeper the aſcent is, the more difficult it is to puſh any weight up it; and the more the aſcent inclines to the horizon, the eaſier the weight may be puſhed up. This is evident from the eaſe with which a rolling weight is forced up a hill, that riſes gently, while it is ſo difficult to roll the ſame weight up a hill which is very ſteep.

The force wherewith a rolling body deſcends upon an inclined plane is to the force of its abſolute gravity, as the height of the plane is to its length. Thus, if the perpendicular height G F of the plane be equal to half its length A B, the cylinder E will roll down the plane with a force equal to half its weight; and it would require a power equal to half its weight to ſuſtain it. If the plane be ſo much elevated as to be perpendicular to the horizon, the cylinder E would deſcend with its whole weight, becauſe the plane contributes nothing to its ſupport or hindrance.

In an inclined plane, a power acts to the greateſt advantage, when its direction is parallel to the ſurface of the plane.

The *wedge* conſtitutes the fifth mechanical power, and may be conſidered as two equally inclined planes, A D F (*fig.* 6) and C B F joined together at their baſes F O; then D C is the whole thickneſs of the wedge A B C D, or the back of the wedge, where the power is applied; E F the height or depth, D F the length of one of its ſides, equal to C F, the length of the other ſide, and O F its ſharp edge, which is driven into the wood intended to be ſplit by the force of a hammer or mallet ſtriking on its back. Thus, A B is a wedge (*fig.* 7) driven into the cleft C E D of the wood F G.

The power gained by the wedge is in proportion to the length of the ſlant ſide to half the thickneſs of the back. Thus, if the back of the wedge be two inches thick, and the ſide 20 inches long, any weight performing on the back will balance 20 times as much acting againſt the ſides. To uſe a wedge to the greateſt advantage, it ſhould be forced, not by preſſure, but by percuſſion, as by the blow of a hammer or mallet; by which means a wedge may be driven in below any weight, and ſo made to lift it up, as the largeſt ſhips, &c.

The wedge has a very great mechanical force, and effects what would be impoſſible by the lever, wheel and axle, or pulley; for the force of the blow ſhakes all the adjacent parts, and thereby makes them ſeparate more eaſily; ſo that not only wood, but even rocks, can be ſplit by it.

To the wedge may be referred the axe or hatchet, the chiſel, the ſpade and ſhovel, knives of all kinds; as alſo the bodkin and needle, and all ſorts of inſtruments which, beginning from an edge or point, become gradually thicker.

The ſixth, and laſt mechanical power, is the *ſcrew*, which is not properly a ſimple machine, becauſe it cannot be uſed without a lever to turn it, called the winch or handle. It is a compound engine of very great force, and is a kind of perpendicular or endleſs inclined plane, ſtill farther aſſiſted by the power of the handle or lever: and the gain of power is in proportion of the circumference deſcribed by the power to the diſtance between one thread and the next in the ſcrew.

Thus, let C be a wheel (*fig*. 8), having a ſcrew *a b*, on its axis, working in the teeth of the wheel D, which ſuppoſe to be 48 in number. Then it is evident, that for one revolution of the wheel C, ſcrew *a b*, and winch A, the wheel D will be moved one tooth by the ſcrew: and therefore in 48 revolutions of the winch A, the wheel D will be turned once round. Then if the circumference of the circle deſcribed by the handle of the winch A be equal to the circumference of a groove round the wheel D, the velocity of the handle will be 48 times as great as the velocity of any given point in the groove; conſequently, if a line G goes round the groove D, and has a weight of 48 pounds ſuſpended from it, below the pedeſtal E F, a power equal to one pound at the handle will ſupport this weight; or, if a groove be made in the wheel C, equal in radius to the circle deſcribed by the handle, the weight H of one pound, ſuſpended therefrom by a line in the groove, will balance the 48 pounds, as before. If the line G, inſtead of going round the groove of the wheel D, go round its axle I, the power of the machine will be as much increaſed, as the circumference of the groove exceeds that of the axle, as ſhewn under the wheel and axle. And if a ſyſtem of pullies were applied to the cord H, the power could be increaſed to an amazing exceſs.

The uſes to which the ſcrew is applied are various; it is chiefly uſed for preſſing bodies cloſe together, as the preſſes for bookbinders, packers, hot-preſſers, &c.

The friction in the ſcrew is very conſiderable, as it is alſo in the wedge, which generally requires a third part more of the power to work them when loaded, than what is ſufficient to conſtitute a balance between the weight and power.

If machines or engines could be made without friction, the leaſt degree of power above what is ſufficient to balance the weight, would be ſufficient to raiſe it. In the lever the friction is little or nothing: in the wheel and axle it is but ſmall: in pullies it is conſiderable: and in the inclined plane, wedge, and ſcrew, it is very great.

Wood greaſed, or metal oiled, have nearly the ſame friction; and the ſmoother they are, the leſs is their friction, provided they be not too highly poliſhed. In poliſhed ſteel moving upon poliſhed ſteel or pewter, the friction is about a fourth part of the weight, on copper a fifth part, and on braſs a ſixth part of the weight: iron or ſteel running in braſs has the leaſt friction of any. And metals of the ſame ſort have more friction than different ſorts; and in general the friction increaſes in the ſame proportion with the weight, but is greater with a greater velocity.

The friction in pullies is now almoſt reduced to nothing, by the contrivance of Mr. Garnett, in his patent friction rollers, which produce a great ſaving of labour and expenſe, as well as wear of the materials, both when applied to pullies and the axles of wheel-carriages. By this contrivance, there is a hollow ſpace left between the nave and axle, or centre and pin-box, which is filled up by ſolid equal rollers, nearly touching each other, and furniſhed with axles, each of which is inſerted into a circular ring at each end, by which their relative diſtances are preſerved; and they are kept parallel by means of wires faſtened to the rings between the rollers, and to which the wires are rivetted.

It is a general property in all the mechanic powers, that when the weight and power balance each other, if they be put in motion, the power and weight will be to each other reciprocally as the velocities of their motion; or the power is to the weight as the velocity of the weight is to the velocity of the power; ſo that their two momenta are equal: viz.—The product of the power, multiplied by its velocity, is equal to the product of the weight multiplied by its velocity. And hence the general rule: viz. *That what is gained in power is loſt in time.* For the weight moves as much ſlower as the power is leſs.

SECT. II.

OF THE APPLICATION OF THE POWERS TO MILLS AND MACHINES.

In order to diſcover the properties of any machine conſiſting of the mechanical powers, it is neceſſary to conſider the weight that is to be raiſed, or the reſiſtance to be overcome; and alſo the power required to raiſe the weight, or overcome the reſiſtance. For this purpoſe, there are two principal problems, the reſolution of which is requiſite to ſhow the powers of any engine.—The firſt problem is, *To determine the proportion that the power and weight ought to have to each other, that they may be in the juſt equilibrium.*—The ſecond is, *To determine what the proportion ſhould be between the power and weight, that the machine may produce the greateſt effect in a given time.*

The firſt problem is ſolved by this general rule, viz.—That the power and weight ſuſtain each other, or are in equilibrium, when the power and weight are reciprocally proportional to the diſtances of the directions in which they act from the centre of motion; or when the product of the power multiplied by the diſtance of its direction is equal to the product of the weight multiplied by the diſtance of its direction. This is the proportion of the weight and power when they are in equilibrium, ſo that the one would not prevail over the other if the engine were at reſt; and if it be ſet in motion, it would continue to proceed uniformly if there were no friction of its parts and other reſiſtances. And in general the effect of any power or force is as the product of that force multiplied by the diſtance of its direction from the centre of motion; or the product of the power, and its velocity when in motion, for the velocity is proportional to the diſtance from that centre.

The ſecond general problem in Mechanics, is of the greateſt importance, though it has been little attended to by mechanical writers, viz.—To determine the proportion between the power and weight, ſo that when the power prevails, and the machine is in motion, the greateſt effect poſſible may be produced by it in a given time. When the power is only a little greater than what is ſufficient to ſuſtain the weight, the motion is uſually too ſlow; and though a greater weight be raiſed in this caſe, it is not ſufficient to compenſate for the loſs of time. And when the power is much greater than what is ſufficient to ſuſtain the weight, the weight is raiſed in leſs time; but it often happens, that this is not ſufficient to compenſate for the loſs which ariſes from the load being reduced; therefore, the only general rule that can be given is, to find when the product of the weight, multiplied by its velocity, is the greateſt; for this product meaſures the effect of the machine in a given time, which is always greater in proportion as both the weight and velocity are greater.

In the conſtruction of compound machines, where it is neceſſary to alter the direction of the motion, recourſe muſt be had to what is called *bevel geer*, the principle of which is as follows:—

Let A and B (*fig.* 15) be two cones revolving on their centres a c and a b; if their baſes be equal, they will each perform their revolution in the ſame time; and any two points in each cone equally diſtant from the centre, as d 1, d 2, d 3, &c. will revolve in the ſame time as f 1, f 2, f 3, &c. reſpectively. But if one cone be twice the diameter of the other, as the cone a d e (*fig.* 20), which is twice the diameter of the cone f a d, then as they turn upon their

centres,

centres, when the cone a f d has made one revolution, the cone a d e will have made but half a revolution, and every part in each cone, equally diſtant from the centre a, will have the ſame proportion in their revolutions to each other, as f 1, f 2, f 3, &c. will have made two revolutions to the points e 1, e 2, e 3, &c. for one revolution of the other cone reſpectively, &c. Now, if the cones are fluted, or have teeth cut in them, diverging from the centre a to the baſes d e, d f (*fig.* 16), they would then become bevel geer. The teeth at the point of the cone being ſmall, and of little uſe, may be cut off; or, inſtead of the two cones, may be uſed two ſhafts, with bevel wheels fixed to them, as the ſhaft a b (*fig.* 18), with the bevel wheel c d, which turns the bevel wheel e f, with its ſhaft b g, and the teeth work freely into each other, as in figure 16. The teeth may be made of any dimenſions, according to the ſtrength required, and by this means a motion may be communicated in any direction, or to any part of a building, with very little trouble and friction.

The method of conſtructing the wheels for any proportion, is as follows:—Draw the line a b (*fig.* 21) to repreſent a ſhaft of a wheel; draw the line e d to interſect the line a b, in the direction that the motion is to be conveyed, and the line e d will repreſent the other ſhaft of the motion.

Then ſuppoſe the ſhaft e d is to revolve three times in the time that the ſhaft a b revolves once; draw the parallel line i i at any diſtance, from a ſcale (ſuppoſe one foot); then draw the other parallel line k k at three feet diſtance; after which, draw the line w x through the interſections of the two ſhafts a b and e d, and likewiſe through the interſections of the two parallel lines i i and k k, in the points x y, which will be the pitch line of the two bevel wheels, or the lines where the teeth of the two wheels act on each other, as may be ſeen in figure 19, where there are three wheels.

Where it is required to communicate a continued uniform motion, and where the angle does not exceed 40 degrees, and alſo where the equality of the motion is not regarded, the univerſal joint may be uſed (*fig.* 22) inſtead of the bevel geer. This joint may be conſtructed by a croſs, as ſhown in the figure; or with four pins faſtened at right angles upon the circumference of a hoop, or ſolid ball. This is of great uſe in ſome machines, where the tumbling ſhafts are continued to a great diſtance from the moving power, as it is in cotton-mills. The ſhafts, by applying this joint, may alſo be cut to any length, which is a great advantage where there is much reſiſtance.

CHAP. XVI.

OF ELECTRICITY.

SECT. I.

THE PRACTICAL PART OF ELECTRICITY.

THE earth, air, and all terreſtrial bodies are ſuppoſed to contain a certain quantity of an elaſtic ſubtle fluid, called by philoſophers, the *electric fluid;* and when any body poſſeſſes more or leſs of this fluid than what naturally belongs to it, ſeveral effects are viſible in it, and the body is ſaid to be electrified.

This certain quantity of electric fluid found in all bodies could never be increaſed or diminiſhed, if all bodies admitted the paſſage of this electric fluid through their pores or along their ſurfaces; but there are many bodies which will not ſuffer this fluid to paſs through them, while others freely permit it. Thoſe bodies through which the electric fluid can paſs are called *conductors* of electricity, of which the moſt perfect are metals of all kinds. And thoſe bodies through which the electric fluid cannot paſs are called *non-conductors* of electricity, of which the moſt perfect are glaſs, reſin, ſealing-wax, ſulphur, bees-wax, and baked wood, among ſolids; and oils and air, among fluids. But all ſubſtances become conductors when they are made very hot. Conducting ſubſtances are alſo called *non-electrics*, and non-conducting ſubſtances are called *electrics*. Into theſe two claſſes all bodies are divided by electricians.

When any body has acquired an additional quantity of electric matter, and is ſurrounded with other bodies through which the electric fluid cannot paſs, or non-conductors, it muſt remain overloaded; or if it have loſt part of its natural ſhare of electric matter, it muſt remain exhauſted; becauſe the bodies which ſurround it prevent any of the electric fluid from entering or coming out of it, and the body is then ſaid to be inſulated.

There are two principal theories of electricity, each of which has had its advocates. The one is, that of two diſtinct electric fluids, repulſive with reſpect to themſelves and attractive of one another; adopted by M. Du Fay, on diſcovering the two oppoſite ſpecies of electricity, viz. the *vitreous* and *reſinous*, which is ſince new-modelled by Mr. Symmer. Upon this hypotheſis theſe two fluids are equally attracted by all bodies, and exiſt in intimate union in their pores; and in this ſtate they ſhow no mark of their exiſtence.

But the friction of an electric body againſt a rubber ſeparates theſe fluids, and cauſes the vitreous electricity of the rubber to paſs to the electric, then to the prime conductor of the machine, while the reſinous electricity of the conductor and electric is communicated to the rubber: thus the quality of the electric fluid poſſeſſed by the conductor and the rubber is changed, while the quantity remains the ſame in each. In this ſeparated ſtate the two electric fluids will exert their reſpective powers; and any number of bodies charged with either of theſe may repel each other, attract thoſe bodies that have leſs of each particular fluid than themſelves, and be ſtill attracted more by bodies that are either only deſtitute of it, or loaded with the contrary. In this theory the electric ſpark makes a double current; one fluid paſſing to the electrified conductor from any ſubſtance preſented to it, while the ſame quantity of the other fluid paſſes from it; and when each body receives its natural quantity of both fluids, the balance of the two powers is reſtored, and both bodies are unelectrified.

The other theory, and that which is commonly received, is that diſtinguiſhed by the name of *poſitive* and *negative electricity*, ſuggeſted by Dr. Watſon, and demonſtrated by Dr. Franklin; in which it is ſuppoſed, that all bodies poſſeſs a certain ſhare of one and the ſame fluid, which is extremely ſubtle and elaſtic, by which the particles of it are ſtrongly attracted, as they are repelled by one another. When bodies poſſeſs their natural ſhare of this fluid, they are ſaid to be in an unelectrified ſtate; but when the equilibrium is deſtroyed, and they have an additional quantity *from* other bodies, or when they loſe part of their natural ſhare by the communication *to* other bodies, they then become electrified, and exhibit electrical appearances; which are generally the ſame in both caſes. In the former caſe they are ſaid to be electrified poſitively, or *plus*; and in the latter caſe negatively, or *minus*. It is alſo ſuppoſed, that electrics always contain an equal quantity of this fluid; ſo that there can be no increaſe on one ſide without a proportional decreaſe or loſs on the other, and *vice verſa*. And as the electric will not ſuffer the fluid to paſs through its pores, there will be an accumulation on one ſide, and a correſponding deficiency on the other; then, connecting both ſides together by proper conductors, the equilibrium will be reſtored, by the ruſhing in of the redundant fluid from the overcharged ſurface to the exhauſted one. Thus, if an electric be rubbed by a conducting ſubſtance, the electricity is only conveyed from one to the other, the one giving what the other receives: and if one be electrified poſitively, the other will be electrified negatively, unleſs the loſs be ſupplied by other bodies connected with it, as in the caſe of the electric and inſulated rubber of a machine. Thus, bodies differently electrified will naturally attract each other, till they mutually give and receive an equal quantity of the electric fluid, and then the equilibrium between them will be reſtored.

The method of diſturbing the equilibrium of the electric fluid in bodies, or of making it paſs from one to another, is chiefly friction, or a ſlight rubbing of them one againſt the other; when the electric fluid will generally leave the rougher ſurface, and paſs upon the ſmoother; or it leaves the leaſt perfect electric, and paſſes to the more perfect one of the two. Thus, if a ſmooth glaſs tube (*fig.* 1, *plate* 20) be drawn through the hand, the effect of the friction makes the electric fluid leave the hand and paſs to the glaſs tube, which is the more perfect electric of the two, where it will remain in addition to its natural quantity. For the electric fluid cannot poſſibly leave the glaſs, becauſe neither the glaſs nor the ſurrounding air are conductors of electricity; but if a conducting ſubſtance, as the finger, or a piece of metal, be preſented to any part of the glaſs, the electric fluid will leave the glaſs and paſs into them; and if the finger, or metal, be preſented to every part of the tube ſucceſſively, the whole of the redundant fluid will leave the tube, and it will retain only its natural ſhare. Here the glaſs is ſaid to be *excited*, becauſe the friction ſeems to excite the electric power which was in the glaſs.

In the ſame manner the friction of the glaſs globe againſt the rubber in the electrical machine makes the electrical fluid which was in the rubber paſs to the glaſs, from whence it is conveyed to the prime conductor, the points of which are preſented to every part of the globe in ſucceſſion, as it is turned in the machine; and as the friction is continued, there will be a conſtant ſupply of electric fluid to the prime conductor (though other bodies be preſented to it), and keep diſcharging all the while in viſible ſparks. The hand, in the former of theſe caſes, and the rubber, in the latter, part with their natural ſhare of electric fluid to the glaſs againſt which they are rubbed, but receive an immediate ſupply from the conducting ſubſtances to which they are connected; and theſe are again ſupplied by the general maſs of fluid that is in the earth.

Again, if a ſtick of ſealing-wax, a piece of ſulphur, or a tube of rough glaſs, be drawn through the hand, the electric fluid belonging to them will paſs from them to the hand, and being ſurrounded by the air, which is a non-conductor, they remain exhauſted, and are ready to take ſparks of electric fire from any bodies preſented to them. The ſulphur, ſealing-wax, &c. in this caſe are ſaid to be excited, as well as the glaſs, which was overloaded with fluid, though the ſtate they are in be the reverſe of one another. It is impoſſible to diſtinguiſh by the eye the courſe of the electric matter, its velocity is ſo great.

There are a variety of inventions for the conſtruction of the electrical machine, but the moſt ſimple is that repreſented in figure 2, which, by reaſon of its ſimplicity, is not liable to be put out of order, as it has neither wheel nor ſtring, though both might be attached thereto, if required. It may alſo be fixed firm on a table, and eaſily taken off:

the globe may alſo be taken out with the greateſt eaſe, in order to be packed up. This machine is the ſame as that uſed by Dr. Prieſtley; and when the inſide of the globe is lined with his compoſition following, it will produce more fire than any of thoſe in common uſe.

A is the baſe, which is a piece of mahogany about nine inches ſquare, and 1¼ thick, in which is fixed the pedeſtal B, to ſupport the globe G, which is fixed in an iron axle C, to which is fixed a braſs cap. The globe is turned by the handle H running in the braſs ſocket E; R is the rubber, made of wood, cut to the curve of the globe, and covered with a leather covering, which is at a little diſtance from the wood in the middle of the curve, that it may the better yield to the preſſure of the globe.

Over this leather is another leather, made to take off by moving a pin. On this leather the amalgam is rubbed; and as it is eaſily taken off, it is more readily brought into order than thoſe which are fixed to the rubber. To this leather is fixed a piece of black ſilk, which extends half round the globe, and greatly increaſes the fire; ſo that this machine will give fire well, if the rubber ſcarcely touch the globe. This machine will alſo ſuit any kind of conductor.

For thoſe who do not chooſe to have the rubber inſulated, there is a ſpring S; but the more curious may have them made with the rubber well inſulated by a glaſs pillar that will hold the rubber to the back of the globe, as in figure 3.

Dr. Prieſtley's Compoſition for lining the Inſide of Globes, or Cylinders.

This compoſition conſiſts of an equal quantity of linſeed oil and reſin, which is boiled over a gentle fire for two hours. When the globe or cylinder is to be lined, it muſt be put into an oven, with a ſufficient quantity of the compoſition broken and put into the inſide; and when it is melted, the globe is turned round every way, in order to ſpread it all over equally.

Inſtead of the above compoſition, ſome uſe a mixture made of four parts of Venice turpentine, one part of reſin, and one of bees-wax; which is prepared and uſed in the ſame manner as the former.

There are alſo other methods for making amalgam: as, 1. By four parts of ſpelter and ſix parts of mercury; 2. alſo by adding ſix ounces of quickſilver to one pound of molten tin, which, when cold and reduced into powder, is to be mixed with ſeven ounces of ſulphur, and ſix ounces of ſal ammoniac: the whole is ſublimated in a mattraſs.

The parts of the machine which are inſulated ſhould be varniſhed over with a varniſh made of highly rectified ſpirits of wine and ſealing-wax; as alſo the glaſs pillars, in order to keep off the moiſture they would imbibe from the damp air.

It is neceſſary for the young practitioner to attend to the following rules in the performing of his experiments, as it will often happen, that though he be in poſſeſſion of very good inſtruments, yet, through ſome inadvertencies, his experiments will not ſucceed according to his expectation, for want of a ſufficient practice in the art.

1. The electrical machine, coated jars, and every part of the electric apparatus, ſhould be kept clean, and free from duſt and moiſture.

2. In clear weather, when the air is dry, and particularly in froſty weather, the machine will always work well: but in hot weather, and damp weather (except it be brought in a warm room, and the apparatus made thoroughly dry), it will not work ſo well.

3. The cylinder ſhould always be wiped clean with a ſoft dry linen cloth that is warm, and then with a clean hot flannel, before the machine be uſed; then applying a little amalgam, turn the winch of the machine, and the electric fluid will come like a wind from the cylinder to the knuckle, and ſome ſparks and cracking will ſoon follow. This indicates that the machine is in good order. But if theſe appearances

be not produced, there is a fault, which is generally in the rubber; to remedy which, remove the rubber from the glaſs pillar, and dry the ſilk part before the fire, then greaſe the leather with a bit of tallow or mutton ſuet.

4. When the table on which the machine ſtands, and to which the chain of the rubber is connected, is very dry, it is a bad conductor, and hinders the operation of the machine. The floor, and walls of the room alſo, in very dry weather have the ſame effect on the machine. In this caſe the chain of the rubber ſhould be connected by a long wire, with ſome moiſt ground, or with the iron-work of a water-pump; by which means the rubber will be ſupplied with a ſufficient quantity of electric fluld.

5. If there be too much amalgam upon the leather of the rubber, the machine will not work well until a little be ſcraped off.

6. If the globe or cylinder contract any black ſpots, as is often the caſe, they ſhould always be wiped off.

7. In charging electric jars, they ſhould be made a little warm before they are uſed, and they will produce a greater effect.

8. When a large battery is to be diſcharged, never diſcharge it through a good conductor, except the circuit be at leaſt five feet long, otherwiſe ſome of the jars would be found broken.

To ſhow the Effects of electrical Attraction and Repulſion.

Suſpend a plate of metal F (*fig.* 2) from the conductor, which is ſupported by two glaſs pillars, and ſupplied with electricity from the globe; and at the diſtance of three or four inches below this, put another plate P of the ſame ſize; upon the bottom plate lay a feather, or ſmall ſlips of paper; and when the machine is ſet in motion, the feather or the papers will be attracted to the upper plate F, from which they will be immediately repelled, and will fly to diſcharge themſelves upon the lower plate P, which is ſupported on the pedeſtal G H; after which they will be attracted and repelled again as before, and fly from one plate to the other with great rapidity, if the electrification be ſtrong. It is uſual to cut the pieces of paper into the figures of men and women, when they exhibit a kind of dance, which affords ſome entertainment to the beholders.

The electrical bells are often uſed in concert with the above experiment, and depend on the ſame principle. Theſe are four bells *a*, *b*, *c*, *d*, which hang from the ends of two braſs rods (*fig.* 5), communicating with the prime conductor, and with another bell *e*, fixed on a pedeſtal A, reaching to the ground. Between the four bells hang four braſs balls, ſuſpended by ſilken ſtrings; each of theſe balls hangs between the centre bell *e* and one of the outermoſt bells. The outermoſt bells being connected with the prime conductor by braſs chains, are electrified, and attract the braſs balls which hang between them and the centre bell; and the attraction being ſtrong, each ball ſtrikes its outer bell with ſome violence, and makes it ring: being then loaded with electricity, it is immediately repelled, and flies to unload itſelf by ſtriking upon the centre bell, which is inſulated by the glaſs pillar B upon the pedeſtal A, and from which pillar the electric matter paſſes to the floor, by means of the braſs pedeſtal A. The balls are then again attracted by the outermoſt bells, as before; and thus the ringing may be continued as long as neceſſary.

When a perſon is to be electrified, and ſtands upon a ſtool with glaſs feet, or baked wood (*fig.* 7), having the chain in his hand, faſtened to the prime conductor, he is then ſaid to be inſulated, and may be conſidered as part of the prime conductor: for every part of his body will exhibit the ſame appearance as the prime conductor. For if the finger of any perſon, ſtanding upon the floor, be preſented to him, a ſpark of fire will be ſeen to iſſue from him, and both he and the perſon who receives it will feel a painful ſenſation; and a ſnapping a ſnapping noiſe will be heard. Every part of his body will then attract all light ſubſtances, as feathers, bits of paper, &c.: the hairs of his head alſo, or of his wig, if they happen to be looſe, will repel each other, and many of them ſtand upright.

Pointed bodies have a remarkable property in electricity; for the more acutely pointed any body is, the more eaſily does it take, or part with, electric matter. Thus, if a needle, or ſharp-pointed wire, be faſtened to the prime conductor, it will retain but a ſmall degree of electricity, and conſequently will give but a ſmall ſpark when the finger or a piece of metal is preſented to it. Again, if the needle or wire be held in a perſon's hand ſtanding upon the floor, and preſented to the conductor, it will be found to receive but a ſmall degree of electricity. In the former caſe, the needle being in contact with the prime conductor, the electric fluid goes off at the point, and is diſperſed in the air. In the latter caſe, the needle being preſented towards the conductor, receives the electric fluid from it at a conſiderable diſtance.

If the ſharp-pointed wire be giving out the electric fluid, the flame will be larger (for a flame will be ſeen at the point of the needle or wire, if the experiment be made in the dark), the parts of which it conſiſts will be fewer, and a ſnapping noiſe will be heard, if the point be not very acute; whereas, if the pointed wire be receiving the electric fluid, the flame will be much ſmaller and more globular; the parts of which it conſiſts will be more in number, and inſtead of the ſnapping noiſe there will be a kind of hiſſing. The flame iſſuing from a body is called a pencil, on account of its oblong ſhape; and when the rays come to a point, they project more equally from the centre; and it is then called a ſtar.

As pointed bodies tranſmit the electric fluid with ſo much eaſe, it affords an opportunity of proving the identity of lightning and the electric fluid; for if a long rod or pole, having a ſharp-pointed wire at the end of it, be ſupported by electric ſubſtances, the point projecting towards the clouds, will draw the electric matter from them, and become ſenſibly charged with electricity, as if it had been connected with the prime conductor of an electrical machine. It will attract all light bodies, and ſparks of electric matter may be drawn from it; and in ſhort, it will exhibit every appearance of common electricity; and on the other hand, by common electricity may be produced, in miniature, all the known effects of lightning.

This diſcovery was effected by Dr. Franklin, by raiſing a kite, called the *electrical kite*; and formed of a large thin ſilk handkerchief, extended and faſtened at the four corners to two ſlender ſtrips of cedar, and accommodated with a tail, loop, and ſtring, ſo as to riſe in the air like a common paper kite. To the top of the upright ſtick of the croſs was fixed a ſharp-pointed wire, extending a foot upwards, above the wood; and to that end of the twine which is next to the hand, was tied a ſilk riband. At the junction of the twine and ſilk was ſuſpended a key, from which, when the kite is raiſed during a thunder-ſtorm, a phial may be charged and electric fire collected, as by means of an electrical machine. From which it appears, that points have a remarkable property, both of throwing off, and receiving the electric fluid; from whence has riſen that uſeful invention of applying metallic conductors to houſes or buildings, in order to preſerve them from the dreadful effects of lightning, as will be hereafter ſhown.

SECT. II.

ELECTRICAL EXPERIMENTS.

1. *The Electrical Star.*—CUT a piece of tin in the form of a ſtar, and let it be ſupported on its centre by a wire projecting from the prime conductor; then, as ſoon as the machine is ſet in motion, and the ſtar electrified, a flame will appear at the extremity of every point of the ſtar, which will have a beautiful appearance, if the experiment be made in the dark. And if the ſtar be made to turn ſwiftly on its centre, an entire circle of fire will be ſeen. This experiment may be rendered more diverting, if the operator now and then touch the prime conductor with his finger, or a piece of metal; for by theſe means he will make it diſappear, and appear again at pleaſure; for in every experiment, if the prime conductor be touched, the effect of the experiment will be ſtopped. And if, inſtead of the ſtar, two ſharp-pointed wires be uſed, with the four ends at right angles, and in the ſame plane, but pointed different ways, and turning upon a centre, when it is electrified, a flame will be ſeen at each of the points; and what is more ſurpriſing, is, that the wires will begin to turn round of themſelves, and in the direction oppoſite to that to which the points are turned; and if the electrification be continued, the motion will become more rapid. It is by this experiment that what is called the *electric horſe-race* is performed; which is done by cutting the figures of horſes in paper, and faſtening them ſo that the points of the wires may be in their tails; by which they will ſeem to purſue one another, though without a poſſibility of any one overtaking another.

2. *The dancing Balls.*—Preſent the point of a wire, which is fixed on the prime conductor, to the inſide ſurface of a glaſs tumbler, graſping it on the outſide with the hands. The glaſs then will ſoon become charged with electric matter; its inſide ſurface acquiring the electricity from the point of the wire, and its outſide ſurface loſing its natural quantity of electric fluid through the hands, and which, in this caſe, ſerves as a coating to the glaſs. Then put a few pith balls upon the table, and cover them with this glaſs tumbler, and they will immediately jump up by the ſides of the glaſs (*fig.* 9), and continue in motion for ſome time, being attracted, and repelled, by the electric fluid upon the ſurface of the glaſs, which they gradually conduct to the table, the outſide of the glaſs acquiring the electric fluid from the ſurrounding air.

3. *Electrics become Conductors when made hot.*—Tie the ſilk ſtring G (*fig.* 10) to a crooked glaſs tube A E F B, by which it may be held; fill the middle part of this tube E F with reſin, ſealing-wax, or any other electric ſubſtance; then fix the two wires A E, B F, in the ſealing-wax, &c. Hold the tube over a clear fire to melt the reſin or ſealing-wax within it; at the ſame time connecting one of the wires A or B with the outſide of a charged jar, and touching the other wire with the knob of the jar. Then endeavour to make the diſcharge through the reſin, wax, &c. and it will be obſerved, that while the reſin is cold, no ſhock can be tranſmitted through it, but as it melts it becomes a conductor; and when perfectly melted, the ſhocks paſs through it freely: by which it may be ſeen, that glaſs and other electrics become conductors, when they are made very hot.

4. *The Thunder Houſe.*—A (*fig.* 11) repreſents the ſide of a houſe, being a board about three quarters of an inch thick. This is fixed perpendicularly upon the bottom board B, upon which is alſo fixed, in a hole in the ſame board, the glaſs pillar C D, about eight inches diſtant from the board A. In the board A is a ſmall ſquare hole I L M K, about a

quarter of an inch deep, and an inch wide, which is filled by another ſmall board nearly of the ſame dimenſions; and made to fit eaſy in the hole, ſo that it may drop out by any ſudden ſhock. This ſmall piece of wood repreſents a ſhutter, or door, in the ſide of the houſe A; L K is a wire faſtened diagonally to this piece of wood. I H is another wire of the ſame thickneſs, having a braſs ball H ſcrewed on its upper point: M N is another wire, turned into a ring at O. Theſe three wires are all fixed to the board A. From the upper extremity of the glaſs pillar C D proceeds a crooked wire, having a ſpring ſocket F, through which is a double-knobbed wire; the lower knob G falls juſt above the knob H of the conductor. The glaſs pillar D C muſt be fixed in the board looſe, ſo that it may be eaſily moved round; by which the braſs ball G may be brought nearer or further from the ball H, without touching the part E F G with the hand. Now, when the ſquare piece of wood M L I K is fixed in the hole, ſo that the wire L K may ſtand in the dotted line I M, then the metallic communication from H to O is complete; and the inſtrument exhibits a houſe furniſhed with a proper metallic conductor: but if the ſquare piece of wood be ſo fixed, that the wire L K ſtands as repreſented in the figure, then the metallic conductor H O, which goes from the top of the houſe to the bottom, is interrupted at I M; in which caſe the houſe is not properly ſecured. Then let the ball G be about half an inch in perpendicular diſtance from the ball H; and by turning the glaſs pillar D C, the former ball will be removed from the latter; then by a wire or chain connect the wire E F with the wire Q of the jar P, and let another wire or chain, faſtened to the hook O, touch the outſide coating of the jar P. Let the wire Q be connected with the prime conductor of the machine, and charge the jar; then by turning the glaſs pillar D C, bring the ball G gradually near the ball H, and when they approach ſufficiently near one another, the jar will explode, and the piece of wood M L I K will be puſhed out of the hole to a conſiderable diſtance. In this experiment, the ball G repreſents a thunder-cloud, which being arrived ſufficiently near the top of the houſe A, the electricity ſtrikes it; and as the houſe is not ſecured with a proper conductor, the exploſion will break part of it, by knocking out the piece of wood I M.

Again, let the piece of wood I M be ſo ſituated that the wire L K may ſtand in the direction I M; in this caſe, the conductor H O is not diſcontinued; and repeating the experiment as before, it will be ſeen, that the exploſion will have no effect upon the houſe, as the piece of wood L M will remain in the hole unmoved; which ſhows the uſefulneſs of a metallic conductor. The inſtrument uſed in this experiment is called the thunder-houſe, as it ſhows the effect thunder has upon a houſe, both ſecured and unſecured.

5. *The electrical Battery*—is the moſt formidable and entertaining part of electricity; and is formed of a number of glaſs-coated jars connected together, ſo that their whole force may be united. And if a battery of no great power is required, as containing about eight or nine ſquare feet of coated glaſs, common pint or half-pint phials will anſwer the purpoſe very well; but when a large battery is required, it is neceſſary to have cylindrical glaſs jars, of about fifteen inches high, and four or five inches in diameter.

The beſt method of coating theſe jars, is to coat them with tin foil on both ſides, which may be fixed upon the glaſs with paſte, made of wheat flour; but in coating the inſide of phials or jars, whoſe mouths are not large enough to admit the tin foil, braſs filings are uſed, mixed with gum-water or bees-wax, &c. And the coatings ſhould not come within two inches of the mouth of the jar, otherwiſe the jar may diſcharge of itſelf. Some kind of glaſs is not capable of holding any charge: the jars or phials ſhould therefore be examined, before any experiment be performed.

A very good battery may be formed of twelve jars, coated on both ſides with tin foil, containing in the whole about twelve ſquare feet of coated glaſs. In the middle of each jar is a cork that ſuſtains a wire, which at the top is faſtened to the wire E, which is knobbed at each end (*fig.* 12), and which connects the inſide coatings of three jars; and by four ſuch wires the inſide coatings of all the jars are connected together. Each of the wires F has a ring at one end, through which one of the wires E paſſes; and the other end has a braſs knob, reſting on the next wire E. If the whole force of the battery be not required, one, two, or three rows of jars may be uſed at pleaſure. The wooden box that contains theſe jars is lined at the bottom with tin foil. It has a hole on one ſide, through which an iron hook paſſes, communicating with the metallic lining, and conſequently with the outſide coating of the jars; to this hook is faſtened a wire, the other end of which is connected with the diſcharging rod.

The diſcharging rod conſiſts of two curved wires, B B (*fig.* 4), which move by a joint C, fixed to the braſs cap of the glaſs handle A. The wires are pointed at the ends. on which points are ſcrewed the two knobs D D, ſo that it may be uſed either with the points or knobs. When a large battery is required, it is better to uſe two, three, or more ſmall batteries, and their force may be united by a wire or chain: but the beſt method of uniting their force, is to have a wire from every jar, connected at the top with a ball, in the form of a wire bird-cage.

The force of electricity, thus accumulated by ſeveral jars or batteries, is aſtoniſhing. Metals, which reſiſt the greateſt effect of chemical fire, are inſtantly made red hot, and melted. But in performing experiments of this kind, the operator ſhould be careful that no perſon touches, or even comes too near, any part of the apparatus; otherwiſe it may produce ſerious conſequences. And it is to be obſerved, in charging a battery, a ſmall conductor is more proper than a large one, as the diſſipation of the electricity is not ſo great.

6. *The animated Spider.*—The experiment called animating the ſpider by electricity, is performed by ſuſpending a piece

of cork B (*fig.* 14) by a ſilk thread; in the cork a few ſhort threads are drawn, to repreſent its legs. It is to be hung in the midway, between the knob E of the wire D E, which is connected with the jar A D, and the knob A: then the jar being charged, by connecting its knob A with the prime conductor, the ſpider will be attracted by the knob A, and then repelled by it to the knob E, where it diſcharges its electricity; and is then again attracted by the knob A, and again repelled to the knob E; and will continue this motion till it has completely diſcharged the jar.

7. *To repreſent any luminous Figures.*—The ſpiral tube is compoſed of two glaſs tubes C D (*fig.* 18), one within the other, the ends being cloſed with two braſs caps A and B. On the outſide of the innermoſt tube is ſtuck a ſpiral row of ſmall round pieces of tin foil, about a twelfth of an inch diſtant from each other; then holding it by one end, and preſenting the other end to the prime conductor, ſmall ſparks will appear between all the pieces of tin foil, and in the dark it will have the appearance of a ſpiral line of fire. If, inſtead of the ſpiral tube, the tin foil be ſtuck upon a flat plate of glaſs A B C D (*fig.* 19), it may be ſo formed as to repreſent any other figures, letters, flowers, &c.

8. *To prove that Electricity prefers a ſhort Paſſage through the Air to a long one through good Conductors.*—A B D (*fig.* 16) is a wire about ten feet long, at the ends of which is fixed a piece of glaſs G, to keep the ends A B at a proper diſtance, and to let them ſlide within half an inch of each other, if required; then connect the chains belonging to the ſliding wires with the hook of the battery, and the diſcharging rod, and ſend the charge of the battery through them. On making the exploſion, a ſpark will be ſeen between A and B; which proves that the electric fluid chooſes a ſhort paſſage through the air, rather than a long one through good conductors; for very little of the electric fluid will paſs through the bent wire A D B.

9. *The electric Spark ſwells Clay.*—Roll a piece of tobacco-pipe clay in the form of a ſmall cylinder, and in the two ends inſert two wires A and B (*fig.* 22), ſo that their ends within the clay may be within a fifth part of an inch of each other. Then if a ſhock be ſent through this clay, by connecting one of the wires with the outſide of a charged jar, and the other wire with the inſide, it will be inflated by the ſpark that paſſes between the two wires, as repreſented in figure 23. If the ſhock be too ſtrong, and the clay not very moiſt, it will be broken by the exploſion, and its fragments ſcattered in every direction; as may be proved by uſing a piece of the tube of a tobacco-pipe inſtead of the clay.

10. *The electric Spark viſible in Water.*—Immerſe two knobbed wires, A and B (*fig.* 17), in a glaſs of water, ſo that the knobs of the wires may be within a little diſtance of each other. Then, if one of theſe wires be connected with the outſide coating of a jar, and the other wire be touched with the knob of it, the exploſion in paſſing through the water, from the knob of one wire to that of the other, will break the glaſs with a ſurpriſing violence. Great caution is neceſſary in performing this experiment, as it is ſometimes attended with danger. If, inſtead of a drinking-glaſs, a glaſs tube be uſed, ſtopped with a cork at each end, through which the wires are inſerted, and the charge be very weak, the electric ſpark will appear in the water paſſing between the wires.

11. *The electrical Thermometer.*—A B (*fig.* 24) is the electrical thermometer, and conſiſts of a glaſs tube about ten inches long, and nearly two inches in diameter, and cloſed air-tight at both ends by two braſs caps. H A is a ſmall tube, open at both ends, paſſing through a hole in the upper cap, and immerſed at the bottom in ſome water at B, in the bottom of the large tube. F G and E I are two wires inſerted through the middle of each of the braſs caps, and having a braſs knob at the head of each, within the braſs

tube. This inſtrument is faſtened to the pillar C D, by a braſs ring C. When the air within the tube A B is rarefied, it will preſs upon the water at the bottom of the tube, and ſo cauſe it to riſe in the ſmall tube; and the riſe and fall of the water ſhow the rarefaction of the air in the glaſs tube A B, which has no communication with the external air.

If the knobs G I of the two wires be brought into contact with each other, and the ring E or F be connected with one ſide of the charged jar, and the other ring with the other ſide, and a ſhock be made to paſs through the wires, the water in the ſmall tube will not be at all moved; which ſhows that the paſſage of the electric fluid, through conductors ſufficiently large, occaſions no rarefaction of the air. But if the knobs G I be placed a little diſtant from each other, and the ſhock ſent through the wires as before, the ſpark between the two knobs will conſiderably rarefy the air, and the water will be ſuddenly forced up the ſmall tube quite to the top.

12. *To ſhow the Courſe of the electric Fluid in the Diſcharge of a Jar, and to make it viſible by the Star and Pencil.*—For this purpoſe, the jar muſt be charged; then taking the diſcharging rod (*fig.* 4) without its knobs, preſent one point within an inch of the knob A (*fig.* 15), and the other point at an equal diſtance from the outſide coating of the jar. By theſe means the jar will be diſcharged ſilently; and if its inſide be electrified poſitively, the point C of the diſcharging rod will be illuminated with a ſtar, becauſe it receives the electric fluid; and the point B with a pencil, becauſe it gives out the electric fluid to the outſide of the jar; and if the jar be electrified negatively on the inſide, the pencil will appear upon the point C, and the ſtar upon the point B.

13. *The univerſal Diſcharger.*—This is an inſtrument of very extenſive uſe, and is compoſed of the following parts: A is a flat board, about fifteen inches long, four broad, and

one thick (*fig.* 8). b B are two glaſs pillars, cemented in two holes in the board. At the top of each is a braſs cap, having a turning-joint, and a ſpring tube, through which ſlides the wire C D. Thus, each of theſe wires has two other motions, viz. an horizontal and vertical one; each wire is alſo furniſhed with an open ring at one end C, and a braſs ball at the other end D, which ball may be taken off at pleaſure. E is a ſtrong circular piece of wood five inches in diameter, on the ſurface of which is a ſlip of ivory; and furniſhed with a ſtrong cylindrical foot, which fits the ſocket, and which, by means of the ſcrew G, may be made faſt, and alſo raiſed higher, or brought down lower.

The *Leyden phial* is an inſtrument to prove the hypotheſis of a ſingle electric fluid, and is formed by coating a ſmall phial about three inches up the outſide with tin foil (*fig.* 20). To the top of the neck a braſs cap is cemented, having a hole with a valve; from this cap proceeds a wire, being blunted at the point, and terminating a few inches within the phial. When the phial is exhauſted of air, a glaſs ball is ſcrewed on the braſs cap, to prevent any air from getting into the phial. This phial ſhows the direction of the electric fluid, both in charging and diſcharging; for if it be held by its bottom with the braſs knob preſented to the prime conductor, which is poſitively charged, the electric fluid will cauſe the pencil of rays to proceed from the wire within the phial, as in figure 21; but if it be diſcharged, a ſtar will appear inſtead of the pencil, as in figure 20. But if the wire be held by the braſs cap, and its bottom be touched by the prime conductor, the point of the wire on its ſide will appear illuminated with a ſtar when charging, and with a pencil when diſcharging. If it be preſented to the prime conductor, electrified negatively, all theſe appearances, both in charging and diſcharging, will be reverſed.

Inflammable air, that will take fire by the electric ſpark, is thus made: A D (*fig.* 25) repreſent two bottles; in the bottle D are put two or three ounces of filings of iron, and ſome oil of vitriol, mixed with four times its quantity of water. The bottle A is filled with water, and the bent glaſs pipe C is fixed with one end, air-tight, into the neck of the bottle D, and the other end a little way up the neck of the bottle A; in a ſhort time the mixture will boil, and emit a fluid, which will paſs through the tube C, into the bottle A, and at length fill it, expelling the water into the baſin B. The bottle A is then to be quickly corked up for uſe.

The *electrical piſtol* is repreſented *fig.* 26, where c a is of thin braſs; to the mouth a b is fitted a cork, and a perforated piece of braſs d ſcrews on the bottom of the piſtol at c, having a glaſs tube, with a wire cemented into it, bent over the glaſs tube, ſo as to reach within one eighth of an inch of the braſs. When the piſtol is to be charged, uncork the inflammable air bottle before mentioned, likewiſe the piſtol, and place the mouth of the piſtol upon the top of the bottle; and the common air which is within the piſtol will deſcend while the other aſcends. Having held the piſtol in this ſituation a few ſeconds, in order to fill it with inflammable air, cork both it and the bottle expeditiouſly, and it is then charged. When it is to be diſcharged, fill a ſmall jar, or a hollow handle, and apply it to the knob of the wire e; it will then explode, and drive out the cork to a conſiderable diſtance, with a report as loud as that of a piſtol filled with gunpowder.

Figure 28 repreſents an inſtrument to cure the tooth-ach, in which A is a flat piece of box wood, about an inch broad, and a quarter of an inch thick. Near its oppoſite edges are made two longitudinal holes, through which are put two braſs wires, a b c and d e f, and fixed in with ſealing-wax, and then bent at c and f, as in the figure, which two points ſerve to receive the tooth and gum between them. When the inſtrument is uſed, hook two chains, g and h, on the lower end of the wires, holding the tooth and gum between the other end of the wires c and f: put the end of the chain g round the bottom of the electric jar, and let a perſon hold the chain h hanging down from his hand, both chains being clear of the table, and not touching each other. Then having charged the jar, let the perſon who holds the chain h ſtrike the end of it againſt the prime conductor: this will diſcharge the jar, and give the patient a ſhock only in his tooth and gum, which ſeldom fails to cure the tooth-ach, if the cauſe be a cold.

The *electrometer* is an inſtrument to ſhow the kind and quantity of electricity, of which there are ſeveral ſorts; the moſt ſimple is that which conſiſts of a linen thread, having a ſmall cork or pith-ball at each end (*fig.* 13). This electrometer is ſuſpended by the middle of the thread on any conductor proper for the purpoſe. And if the conductor be charged poſitively, by applying a ſtick of ſealing-wax, excited, the balls collapſe together; and by applying an excited ſmooth glaſs tube, they will recede further aſunder; and if the conductor be charged negatively, the reverſe will take place.

But the moſt perfect of theſe inſtruments is that called the quadrant electrometer, which ſhows the exact degree to which any body is electrified, and is as follows;—A is a fine rod, that turns on B, the centre of a ſemicircle (*fig.* 27), ſo as always to keep near its graduated limb, which is divided into 180 degrees. At the end of the rod is a cork ball C. The pillar D may be fixed either to the prime conductor, or to the braſs knob of a jar or battery, or be ſet on a ſtand by itſelf. The inſtrument ſhould be made of box wood, and the ſemicircle of ivory.

When this inſtrument begins to be electrified, the rod A is repelled by the pillar D, and conſequently begins to move over the edge of the ſemicircle, and ſhows very exactly the degree to which the conductor is electrified, or how high any jar or battery is charged. This inſtrument ſhould always be made very dry before the fire, when it is uſed, taking care that it be not heated.

If the jar or battery be charged with poſitive electricity, and it be required to know the exact time that it becomes diſcharged, while you are attempting to charge it negatively, obſerve the moment the index comes to the perpendicular ſtation, and at that moment there will not be the leaſt ſpark left in the jar. If the operation be continued, the index will again advance along the ſemicircle; and thus ſhow the exact quantity of negative electricity which the jar has acquired.

SECT. III.

OF MEDICAL ELECTRICITY.

Electricity was no ſooner brought to any degree of perfection, than it was applied to medical purpoſes. For by late obſervations, it has been found to poſſeſs the invariable properties of increaſing the ſenſible perſpiration, quickening the circulation of the blood, and promoting all the glandular ſecretions. And among all the variety of caſes in which it has been uſed, there are none in which it has been found prejudicial, except thoſe of pregnancy, and venereal diſeaſe; and there are a number of caſes, in which it has been applied with conſiderable ſucceſs. In moſt diſorders where it has been uſed with perſeverance, it has given at leaſt a temporary and partial relief; and in ſome caſes it has effected a total cure. Of which, numerous inſtances may be ſeen in the Philoſophical Tranſactions, and the writings of Meſſrs. Lovet, Ferguſon, Weſtley, Cavallo, &c. &c.

To know what caſes are proper to be electrified, experience ſhows in general, that all kinds of obſtructions, whether of motion, of circulation, or ſecretion, are very often removed, and in general alleviated, by electricity. Likewiſe, nervous diſorders have very often been cured; and rheumatic diſorders, even of a long ſtanding, are always relieved, and very often quite cured, by only drawing the electric fluid with a point from the affected part, or by drawing ſparks from the conductor. It has alſo been found very beneficial in diſeaſes of a long ſtanding; and has not unfrequently been found a powerful remedy in muſcular contractions.

There are three inſtruments generally uſed for adminiſtering medical electricity, beſides the electrical machine, viz.—An electric jar, with Mr. Lane's electrometer; an inſulated chair or ſtool, upon which a common chair may be occaſionally ſet; and the directors.

The jar uſed on this occaſion ſhould be coated with tin foil, and ſhould be about four inches in diameter, and ſix in height, which would contain above 72 ſquare inches of coated ſurface. Through the covering of the jar paſſes a braſs wire B (*fig.* 1, *plate* 21), touching the inſide coating of the jar, and having a braſs ball F, to which the electrometer F D C is faſtened, and terminating at the top in a braſs ball B, which is to touch the prime conductor, and which is ſuppoſed to ſtand before the electrical machine. The electrometer C E D F conſiſts of a piece of glaſs F D, cemented to the two braſs caps D and F; from the former of which proceeds a ſtrong perpendicular braſs wire, having at the top an horizontal ſpring ſocket, through which ſlides the wire C E, having the braſs ball C at one end, and an open ring E at the other end; and ſo fixed, that the ball C is exactly the ſame height as the ball B, and may be ſet at any required diſtance from the ball B. This diſtance ſeldom exceeds half an inch, therefore the electrometer may be made very ſmall. Sometimes there is a ſcale on the wire C E, which ſerves to ſet the balls B C to any given diſtance from each other, with more certainty. When this inſtrument is uſed, the jar is ſo placed, that the ball B may touch the conductor. Then, ſuppoſe the ball C to be ſet at one tenth of an inch diſtance from the ball B, and the electrometer E be connected to the outſide coating of the jar at I by a chain E I. In this caſe, if the electrical machine be put in motion, the jar will be charged; and when the charge is ſo high, that the electric fluid accumulated within the jar, can paſs from the ball B to C, which is here ſuppoſed to be one tenth of an inch; the diſcharge will take place, the ſpark will appear between the balls, and the ſhock will paſs through the chain E I from E to I; for the part F D being of glaſs, and generally covered with ſealing-wax, is impervious to the electric fluid; therefore the electric fluid has no way to paſs from the inſide to the outſide of the jar, but from the ball B to the ball C, and along the wire C E, and from thence along the chain E I.

When the electrical ſhock is to be adminiſtered to any part of the human body (as, for example, to the arm), inſtead of the chain, which muſt now be taken away, two ſmall pliable wires E L, I L, are to be faſtened, one to the ring E, and the other to a hook I, of the ſtand H I, which communicates with the outſide coating of the jar (if the jar have not the ſtand H I, the extremity of the wire I may be put in contact with the outſide coating of the jar in any other convenient manner); the other end of the ſaid wires is faſtened to the braſs wires L L of the directors K L. Each director conſiſts of a knobbed braſs wire L, connected to a glaſs handle K, by means of a braſs cap. Then the operator, holding the directors by the extremities of the glaſs handles, brings their balls into contact with the extremities of that part of the patient's body through which the ſhock is to be ſent. Then it is evident, from a view of the figure, that the diſcharge of the jar muſt be made through that part of the patient's arm which lies between the two knobs of the directors, as, in the former caſe, the diſcharge was made through the chain E I. Thus, the operator has nothing more to do, but to hold the knobs of the directors to the extremities of that part of the body through which the ſhock is to paſs, while an aſſiſtant keeps the machine in motion. Care muſt be taken that the two wires E L and I L do not touch each other; otherwiſe the ſhock will not paſs through the patient's body. By theſe means, any number of ſhocks, preciſely of the ſame ſtrength, may be given without altering any part of the apparatus. And when it is required to increaſe or diminiſh the force of the ſhock, it is only neceſſary to increaſe or diminiſh the diſtance between the balls B and C, which is done by moving the wire C E through the ſocket.

It is of little conſequence whether the patient ſtands upon the ground, upon the inſulated ſtool, or in any other ſituation. Neither is it neceſſary to remove the clothes from the part that is to be electrified, for the ſhocks will readily go through them, except there be too many coverings.

In the application of electricity, the chief difficulty conſiſts in diſtinguiſhing the proper ſtrength of the electric force that is requiſite for a given diſorder. For this purpoſe, it is impoſſible to give any general rules, the circumſtances being of ſo complex a nature, that nothing but long experience, and ſtrict attention to every particular phenomenon, can direct the operator. It need hardly be ſaid, that regard muſt be had to the ſex and age of the patient. The ſureſt rule that can be given, is to begin with more gentle treatment, at leaſt ſuch that, conſidering the circumſtances, may be thought rather weak than ſtrong. If, after a few days trial, this gentle treatment be found ineffectual, then the operator may gradually increaſe the force of the electricity, until he finds the proper degree. But when any limb of the body is deprived of motion, it muſt be obſerved, that the cauſe is ſometimes a contraction of the muſcles; in which caſe, electricity has often proved an effectual remedy: but the loſs of motion is ſometimes occaſioned by a relaxation, as well as contraction: as, when the hand is bent inwardly, and the patient has no power to ſtraighten it. In theſe caſes it is often difficult to diſcover the real cauſe: but the ſureſt method is, to electrify both thoſe muſcles which are contracted, and alſo their antagoniſts; for no injury can attend electrifying a ſound muſcle. In rheumatic diſorders, the electric fluid ſhould be drawn from the parts afflicted; or the ſparks may be drawn from the conductor.

The operation ſhould be continued for four or five minutes, and may be repeated once or twice a day. When the ſhocks are ſtrong, their greateſt number at one operation ſeldom exceeds a dozen or fourteen, except they be given to different parts of the body.

The *electrophorus* (*fig.* 2) conſiſts of two plates, from ſix to 18 inches diameter, in general, and ſometimes much larger. The upper plate is generally made of braſs; but a tin plate will ſerve the purpoſe, having a wire turned upon its edge, in the common manner; on the centre of this plate is fixed the ſocket O, in which the glaſs handle I is fixed, which is nine or ten inches long. When the electrophorus is to be of a large diameter, a thin board, covered with tin foil, and ſuſpended by ſilken ſtrings, will anſwer exceedingly well.

The lower plate (only the edge of which could be ſhown in the figure) may be made either of glaſs, ſealing-wax, or the following compoſition, viz.—reſin four parts, pitch three parts, ſhell-lac three parts, Venice turpentine two parts, melted together over a gentle fire; then poured and ſpread upon a thin linen cloth, about a quarter of an inch thick; the linen cloth is then ſtretched upon a hoop, and made as tight as poſſible.

To charge a jar with this machine, rub the coated ſide of the under plate with a piece of fine new flannel, or a hare's or cat's ſkin; and when it is excited as much as poſſible, ſet it upon a table, and place the upper plate B upon it, and put your finger upon the upper plate; then, taking your finger off, take hold of the glaſs handle I, and apply it to the knob of a coated jar. When this operation is repeated 30 or 40 times, the jar will become charged.

It was with a machine of this kind that Mr. Cavallo charged a coated phial ſeveral times by only once exciting, and ſo ſtrong, as to pierce a hole through a card at every diſcharge. If a plate of glaſs be coated with ſealing-wax, and excited, and then laid with the wax ſide downwards; then, on making the above experiment, by putting the plate upon it, and taking the ſpark with the finger, and applying it to the glaſs handle, &c. it will have the contrary electricity to what it had before.

When it is required to diſcover whether a ſmall degree of electricity be poſitive or negative, or to know how the charge advances in uſing large batteries, and of what ſtrength it is, the moſt uſeful electrometer is Mr. Canton's balls, which are made of pith of elder, turned perfectly globular, and ſuſpended by fine threads from the conductor (*fig.* 3).

To know whether the inſide of a jar or battery be charged poſitively or negatively, the balls are to be preſented to the jar or battery which ſtands upon the table, and they will be immediately attracted by the wire, and diverge from each other. This is always the caſe in both poſitive and negative electricity. And the greater the diſtance to which the balls ſeparate, and the more they repel one another, the higher is the charge. To determine whether the electricity is poſitive or negative, rub a ſmall piece of glaſs againſt the hand or coat, which will excite it poſitively, and then preſent it to the balls in their diverging ſtate, and if it makes the balls converge, it ſhows they are electrified poſitively; but if it increaſes their divergency, it ſhows their electricity to be negative. And it muſt be obſerved, that the electricity of the balls is always contrary to that with which they are charged, for they do not receive any electricity from the wires of the jar or battery; for all bodies placed within the influence of electrified bodies are affected with the contrary electricity.

But to diſcover the kind of electricity, when the charge is very ſmall, inſtead of the pith-balls, a piece of downy feather ſhould be uſed, ſuſpended by a ſingle ſilken thread, as it comes from the worm, or at leaſt by a very few of thoſe threads, to render it as light as poſſible. If any electrified body be preſented to this, the feather will be repelled by it, if it be of the ſame kind with its own, and attracted by it, if the electricity be contrary to it. For this light body, when once electrified, either poſitively or negatively, will retain its virtue a long time, with very little loſs.

Notwithſtanding electricity might be rendered ſo generally uſeful in the application of it to medical purpoſes, yet it is frequently found to be ineffectual, where it might be expected to prove the moſt ſalutary, which is very often the effect of ignorance in the operator; for many perſons, particularly in the metropolis, undertake to adminiſter electricity, who are entirely deſtitute of any medical knowledge, and, conſequently, of the cauſe and ſituation of diſorders; hence, the failure of it is owing, generally, to an error in the application.

The technical Terms uſed by Writers on Electricity.

Battery, electrical, a number of jars combined together, to be all charged and diſcharged at the ſame time (*fig.* 12, *plate* 20).

Charging, throwing an additional quantity of electric fluid upon one ſide of a plate of glaſs, or a jar, while the other ſide is exhauſted in the ſame proportion. All electric ſubſtances may be charged as well as glaſs.

Circuit, thoſe conducting ſubſtances uſed to connect the two coatings of a jar or battery together, and through which the electric fluid muſt paſs.

Conductor, a piece of metal furniſhed with points, to receive the electric matter from the globe. It muſt always be inſulated, or unconnected with the earth, by means of electric ſubſtances, as glaſs, baked wood, &c. Whenever it is indefinitely mentioned, the prime conductor is underſtood.

Diſcharging, is reſtoring the equilibrium of the electric fluid, after it has been diſturbed by charging. It is effected by forming a communication between the overloaded and exhauſted ſides of a jar, battery, &c. by ſome conducting ſubſtance.

Diſcharging rod, a braſs rod, or any other inſtrument (*fig.* 4 and 8, *plate* 20), uſed to effect a diſcharge.

Electric matter or *fluid*, that ſubtle fluid inherent in all bodies, and ſuppoſed to be the cauſe of all thoſe appearances which we term electric.

Electrics, thoſe bodies in which the electric powers of attraction, repulſion, &c. may be excited by friction. They are called *non-conductors*, becauſe the electric fluid cannot paſs through them. And *non-electrics* are called *conductors*, becauſe the electric matter may paſs through them; but no electric powers can be excited in them.

Electrometers, inſtruments to meaſure the quantity of electric matter.

Excitation, calling forth the electric powers from electric ſubſtances by friction.

Inſulating, placing bodies where they are not in contact with any conducting ſubſtance; as, by ſuſpending them in the air by ſilken ſtrings, placing them on glaſs ſtands, &c.

Negative electricity, a leſs quantity of the electric fluid than is natural to any body.

Poſitive electricity, a greater quantity of the electric fluid than its natural ſhare.

Rubber, or *Cuſhion*, a piece of leather, or any other ſubſtance, againſt which the glaſs globe or other electric body is rubbed, in order to excite them.

Pencil, the appearance of the electric fluid iſſuing from the point of a body electrified poſitively.

Star, the appearance of the electric fluid iſſuing from the point of a body electrified negatively.

Shock, electric, the convulſion given to the animal muſcles by the diſcharge of a jar or battery.

Wire of a jar, &c. the wire or metal rod which touches the inſide coating of a jar, &c.

CHAP. XVII.

OF PNEUMATICS.

SECT. I.

OF THE PROPERTIES OF AIR.

PNEUMATICS is that part of natural philoſophy which treats of the weight, preſſure, and elaſticity of the air, with the effects ariſing from them.

The air is that thin, tranſparent, fluid body which ſurrounds the whole earth to a conſiderable height; and which, together with the clouds and vapours that float in it, is called the atmoſphere. That the air is a fluid, is evident from the following properties, which it poſſeſſes in common with all other fluids: viz.—1. It yields to the leaſt force impreſſed on it.—2. Its parts are eaſily moved among one another.—3. It preſſes according to its perpendicular height.—4. And its preſſure is every way equal.

But the air differs from all other fluids, in the four following particulars:—1. It can be compreſſed into a much leſs ſpace than what it naturally poſſeſſes, which no other fluid can: 2. it cannot be fixed or congealed as other fluids can: 3. it is of a different denſity in every part upward from the earth's ſurface, its weight decreaſing the higher it riſes, conſequently it muſt alſo decreaſe in denſity: 4. it is of an elaſtic nature, and the force of the ſpring is equal to its weight.

It is evident, that air is a body, for it excludes all other bodies out of the ſpace it poſſeſſes: thus, if a glaſs veſſel or jar be inverted, and plunged into a veſſel of water with a ſteady hand, ſtill preſſing it downwards, there will very little water get into the jar, becauſe the air, of which it is full, keeps the water out. It is upon this principle that diving-bells are conſtructed.

Air being a body, muſt neceſſarily have gravity or weight; and its weight is determined by the following experiment:—let a bottle that holds a wine quart be emptied of its air, by means of the air pump; then weighing the bottle, it will be found to be about 16 grains lighter than when the air is let into it again: which ſhows that a quart of air weighs 16 grains. And to find the proportion of the weight of air to that of water, divide the weight of a certain quantity of water by the weight of the ſame quantity of air; thus, a quart of water weighs 14621 grains, which, divided by 16, the weight of a quart of air, quotes 914, in round numbers: which ſhows that water is 914 times as heavy as air, near the ſurface of the earth. This is in general the greateſt weight of the air; for commonly it is reckoned only 840 times leſs denſe than water, at a mean rate; the denſity of the air being very various, according to the ſituation of the climate, ſeaſon of the year, and many other circumſtances.

The air has a different denſity as we riſe from the ſurface of the earth, and grows continually rarer and lighter the farther it is from the earth, which is owing to its being of an elaſtic nature, and capable of being compreſſed into a leſs ſpace, for the lowermoſt parts of the atmoſphere, being preſſed with the weight of all that is above them, muſt conſequently be rendered more denſe and compact at the earth's ſurface than at any height above it. And that air, towards the upper part of the atmoſphere, being leſs preſſed, is conſequently leſs denſe and compact than that near the earth; for the denſity of the air is always as the force that compreſſes it. The following table given by Dr. Cotes, ſhows that the rarity of the air at a diſtance from the earth's ſurface increaſes in a geometrical proportion, while its height from the earth increaſes in an arithmetical proportion:

At the altitude of	Miles above the ſurface of the earth the air is	times thinner and lighter than at the earth's ſurface.
7	4	
14	16	
21	64	
28	256	
35	1024	
42	4096	
49	16284	
56	65536	
63	262144	
70	1048576	
77	4194304	
84	16777216	
91	67108864	
98	268435456	
105	1073741824	
112	4294967296	
119	17179869184	
126	68719476736	
133	274877906944	
140	1099511627766	

From this table, it appears, that at the height of ſeven miles from the earth, the air is four times rarer and lighter than at the earth's ſurface; at the height of 14 miles it is 16 times rarer and lighter; at 21 miles, 64 times rarer, &c. From hence it may be proved, that a cubical inch of ſuch air as we breathe, near the earth's ſurface, would be ſo much more weight on each ſquare foot, gives 12,043,468,800,000,000,000 pounds, for the preſſure of the whole atmoſphere.

All common air is impregnated with a certain kind of what is called *vivifying ſpirit*, which is eſſential to preſerve animal life; and in a gallon of air there is enough of it for one man during the ſpace of a minute, but not much longer. This ſpirit is alſo in that air which is in water, as appears by the fiſh dying, when they are excluded from freſh air, as in a pond that is frozen over.

This ſpirit in air is loſt by paſſing through the lungs of any animal, and is the reaſon why an animal dies ſo ſoon when deprived of freſh air. The little eggs of inſects, alſo, when ſtopped up in a glaſs, and excluded from the air, do not produce their young, though they be aſſiſted by warmth. The ſeeds alſo of plants, though mixed in good earth, will not grow if they be deprived of air.

The vivifying quality is alſo deſtroyed by the air paſſing through fire, particularly charcoal fire, or the flame of ſulphur.

Air may alſo become vitiated, by being cloſely confined in any place for a conſiderable time, or by being mixed with malignant ſteams: and laſtly, by the corruption of the vivifying ſpirit; as in the holds of ſhips, in oil-ciſterns, wine-cellars which have been ſhut ſome time, or brewers' vats. In any of them, the air may be ſo much vitiated as to be immediate death to any animal that enters them.

When the air has loſt its vivifying ſpirit, it is called *damp*, becauſe it abounds with humid and moiſt vapours, and becauſe it deadens fire, extinguiſhes flame, and deſtroys life. The effects of theſe damps are ſufficiently known to thoſe who work in mines.

When part of the vivifying ſpirit of air in any country begins to putrefy, the inhabitants of that country will be ſubject to an epidemical diſeaſe, which will rage till the putrefaction is over. And as the putrefying ſpirit occaſions the diſeaſe, ſo if the diſeaſed body contributes towards the putrefaction wards that part which is moſt heated by the ſun. But the courſe of the ſun being from eaſt to weſt, with reſpect to the earth, the common courſe of the air which ſupplies the place of the rarefied air muſt be in the ſame direction, viz. from eaſt to weſt; but on the north ſide its courſe will be directed a little towards the north, and on the ſouth ſide as much towards the ſouth.

This would be the general courſe of the wind about the equator, if it were not affected by other cauſes, which change its direction: as, 1. By exhalations that ariſe out of the earth, at different times and different places, occaſioned by ſubterraneous fires, volcanoes, &c.; 2. by a ſudden inundation of rain, which cauſes a contraction of the air; 3. by the violent heat of ſome burning ſands, which cauſe an extraordinary rarefaction of the contiguous air; 4. by high mountains, which alter the direction of the wind; 5. by the declination of the ſun towards the north or ſouth, thereby cauſing a greater heat in the air on the ſame ſide of the equator.

Theſe are the principal cauſes which create ſuch a great variety and uncertainty in the winds in moſt countries diſtant from the equator; as, 1. The variations of the winds in the different parts of Europe; 2. the monſoons which are found in the Indian ſeas; 3. thoſe winds which always blow from weſt to eaſt, on the weſtern coaſt of America, and on the coaſt of Guinea; and the ſea breezes, which, in hot countries, blow from ſea to land in the daytime; and the land breezes, which blow towards the ſea in the night; and all thoſe other irregularities in the wind, as ſtorms, whirlwinds, hurricanes, &c.

SECT. III.

OF THE CAUSES OF THUNDER, LIGHTNING, &c.

The effluvia and vapours ariſing from different bodies meet and unite together in the atmoſphere, which is the common receptacle of all vaporous bodies, as the ſteams from moiſt bodies, the ſmoke from bodies burnt, and the effluvia emitted from ſulphurous, nitrous, acid, and alkaline ſubſtances. And every volatile body riſes to a certain height in the atmoſphere, according to its own ſpecific gravity. And when the effluvia which ariſe from an acid and alkaline body meet each other in the air, there will be a conflict between theſe two vapours, or what is vulgarly called a fermentation between them. If this fermentation be great, it will produce a fire; and if the effluvia be of a combuſtible nature, the fire will run from one part of the air to another, following the inflammable matter.

Theſe things may be demonſtrated by the following experiment:—Mix ſome oil of cloves and Glauber's ſpirit of nitre together, which will immediatly produce a ſudden fermentation, with a fine flame; and if the ingredients be neat, there will be a ſudden exploſion. Theſe are the effects of the union of an acid and alkaline fluid.

From this experiment, we may account for the effects of thunder and lightning, which is occaſioned by the effluvia of ſulphurous and nitrous bodies meeting each other in the air, where, aſſiſted by the ſun's heat, a fermentation, fire, and exploſion enſue. When the inflammable matter is thin and light, it will aſcend to the upper parts of the atmoſphere, before the fermentation takes place; but when it is more denſe, it will hover near the ſurface of the earth, where, when an exploſion takes place, the fire is viſible, and often dangerous; the exploſion alſo has a violent force; and the heat being great, will rarefy and drive away all the adjacent air, kill men and cattle, ſplit trees, rocks, &c.

Lightning differs from all other fires; for it has often been known to paſs through wood, leather, cloths, and other ſubſtances, without hurting them; at the ſame time melting iron, ſteel, ſilver, gold, and other hard bodies. It has melted or burnt aſunder a ſword, without hurting the ſcabbard; and melted money in a man's pocket, without hurting his clothes. So fine are the particles of this fire, that they paſs through ſoft, looſe bodies, without injuring them, and ſpend their force upon thoſe that are more denſe.

Any ſteel inſtruments, as knives, forks, &c. that have been ſtruck with lightning, have a ſtrong magnetic virtue, which they retain many years. The lightning ſtriking the mariner's compaſs has often turned it quite round, and made it ſtand the contrary way, that is, with the north pole towards the ſouth.

Thoſe exploſions which ſometimes happen in mines, and called fire-damps, are of the ſame nature with lightning, and occaſioned by ſulphurous and nitrous vapour riſing from the mine, which, mixing with the air, take fire from the lights uſed in the mine. This fire, when once kindled, continues to run from one part of the mine to another, as the combuſtible matter happens to be; and as the elaſticity of the air is increaſed by the heat, the air in the mine will ſwell conſiderably; and, for want of room, will at length explode, with a degree of force equal to the violence of the fire, the quantity of effluvia, and denſity of the vapours. This is ſometimes ſo ſtrong as to blow up the mine; at other times it is ſo weak, that when it has taken fire it may be eaſily blown out.

Air that will take fire from the flame of a candle, may be produced thus:—Having pumped the air out of the receiver of the air-pump, let the air run into it through the flame of the oil of turpentine; then remove the cover of the receiver, and holding a candle to that air it will take fire.

When combuſtible vapours are kindled in the bowels of the earth, where there is little or no vent, they produce earthquakes, and violent ſtorms or hurricanes of wind, as ſoon as they break forth in the open air.

An artificial earthquake may be produced thus:—Take 10 or 15 pounds of ſulphur, and as much of the filings of iron, and knead them with common water into the conſiſtence of a paſte: this, being buried under ground, will, in eight or ten hours times, burſt out into flames, and cauſe the earth to tremble around it to a conſiderable diſtance.

It is owing to ſubſtances of this nature that we have volcanoes.

SECT. IV.

THE CONSTRUCTION AND USE OF THE AIR-PUMP, BAROMETER, AND AIR-GUN.

1. *Of the Air-pump.*

THE air-pump is a machine to pump the air out of any veſſel, and conſtructed on the ſame principle as the water-pump.

The air-pump, with all its apparatus, is ſhown *fig.* 1, *plate* 22, where L L is the plate, on which is placed a wet leather, and the large glaſs receiver M, placed upon the leather, ſo that the hole *i* in the plate may come within the glaſs. Then, by turning the handle F (*fig.* 2), the air will be pumped out of the receiver, which will be held down to the plate by the force of the external air or atmoſphere. For, as the handle F is turned backwards, or towards D, it raiſes the piſton *d e* in the barrel B K, by means of the wheel E, and rack work D and C; and as the piſton is leathered ſo tight, as to fit the barrel exactly, no air can get between the piſton and barrel, and therefore all the air above *d*, in the barrel, is lifted up towards B, and a vacuum is made in the barrel from *b* to *e*; upon which part of the air in the receiver M (*fig.* 1), by its ſpring, ruſhes through the hole *i* in the braſs plate L L, through the pipe G C G, which communicates with both barrels, by means of the hollow trunk I H K (*fig.* 2), and puſhing up the valve *b*, enters into the vacant place *b e* of the barrel B K. For wherever the reſiſtance or preſſure of the air is diminiſhed, the air will run to that place, if it can find a paſſage. Then, if the handle F be turned the contrary way, the piſton *d e* will be lowered in the barrel; and as the air which came laſt into the barrel cannot be puſhed back through the valve *b*, it will aſcend through a hole in the piſton, and make its eſcape through a valve at *d*; and by that valve be prevented from returning into the barrel, below the piſton. At the next raiſing of the piſton, a vacuum is again made in the barrel, between *b* and *e*, as before, when more of the air that is left in the receiver M (*fig.* 1) eſcapes by its ſpring into the barrel B K, through the valve *b*. What is here explained concerning the barrel B K muſt be underſtood with regard to the other barrel A I. And as the handle F is turned backwards and forwards, it raiſes and depreſſes the piſtons in each barrel alternately, raiſing one, while it depreſſes the other. And as there is a vacuum made in each barrel, when the piſton is raiſed, the particles of air in the receiver M (*fig.* 1) puſh out one another by their elaſticity, through the hole *i*, and pipe G G, into the barrels, until the receiver is ſo much exhauſted of the air, and the elaſticity of the air is ſo much weakened, that it will no longer have ſufficient force to paſs through the valves *b d*; and then no more air can be taken out. It is impoſſible to make a perfect vacuum in the receiver, or to entirely exhauſt it of air; for the quantity of air taken out at any one ſtroke will always be as the denſity of the air in the receiver; and therefore it is impoſſible to take it all out: for if the receiver and barrels be both of equal capacity, there will always as much remain in the receiver, as was taken out at the laſt turn of the handle.

At *k* (*fig.* 1), juſt under the pump plate, there is a cock, by the turning of which the air may at any time be let into the receiver again, through the hole *i*, and then the receiver becomes looſe, and may be taken off the plate. The barrels (*fig.* 2) are fixed to the frame E *e e* (*fig.* 1), by the two ſcrew-nuts *f f*, which preſs the piece E upon the barrels; and the hollow trunk H (*fig.* 2) is covered by the box G H (*fig.* 1).

l m m m n is a glaſs tube, open at both ends, and about 34 inches long, the upper end communicating with the hole in the pump plate, and the lower end immerſed in the veſſel N, which is nearly filled with quickſilver. This tube has a wooden ruler *m m*, called the gauge, and divided into inches, and parts of an inch, from the bottom at *n*, at the ſurface of the quickſilver, and continued upwards to *m*, about 31 inches.

The uſe of this rule is, to diſcover the quantity of air that remains in the receiver M: for as the air is pumped out of the receiver M, it is alſo pumped out of the tube *l m n*, becauſe the tube opens in the receiver; and as the tube is gradually emptied of its air, the quickſilver in the veſſel N is forced up the tube by the preſſure of the atmoſphere upon the quickſilver in the veſſel: and if the receiver could be perfectly exhauſted of air, the quickſilver would ſtand as high in the tube, as it does at that time in the barometer; for the quickſilver in both caſes is ſupported by the ſame power, viz. the weight of the atmoſphere on the quickſilver in the open veſſel.

Every turn of the handle F exhauſts a portion of air from the receiver, and conſequently raiſes the quickſilver in the tube; and the aſcent of the quickſilver is always proportionable to the quantity of air exhauſted; and the quantity of air remaining in the receiver is proportionable to the defect of the height of the quickſilver in the gauge, from its height in the barometer.

There are ſeveral experiments made with this air-pump, to ſhow the reſiſtance, weight, and elaſticity of the air.

The reſiſtance of the air is meaſured by a ſmall machine, having two mills, *a* and *b* (*fig.* 3), which are of equal weights, and each turning freely on its own axis, independent of each other. Each mill has four thin ſails fixed to its axis; thoſe of the mill *a* have their planes at right angles to its axis; and thoſe of *b* have their planes parallel to it. Therefore, when theſe mills turn round in common air, the mill *a* has but little reſiſtance from the air, becauſe its ſails cut the air with their thin edges; but the mill *b* is greatly reſiſted by the air, becauſe it expoſes the whole plane of its ſails againſt the air. In each axle is a fine pin near the middle of the frame, which goes quite through the axle, and ſtands out a little on each ſide of it. Upon theſe pins the ſlider *d* is made to bear, to hinder the mills from going round, when the ſtrong pin C is ſet on bend againſt the lower end of the pins.

Set this machine in motion, by drawing up the ſlider *d* to the pins on one ſide, and ſetting the ſpring C on bend at the oppoſite ends of the pins; then puſhing down the ſlider *d*, the ſpring C, acting with equal force upon each mill, will ſet them both in motion; but the mill *a* will run much longer than the mill *b*, becauſe the air makes leſs reſiſtance againſt its ſails than againſt thoſe of *b*.

Again, draw up the ſlider *d*, and ſet the ſpring C againſt the pins as before; then place the machine under the receiver M, upon the pump plate; and having exhauſted the receiver of its air, puſh the wire P P, which runs through the collar of leathers on the neck *q*, upon the ſlider *d* (*fig.* 3), which will diſengage it from the pins, and thereby ſuffer the ſpring C to ſet the mills a-going; and as there is no air in the receiver to make any ſenſible reſiſtance, they will both move a conſiderable time longer than they did in the open air, and they will both ſtop at the ſame moment. This experiment ſhows the reſiſtance of air on bodies in motion; and that equal bodies meet with different degrees of reſiſtance, according as they expoſe a greater or leſs ſurface to the air in the planes of their motions.

Again, put a guinea and a feather on the braſs flap *c*, in the tall cylindrical receiver A B (*fig.* 4), which is to be placed over the hole *i*, on the pump plate; turn up the braſs flap *c*, ſo as to confine both the guinea and the feather; then, putting a wet leather over the top of the receiver, and covering it with the plate *g*, from which the guinea and feather tongs *e d* will hang within the receiver; pump the air out of this receiver, and, by means of the wire *f*, open the tongs *e d*, and the flap *c* falling down, the guinea and feather will deſcend with equal velocity in the receiver, and both fall upon the plate at the ſame inſtant.

To ſhow the weight of the air, no more is neceſſary than a thin bottle or Florence flaſk (whoſe contents are exactly known) having a braſs cap with a valve tied over it, fixed to the mouth. This braſs cap is to be ſcrewed into the hole *i* of the pump plate, and the bottle exhauſted of its air. The bottle is then to be accurately weighed; when it will be found, that for every quart the bottle contains, it will weigh

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16 grains leſs than when it was full of air, when the quickſilver ſtands at 29½ inches in the barometer.

If the receiver O (*fig.* 1), or M, be placed over the hole *i* in the pump plate, and the air be exhauſted therefrom, this ſmall receiver will be preſſed down to the pump plate, by the weight of the atmoſphere, which will be found to be equal to as many fifteen pounds as there are ſquare inches in that part of the plate which the receiver covers; and which will hold down the receiver ſo faſt, that it cannot be removed until the air be let into it, by turning the cock *k*, when it will be perfectly looſe.

Place the ſmall glaſs A B (*fig.* 5), which is open at both ends, over the hole *i*, on the pump plate L L (*fig.* 1); and having put your hand cloſe upon the top of it, at B, exhauſt the air out of the glaſs, and your hand will be preſſed down upon the glaſs with a weight equal to as many times 15 pounds as the end of the glaſs B contains ſquare inches, as before.

If a piece of wet bladder be tied over the end of the glaſs (*fig.* 6), and, when it is dry, the glaſs be exhauſted of its air; the outer air will preſs upon the bladder, which will have a ſpherical concave figure, and will grow more concave as more air is pumped out of the glaſs, till, at length, it will break with a report as loud as that of a gun. If, inſtead of the bladder, a flat piece of glaſs be laid on the top of this receiver, and joined to it by a ring of wet leather between them, to exclude the air upon exhauſting the air out of the receiver, the preſſure of the outward air will ſoon break the flat piece of glaſs to pieces.

Let the two braſs cups A and B (*fig.* 7) be joined together with a wet leather between them, having a hole in the middle of it; then fix the end of the pipe D into the hole *i* of the pump plate, and exhauſt the air out of them, having turned the cock E, which permits the air to come through the pipe C D. Then turn the cock E again, to keep

keep out the air, and unſcrew the pipe D from the pump plate, and ſcrew on the handle F; then it will require a great force to pull theſe two cups aſunder; for if the diameter of the cups be four inches, they will be preſſed together, by the external air, with a force equal to 190 pounds. But if they be put under the large receiver M (*fig.* 1); and the air exhauſted out of the receiver, they will fall aſunder, having no external air to keep them together.

Place the veſſel A (*fig.* 8) on the pump plate, having ſome quickſilver in it, and cover it with the receiver B, in which is inſerted through the collar of leathers, in the braſs neck C, the tube *d e* open at the lower end; then exhauſt the air out of the receiver, and it will alſo be exhauſted out of the tube. When the receiver is ſufficiently exhauſted, puſh down the tube, ſo as to immerſe the lower end into the quickſilver. In this experiment, though the tube be exhauſted of air, yet none of the quickſilver will riſe in it, becauſe there is no air in the receiver, to preſs upon its ſurface; but if the air be let into the receiver, by the cock *k* in the pump plate, the quickſilver will immediately riſe in the tube, and ſtand nearly as high as it does at that time in the barometer.

This experiment ſhows, that the quickſilver is ſupported in the tube, merely by the preſſure of the air on its ſurface, in the open veſſel, in which the tube is immerſed; and that the more denſe and heavy the air is, the higher the quickſilver riſes; and, on the contrary, the thinner and lighter the air is, the leſs it will riſe. This is the reaſon why the quickſilver in the barometer falls before rain or ſnow, and riſes before fair weather; for, in the former caſe, the air is too thin and light to bear up the vapours; and in the latter caſe, too denſe and heavy to let them fall.

Note. In all experiments made with mercury, by the air-pump, there ſhould be a ſhort pipe ſcrewed in the hole *i* of the pump plate, ſo as to riſe about an inch above the plate, to prevent any quickſilver from getting into the air-pipe and barrels; for, ſhould any get looſe into the pipe or barrels, it ſpoils them, by looſening the ſolder, and corroding the braſs.

To ſhow the elaſticity or ſpring of the air, ſcrew the pipe A (*fig.* 9) into the pump plate, and place the receiver G H, upon the plate *c d*, which is fixed to the pipe, and exhauſt the air out of the receiver; then turn the cock *e*, to keep out the air, and unſcrew the pipe from the pump, and ſcrew it into the mouth of the copper veſſel A (*fig.* 10), which is half filled with water. Then, upon opening the cock *e*, the elaſticity of the air which is confined in the upper part of the copper veſſel A, will force the water up through the pipe A B in a jet or fountain, into the exhauſted receiver.

There are a great number of other experiments to be made with this uſeful machine, the air-pump, as:—1. To ſhow how neceſſary air is for the ſupport of animal life; by putting any ſmall animal under a receiver, and exhauſting the air. 2. The different effects it has on different bodies; by increaſing their gravity.—3. How long it will ſupply flame or fuel; by putting a lighted candle under the receiver.—4. The property of air in conveying ſound; by putting a bell under the receiver, and ſtriking it when the air is exhauſted, &c.

2. *Of the Barometer.*

This is an inſtrument uſed for meaſuring the weight of the atmoſphere, foretelling the changes of weather, and meaſuring the height of mountains, &c.

The common barometer is formed of a glaſs tube, hermetically ſealed at one end, and filled with quickſilver, defecated, and purged of its air. The open end of the tube is then immerſed in a veſſel of quickſilver; and by the preſſure of the atmoſphere on the quickſilver, in the open veſſel, the mercury in the tube will riſe to the height of twenty-nine inches and a half, when the weight of the atmoſphere is at a mean rate. When the weight of the atmoſphere is greater, then the mercury in the tube will riſe higher; and when the weight of the atmoſphere is leſs than its mean weight, the mercury in the tube will fall lower.

To conſtruct the Barometer. Being provided with a glaſs tube of one third or one half of an inch wide (the wider the better), and about thirty-four inches long, being cloſe at the top, or hermetically ſealed, pour into it well-purified quickſilver, with a ſmall funnel, of either glaſs or paper, till it wants about half an inch of being full; then ſtopping it cloſe with the finger, invert it ſlowly, and the air in the empty part will aſcend gradually to the other end, and collect in its way any ſmall air-bubbles, which will unavoidably get in, in filling the tube: then again invert it: and thus continue to invert it ſeveral times, turning the two ends alternately upwards, till all the air bubbles are collected, and brought up to the open end of the tube; then the tube will appear like a fine poliſhed ſteel rod, without a ſpeck in it. Then pour in a little more quickſilver to fill the tube quite up, and ſtopping the open end of the tube with the finger, invert the tube, and immerſe the finger and end of the tube, thus ſtopped, into a baſon of purified quickſilver; withdraw the finger, and the mercury will deſcend in the tube to ſome place between 28 and 31 inches above the mercury in the open veſſel, as theſe are the limits between which it always ſtands in this country, on the common ſurface of the earth. Meaſure from the ſurface of the quickſilver in the open veſſel, to the height of twenty-eight inches, and alſo to the height of thirty-one inches, dividing the three inches between theſe two numbers, into inches and tenths of an inch, which are marked on a ſcale placed againſt the ſide of the tube; and the tenths ſubdivided into hundredth parts of an inch, by a ſliding index, carrying a *vernier* or *nonius*. Theſe three inches, between twenty-eight and thirty-one, divided thus, will anſwer all the purpoſes of a ſtationary or chambe barometer; but for experiment on altitude and depths, it is neceſſary to have the ſcales continued higher up, and a great deal lower.

Several precautions are neceſſary in filling the tube, and fitting up the barometer, as:—1. The bore of the tube ſhould be pretty wide, to allow a free motion to the quickſilver.—2. The baſon at bottom ſhould alſo be pretty large, that the ſurface of the mercury in it may not ſenſibly riſe or fall with that in the tube.—3. The bottom of the tube ſhould be cut off rather obliquely, that, when it reſts on the bottom of the baſon, there may be a free paſſage for the quickſilver. And, laſtly, It is beſt to boil the quickſilver in the tube, which will expel all the air from it, and render it very pure.

This inſtrument owes its invention to *Toricelli*, the diſciple of Galileo, who lived about the beginning of the ſeventeenth century, and who having diſcovered that water could not be raiſed in a pump unleſs the ſucker was within 33 feet of the ſurface of the well, deſired Toricelli to inveſtigate the cauſe of it. After ſome time, Toricelli diſcovered, that the preſſure of the atmoſphere was the cauſe of the aſcent of the water in the pump; and that a column of water, 33 feet high, was the juſt counterpoiſe to a column of air of the ſame baſe, and which extended up to the top of the atmoſphere: and this was the true reaſon why the water did not aſcend any higher. He alſo diſcovered, that a column of quickſilver, about 2½ feet high, would be a counterpoiſe to a column of water of the ſame baſe, and 33 feet in height; as quickſilver is nearly 14 or rather 13.6 times heavier than water. This ſuppoſition he ſoon verified, by filling a glaſs tube with quickſilver, and inverting the open end of it into a baſon of the ſame, when the mercury deſcended till its height above that in the baſon was above 2½ feet, juſt as he expected.

It

6. In fair weather, when the mercury falls much and low, and continues ſo for two or three days before the rain comes, then expect a great deal of wet, and probably high winds.

7. The unſettled motion of the mercury denotes uncertain and changeable weather.

8. You are not ſo ſtrictly to obſerve the words engraved on the plate, as the mercury's riſing and falling: though, in general, it will agree with them. For, if it ſtand at the words *Much Rain*, then riſes up to *Changeable*, it preſages fair weather; though not to continue ſo long as if the mercury had riſen high: and on the contrary, if the mercury ſtood at *Fair*, and falls to *Changeable*, it preſages foul weather; though not ſo much of it as if it had ſunk lower.

Upon theſe rules of Mr. *Patrick*, Mr. *Rowning* remarks, that it is not ſo much the abſolute height of the mercury in the tube that indicates the weather, as its motion up and down; therefore, to paſs a right judgment of what weather is to be expected, we ought to know whether the mercury is actually riſing or falling; to which end the following rules ſhould be obſerved:

9. If the ſurface of the mercury be convex (ſtanding higher in the middle of the tube than at the ſides), it is a ſign that the mercury is then riſing.

10. But, if the ſurface be concave (or hollow in the middle), it is then ſinking.

11. If it be plain, or rather a little convex, the mercury is ſtationary; for, mercury being put into a glaſs tube, eſpecially a ſmall one, naturally has its ſurface a little convex; becauſe the particles of mercury attract each other more forcibly than they are attracted by the glaſs.

12. Sometimes the mercury will ſtick to the ſides of the tube; therefore, when an obſervation is to be made with ſuch a tube, the tube ſhould be ſhaken a little; then, if the air be grown heavier, the mercury will riſe about a twentieth of an inch higher: but if the air be lighter, it will ſink as

much: and if it be the wheel barometer, tap it gently with the finger, which will give the mercury a free motion.

To the foregoing rules may be added the following, taken from later and cloſer obſervations:

13. In winter, ſpring, and autumn, the ſudden falling of the mercury, and that for a large ſpace, denotes high winds and ſtorms; but in ſummer, it denotes heavy ſhowers, and often thunder; and it always ſinks loweſt of all for great winds, though not accompanied with rain; for wind and rain together, it falls more than for either of them alone. Alſo, if, after rain, the wind change into any part of the North, with a clear and dry ſky, and the mercury riſe, it is a certain ſign of fair weather.

14. After very great ſtorms of wind, when the mercury has been low, it commonly riſes very faſt. In ſettled, fair, and dry weather, expect but little rain, except the barometer ſink much; for a ſmall ſinking then, only denotes a little wind, or a few drops of rain; and the mercury ſoon riſes again to its former ſtation. In a wet ſeaſon, ſuppoſe in hay-time and harveſt, the ſmalleſt ſinking of the mercury muſt be noticed, for, when the conſtitution of the air is much inclined to ſhowers, a little ſinking then denotes more rain, as it never then ſtands very high; and if in ſuch a ſeaſon it riſes ſuddenly, very faſt and high, expect not fair weather more than a day or two, but rather that the mercury will fall again very ſoon, and rain immediately to follow: the ſlow gradual riſing, and keeping on for two or three days, being moſt to be depended on for a week's fair weather; and the unſettled ſtate of the quickſilver always denoting uncertain and changeable weather, eſpecially when the mercury ſtands any where about the word *Changeable* on the ſcale.

15. The greateſt heights of the mercury in this country are found upon eaſterly and north-eaſterly winds; and it may often rain or ſnow, the wind being in thoſe points, and the barometer ſink little or none, or may even be in the

riſing ſtate the effect of thoſe winds counteracting. But the mercury ſinks for wind as well as rain in all the other points of the compaſs; but riſes as the wind ſhifts about to the north, or to the eaſt, or between thoſe points; but if the barometer ſhould ſink, with the wind in that quarter, expect it ſoon to change from thence; or elſe, ſhould the fall of the mercury be much, a heavy rain is likely to enſue.

16. The barometer being lower, and continuing ſo longer than what can be accounted for, by immediate falls, or ſtormy weather, indicates the approach of very cold weather for the ſeaſon; and alſo cold weather, though dry, is always accompanied by a low barometer, till near its termination.

17. Warm weather is always preceded, and moſtly accompanied, by a high barometer; and the riſing of the barometer, in the time of cold weather, is a ſign of the approach of warmer weather: and alſo, if the wind be in any of the cold points, a ſudden riſe of the barometer indicates the approach of a ſoutherly wind, which, in winter, generally brings rain.

The barometer is alſo found to ſink in a certain ratio to its diſtance from the ſurface of the earth: it is, therefore, uſed for meaſuring any acceſſible heights. Various rules have been given by writers on the barometer, for applying it to this purpoſe, or computing the height aſcended, from the fall of the mercury in the tube, the moſt accurate of which is that of Dr. Halley, now greatly improved by De Luc, by introducing into it the correction of the columns of the mercury and air, on account of heat, with other corrections and modifications. This rule is as follows, viz. 10,000 × logarithm of $\frac{M}{m}$ is the altitude in fathoms, in the mean temperature of 31 degrees; and for every degree of the thermometer above that, the reſult muſt be increaſed by ſo many times its 435th part, and diminiſhed when below it. In this theorem M denotes the length of the column of mercury in the barometer tube at the bottom of the hill or eminence; and m

denotes the ſame at the top of the hill or eminence; and it is to be obſerved, that the reſult is always in fathoms of ſix Engliſh feet each.

As the ſcale of variation in the barometer is but ſmall, being included within three inches, viz. from 28 to 31 inches, ſeveral contrivances have been deviſed for enlarging the ſcale, to render the ſmall variations of the mercury more apparent; this has given riſe to the invention of ſo many different kinds of barometers:—a few of the moſt improved are the following:

1. *The Diagonal Barometer.*

This is a method of enlarging the common ſcale of three inches perpendicular height, by extending it to any length, B C (*fig.* 11), in an oblique direction. This barometer was invented by Sir Samuel Moreland. The perpendicular height of the diagonal part B C, is equal to the ſcale of variation of three inches, or C I; and conſequently, while the mercury in the common barometer riſes the whole length of the ſcale, which is three inches, and equal to I C; in this barometer it will move from B to C: thus the ſcale is enlarged in this barometer, in the proportion of B C to I C. But it is found, that the diagonal part B C cannot be bent from the perpendicular, more than in an angle of 45 degrees, which increaſes the ſcale only in the proportion of 7 to 5. This form is liable to ſome inconveniences, on account of the obliquity of the part B C, which makes the mercury frequently divide into ſeveral parts, and renders it neceſſary to fill the tube again.

2. *The Horizontal Rectangular Barometer.*

This barometer (*fig.* 12) was the invention of *J. Bernoulli*, and *Caſſini*, and conſiſts of a tube A C D E, ſealed at the upper end A, and bent to a right angle at D; the end

F being open. The mercury, in this, ſtands in both legs from E to C. The ſcale of variation from D to F is here made larger; and it is evident, in moving three inches from A to C, it will move as much more in the ſmall leg D F, as the area of the tube at A C is greater than that of D F: wherefore the motion of the mercury at E muſt be very ſenſible. Though the end of the tube F be open, yet the mercury cannot run out, being oppoſed there by the preſſure of the atmoſphere. This inſtrument is founded on that theorem in Hydroſtatics, that fluids of the ſame baſe preſs according to their perpendicular altitude, and not according to the quantity of their matter. So that the ſame preſſure ſuſtains the quickſilver, that fills the tube A D F, and the ciſtern C, as would ſupport the mercury in the tube alone. This form is, however, liable to ſome inconveniences; for the attrition of the mercury againſt the ſide of the glaſs, and the quick motion of it in the part D F, is apt to break the mercury, and render its motions unequal: it is alſo apt to be thrown out at the open end F, by any ſudden ſhock.

3. *Dr. Hook's Wheel Barometer* (*Fig.* 13).

This barometer tube has a large ball A at the top, and is bent up at the lower end, which is open, where an iron ball floats on the top of the mercury in the tube, and which ball is connected to another ball H, hanging freely over a pulley, and turning an index K about its centre. When the mercury riſes in the part K, it raiſes the ball, and the other ball H deſcends, and turns the pulley with the index round a graduated circle from M towards N; and the contrary way when the quickſilver and the ball ſink in the bent part of the tube. This ſcale is eaſily enlarged 10 or 12 fold, being increaſed in proportion to the axis of the pulley to the length of the index K. If this inſtrument could be conſtructed without any friction of the pulley and axis, it would anſwer

The Air-gun.

This inſtrument is an ingenious pneumatical invention, for driving a bullet, with great violence, by means of condenſed air, forced into an iron ball by a condenſor.

The condenſor (*fig.* 15) has at the end *a*, a male ſcrew, on which the hollow ball *b* is ſcrewed, in order to be filled with condenſed air. In the inſide of this ball there is a valve, to prevent the air from eſcaping (after it is injected into it), until it be forced open by a pin, *a* (*fig.* 16).

When the air is to be condenſed into the ball, place your feet on the iron croſs, *h h*, in order to hold down the piſton rod *d c*; then lift up the barrel *e a* by the handles *i i*, until the piſton *c* be brought below the hole *e*; the barrel *a c* and ball *b* will then be filled with air through the hole *e*. Then thruſt down the barrel *a c*, until the piſton *d c* reach the neck of the iron ball at *a*; then all the air between *e* and *a* will be forced up through the valve into the ball; and when the handles *i i* are again lifted up, the valve in the ball will cloſe, and ſo keep in the air: thus by rapidly continuing the ſtrokes up and down, the ball will preſently be filled: then unſcrew the ball from the condenſor, and ſcrew it upon another male ſcrew, which is connected with the barrel, and goes through the ſtock of the gun (*fig.* 16). A bullet then being depoſited in the barrel of the gun, the hammer of the lock at *a* ſtrikes againſt the pin, which opens the valve in the ball, and lets out as much air as will drive a muſket-ball to a conſiderable diſtance.

There are ſeveral kinds of air-guns; but that here deſcribed, is the moſt improved and uſeful, as the gun need not be any larger than a ſmall fowling-piece: and ſeveral balls, filled with condenſed air, may be taken to any diſtance from home with very little trouble, and which will ſave the trouble of filling the ſame ball every time it is wanted. A ball of 3½ inches diameter may be made to contain twelve penny-weights of air, which will diſcharge 12 or 15 bullets with conſiderable force.

There are ſome air-guns that have a ſmaller barrel contained within a larger one; and the ſpace between the two barrels holds the condenſed air. In this inſtrument there is a valve fixed at *a* (*fig.* 16), with a condenſor fixed to the barrel at *a*, and continued through the but end to *e*, where the piſton rod may be left in. Here the whole gun ſerves inſtead of the handles *i i* (*fig.* 15), to condenſe the air into the barrel.

The magazine air-gun differs from the others, by having a ſerpentine barrel, which contains 10 or 12 bullets: theſe are brought into the barrel of the gun ſucceſſively, by means of a lever; and they may be diſcharged as faſt as if they were in ſeparate guns.

CHAP. XVIII.

OF HYDROSTATICS.

Definitions.

1. HYDROSTATICS treat of the equilibrium of fluids; or the gravitation of fluid bodies remaining at reſt. When this equilibrium is removed, and the fluid body ſet in motion, the effects it then produces are called *Hydraulics.*

2. A *Syphon* is a bent tube (*fig.* 5, plate 23).

3. A *Valve* is a kind of flap or cover, fixed to a pipe, or to the aperture of any body, and which, by opening only one way, ſuffers water or any fluid body to paſs, but not to return.

4. A *Piſton* is a ſmall cylinder, fixed to the end of a rod, and fitted to the bore of a pipe, and frequently contains a valve.

Axioms.

1. All fluids, except air, are incompreſſible, or incapable of being compreſſed into a leſs ſpace.

2. In a veſſel of water, or any other fluid body, the preſſure of the upper parts on the lower is in proportion to the depth; and is the ſame at the ſame depth, whatever the diameter of the veſſel may be.

3. The preſſure of a fluid upwards, is equal to its preſſure downwards, at any given depth.

4. The bottom and ſides of a veſſel are preſſed by the fluid it contains, in proportion to its height, without any regard to the quantity.

5. If fluids of different gravities be contained in the ſame veſſel, the heavieſt will be at the bottom, the lighteſt at the top, and the others will be farther diſtant from the top in proportion to their ſpecific gravities.

6. The direction of the preſſure of a fluid againſt the ſides of the veſſel which contains it, is in lines perpendicular to the ſides of ſuch veſſel.

7. A body that is heavier than an equal quantity of a fluid, will ſink in that fluid; but, if it be lighter, it will ſwim at the top of the ſame fluid; and if it be of the ſame gravity, it will neither ſink nor ſwim, but will remain ſuſpended in any part of the fluid.

8. A ſolid immerſed into a fluid, is preſſed by that fluid on all ſides, in proportion to the height of the fluid above the ſolid. And bodies very deeply immerſed in any fluid, may be conſidered as equally preſſed on all ſides.

SECT. I.

OF FLUIDITY.

A FLUID body, in Sir Iſaac Newton's definition, is *a body yielding to any force impreſſed, and which has its parts very eaſily moved one among another.* This is the definition of a perfect fluid: if the fluid require ſome ſenſible force to move its parts, it is an imperfect fluid; and the imperfection is in proportion to that force: ſuch perhaps are all the fluids with which we are acquainted.

Fluids are either elaſtic, ſuch as air; or non-elaſtic, as water, mercury, &c. The latter are incompreſſible, and occupy the ſame ſpace under all preſſures or forces; but the former dilate and expand themſelves continually, by taking off the external preſſure from them. The properties of the former fluids conſtitute the doctrine of Pneumatics, before treated of; the latter contain the principles of Hydroſtatics.

Fluidity differs from liquidity, or humidity; the latter implying wetting or adhering. Thus, air, ether, mercury, and other melted metals, and even ſmoke and flame, are fluid bodies, though not liquid ones; while water, beer, milk, &c. are both fluids and liquids.

The modern opinion concerning the original and conſtituent parts of fluids, is, that they are ſmall, ſmooth, hard, globular particles; conſequently, each particle muſt be a ſolid globular body; and conſidered ſingly, is no fluid; but becomes a fluid, by being joined with other particles of the ſame or a ſimilar kind.

That the particles of fluid bodies are very ſmall, is evident, from their texture having never been diſcovered by the fineſt microſcope: that they are ſmooth, appears from that freedom wherewith they glide over one another, when ſet in motion: that they are hard and impenetrable, is plain from their being incapable of compreſſion: and that they are ſpherical, is obvious, from their being ſo eaſily put in motion; and from the interſtices or vacancies, which are hereafter proved to ſubſiſt between them; which could not be the caſe, unleſs they were ſpherical, and touched each other only in ſome ſingle points of their ſurfaces. For, upon mixing ſalt with water, a certain quantity of the ſalt will be diſſolved, without increaſing the dimenſions of the water; which demonſtrates the vacuities between the particles of the water. When a fluid becomes more buoyant, it is a proof that its ſpecific gravity is increaſed, and conſequently, many of its vacuities filled up; and even then it may receive a certain quantity of other diſſoluble bodies, the particles whereof are adapted to the remaining vacancies, without adding any thing to its bulk, though the abſolute weight of the whole fluid be thereby increaſed. This is demonſtrated by taking the weight of a phial of rain water with a nice balance: when the water is poured out, and ſome ſalt added to it, and the phial again filled with the water, it will be found to weigh more than when before the ſalt was put in, from the vacuities of the freſh water being filled with ſaline particles.

It has alſo been found by experiment, that the particles whereof fluids are compoſed, conſiſt of ſpheres of different diameters, whoſe interſtices may be ſucceſſively filled with proper ingredients; and where theſe interſtices are ſmaller, the gravity of the fluid will be greater, and *vice verſa.*

For example: if a barrel be filled with any large ſpherical bodies, as bullets, many ſmall ſhot may afterwards be placed in the interſtices of theſe bullets; the vacuities of the ſhot may then be filled with ſea ſand; the interſtices of which may again be filled with water, which will alſo admit of a certain quantity of ſalt in the vacuities; and thus the weight of the barrel may be greatly increaſed, without increaſing the ſpace occupied by theſe materials. This reaſoning alſo holds good in fluid bodies, as well as in thoſe which are ſolid; for river water will diſſolve a certain quantity of ſalt; after which it will diſſolve a certain quantity of ſugar; and after that a certain quantity of alum; and then perhaps will receive other diſſoluble bodies, without increaſing the dimenſions of the whole.

If fluids were not compounded of ſuch primary particles, but made up of one homogeneous ſubſtance, equally denſe, without conſiſtence, there would be no difference in their ſpecific gravities, and all fluids would be of the ſame weight, which is not the caſe.

That a fluid has vacuities, is evident from the following conſideration, viz. if all ſpace were abſolutely full of matter, that matter muſt be either fluid or fixed. If it were fixed, no motion could poſſibly be therein, as is evident from reaſon and experience; it muſt therefore be fluid. But a fluid without vacuities would be denſer, and conſequently heavier, than all other fluids; and if denſer, all bodies will emerge and ſwim at the top, by hydroſtatical laws, and there would be no ſuch thing as gravity. But as gravity exiſts, all ſpace therefore cannot be filled, even with a fluid.

By the experiments of *Borcelli*, it has been demonſtrated, that the conſtituent parts of all fluids, are not fluids themſelves, but conſiſtent bodies; and that the elements of all bodies are perfectly firm and hard. The incompreſſibility of water, proved by the Florentine experiment, is a ſufficient evidence that each primary particle of this fluid is a perfect impenetrable ſolid.

This famous experiment was firſt attempted by the ingenious Lord *Verulam*, who encloſed a quantity of water in a piece of lead, and found, that the water would ſooner make its way through the pores of the lead, than be reduced to leſs compaſs, by any force that could be applied. This experiment was afterwards made at Florence, with a globe of ſilver; which being filled with water, and well cloſed, was gently preſſed, when a ſmall quantity of water iſſued through the pores of the ſilver in the form of dew.

The

3. The ſpecific gravities in bodies of the ſame abſolute weight are reciprocally as their bulks.

4. The ſpecific gravities of all bodies are in a ratio compounded of the direct ratio of their weights, and the reciprocal ratio of their magnitudes.

5. The abſolute gravities, or weights of bodies, are in the compound ratio of their ſpecific gravities, and magnitudes or bulks.

6. The magnitudes of bodies are directly as their weights, and reciprocally as their ſpecific gravities.

7. When a body is immerſed in a fluid that is ſpecifically lighter than the body, the body loſes as much of its weight as is equal to the weight of a quantity of the fluid of the ſame bulk or magnitude.

Therefore, the ſpecific gravities of two bodies are as the abſolute gravities, under the ſame bulk. The ſpecific gravity of the fluid will be to that of the body immerſed in it, as the part of the weight loſt by the ſolid is to the whole weight. And the ſpecific gravities of two fluids are as the weights loſt by the ſame ſolid immerſed in them.

8. The ſpecific gravity of any fluid or ſolid body is found as follows:—Suſpend a globe of lead by a fine thread from one arm of a balance, and to the other arm faſten an equal weight. Immerſe the globe of lead into the fluid, and obſerve what weight it will require then to balance it, and conſequently what weight it has loſt, which is proportional to the ſpecific gravity of the fluid, compared with the whole weight of the ſolid. Thus the proportion of the ſpecific gravity of one fluid to another, is determined, by immerſing the globe ſucceſſively in all the fluids, and obſerving the weight the globe has loſt each time, which will be the proportions of the ſpecific gravities of the fluids.

This operation alſo gives the ſpecific gravity of the ſolid immerſed, whether it be a globe, or of any other ſhape, if the gravity of the fluid be known. For the ſpecific gravity of the fluid is to that of the ſolid, as the weight loſt is to the whole weight.

9. Th

The foregoing theorems have been ſufficiently demonſtrated by various authors, from the principles of mechanics: they are alſo exactly conformable to experience; as hath been ſufficiently proved from ſeveral courſes of philoſophical experiments,

Various tables have been given, by different authors, of the ſpecific gravities of many kinds of bodies. It will be ſufficient in this place to give the ſpecific gravities of ſome of the moſt uſeful bodies that have been determined with greater certainty and accuracy. The numbers in this table expreſs the number of avoirdupois ounces contained in a cubical foot of each body; that of common water being juſt 1000 ounces, or 62½ pounds.

Table of ſpecific Gravities.

1. *Solids.*

Platina, pure	23000	Crude antimony	4000
Fine gold	19640	Diamond	3517
Standard gold	18888	Granite	3530
Lead	11325	White lead	3160
Fine ſilver	11091	Iſland cryſtal	2720
Standard ſilver	10535	Marble	2705
Copper	9000	Pebble-ſtone	2700
Copper halfpence	8915	Rock cryſtal	2650
Gun-metal	8784	Pearl	2630
Fine braſs	8350	Green glaſs	2600
Caſt braſs	8000	Flint	2570
Steel	7850	Onyx ſtone	2510
Iron	7645	Common-ſtone	2500
Pewter	7471	Cryſtal	2210
Caſt iron	7425	Clay	2160
Tin	7320	Oyſter-ſhells	2092
Lapis calaminaris	5000	Brick	2000
Loadſtone	4930	Common earth	1984
Mean ſpecific gravity of		Nitre	1900
the whole earth	4500	Vitriol	1880

Alabaſter

9. The ſpecific gravity of a ſolid that is lighter than the fluid in which it is immerſed, is found by the following proceſs. To the lighter body, whoſe ſpecific gravity is required, annex another body, that is much heavier than the fluid, ſo that the compound maſs may ſink in the fluid. Weigh the heavier body, and the compound maſs, ſeparately, both in water and out of it; then find how much each loſes in water, by ſubtracting its weight in water from its weight in air; and ſubtract the leſs of theſe remainders from the greater; then ſay, as this laſt remainder is to the weight of the lighter body in air, ſo is the ſpecific gravity of the fluid to the ſpecific gravity of that body.

10. The ſpecific gravities of bodies of equal weights are reciprocally proportional to the quantities of weight loſt in the ſame fluid. Hence is found the ratio of the ſpecific gravities of ſolids, by weighing in the ſame fluid, maſſes of them that weigh equally in air, and noting the weight loſt by each.

11. A body deſcends in a fluid that is ſpecifically lighter, but aſcends in a fluid that is ſpecifically heavier, with a force equal to the difference between its weight an the weight of an equal bulk of the fluid.

12. A body ſinks in a fluid that is ſpecifically heavier, far, as that the weight of the body is equal to the weigh a quantity of the fluid of the ſame bulk as the part of body which is immerſed in the fluid. Therefore, as ſpecific gravity of the fluid is to that of the body, ſo is whole magnitude of the body to the magnitude of the immerſed. By this theorem is found the abſolute weigh any body immerſed in a fluid: for the weight of t quantity of the fluid which is diſplaced by the ſolid bo always equal to the whole weight of the ſolid; thus, boat on the water be ſo loaded, that it diſplace three cub feet of water, its whole weight will be 3000 [illegible] pounds, that being the weight of three cubical [illegible]

13. In equal ſolids the ſpe[illegible]vities ar[illegible] immerſed in the ſame fluid.

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Alabaſter	1874	Brazil wood	1031
Horn	1840	Box wood	1030
Ivory	1825	Bees wax	955
Sulphur	1810	Butter	940
Chalk	1793	Oak	925
Solid gunpowder	1745	Gunpowder, ſhaken	922
Alum	1714	Logwood	913
Dry bone	1660	Ice	908
Sand	1520	Aſh	800
Lignum vitæ	1327	Maple	755
Coal	1250	Beech	700
Jet	1238	Elm	600
Ebony	1177	Fir	550
Pitch	1150	Saſſafras wood	482
Roſin	1100	Cork	240
Mahogany	1063	New-fallen ſnow	86
Amber	1040		

2. *Fluids.*

Quickſilver	13600	Ale	1028
Oil of vitriol	1700	Vinegar	1026
Oil of tartar	1550	Tar	1015
Honey	1450	Common water	1000
Spirit of nitre	1315	Diſtilled water	993
Aqua fortis	1300	Red wine	990
Treacle	1290	Proof ſpirits	931
Aqua regia	1234	Olive oil	913
Human blood	1054	Pure ſpirits of wine	866
Urine	1032	Oil of turpentine	800
Cows milk	1031	Æther	726
Sea water	1030	Common air	1.232, or, $1\frac{7}{30}$

As theſe numbers are the weights of a cubic foot, or 1728 cubic inches of each of the foregoing bodies in avoirdupois ounces, the quantity, in any other weight, or the weight of any other quantity, may be found by proportion.

For example, required the content of an irregular block of common ſtone, weighing one hundred weight, or 1792 ounces: here, as 2500, the ounces in a cubic foot of common ſtone, is to 1792, ſo is 1728, the inches in a cubical foot, to 1238⅘ cubical inches, the contents.

Again, what is the weight of a block of granite, the length whereof is 63 feet, and the breadth and thickneſs each 12 feet, being the dimenſions of one of the ſtones of granite in the walls of Balbeck? Here the ſolid content of this ſtone is 9072 feet; therefore, as 1 is to 9072, ſo is 3500 ounces to 31,752,000 ounces, or 885 tons 18 cwt. 3 qrs. the weight of the ſtone.

SECT. II.

THE CONSTRUCTION AND USE OF THE HYDROSTATIC BALANCE, HYDROMETER, AND HYDROSTATIC BELLOWS.

The Hydroſtatic Balance.

THE hydroſtatic balance is the moſt convenient inſtrument of any hitherto invented, for diſcovering the ſpecific gravity of all ſubſtances, whether fluid or ſolid. It is conſtructed in various forms; but that which is moſt generally retained is the following:

V C G (*fig.* 1, *plate* 23) is the ſtand or pillar of the inſtrument, which is to be fixed in a table. From the top A, by two ſilken ſtrings, hangs the horizontal bar B B, from which is ſuſpended by a ring *i*, the fine beam of a balance *b*, which is prevented from deſcending too low on each ſide by the gently ſpringing piece *l*, *x*, *y*, *z*, fixed on the ſupport M. The harneſs is annulated at *o*, to ſhow exactly the perpendicular poſition of the examen, by the ſmall-pointed index fixed above it. On each ſide of the piece A is a pulley, over which paſſes a ſtring, which goes down to the bottom on the other ſide, and hangs over the hook at V, which hook is moveable about an inch and a quarter, backward and forward, by means of the ſcrew P, ſo that the balance may be raiſed or depreſſed ſo much. But when a greater elevation or depreſſion is required, the ſliding piece S, which carries the ſcrew-pin, is readily removed to any part of the ſquare braſs rod V K, and fixed by means of the ſcrew.

By theſe means the motion of the balance is adjuſted; the other parts of the apparatus are as follow: D is a piece to ſupport the ſmall board H H, fixed under the two ſcales *d* and *e*, and is moveable up and down by a long ſlit in the pillar above C, in which D ſlides, having a ſcrew in the back part to faſten it when neceſſary. From the bottom of the middle of each ſcale *d* and *e*, hangs a braſs wire *a d* and *a c*, by a fine hook; theſe wires paſs through two holes, *m m*, in the table. To the wire *a d* is ſuſpended a curious cylindrical wire *r s*, perforated at each end for that purpoſe. This wire *r s* is covered with paper, graduated by equal diviſions, and is about five inches long.

In one corner of the board, at E, is a fixed tube of braſs, on which a round wire *h l* is ſo adapted as to move ſomewhat freely, by its flat head I. Upon the lower part of this moves another tube Q, which has ſufficient friction to make it remain in any poſition required; to this is fixed an index T, which moves horizontally when the wire *h l* is turned round; and therefore may be eaſily ſet to the graduated wire *r s*. To the lower end of the wire *r s* hangs a weight L, which has a wire *p n*, with a ſmall braſs ball *g* at the end, about a quarter of an inch in diameter. On the other wire *a c*, from the other ſcale, hangs, by means of a horſe-hair, a large glaſs bubble R.

To apply this inſtrument to uſe, let the weight L be taken away, and a wire *p n* be ſuſpended from the hook *s*; and let the bubble R be taken away from the other ſcale, and a weight ſuſpended in the room thereof. Suppoſe the weight to be ſufficient to keep the parts belonging to the other ſcale in equilibrium; and that the middle point of the wire *p n* is at the ſurface of the water in the veſſel N. And, *Note*, the wire *p n* is to be of ſuch a ſize, that the length of one inch ſhall weigh four grains.

It is evident, that as braſs is eight times heavier than water, for every inch the wire ſinks in the water, it will become an eighth part lighter, which is half a grain; and heavier in the ſame proportion, for every inch it riſes out of the water: thus, by ſinking it two inches below the middle point, or raiſing it as much above it, the wire will become one grain lighter or heavier. Therefore, when the middle point of the wire is at the ſurface of the water, if the balance be in equilibrium, and the index T ſet to the middle point *a* of the graduated wire *r s*, and the diſtance on each ſide *a r* and *a s* contain an hundred equal parts; then, if, in weighing bodies, the weight is required to the hundredth part of a grain, it may be eaſily found by the following method: let the body to be weighed be placed in the ſcale *d*, put the weight *x* in the ſcale *e*, and let this be ſo adjuſted, as one grain more ſhall be too much, and one grain leſs too little. Then, by moving the balance gently up or down by the ſcrew P, till the equilibrium be exactly ſhown at O; if the index T be at the middle point *a* of the wire *r s*, it ſhows that the weight put in the ſcale *e* is juſt equal to the weight of the body.

The foregoing method diſcovers the abſolute weight of the body; but to find the relative or ſpecific weight, it muſt be weighed hydroſtatically in water, as follows:—Inſtead of putting the body to be weighed into the ſcale *d* as before, let it hang by a horſe-hair, as the weight R, ſuppoſing the veſſel of water O were removed. Then the equilibrium being made, the index T ſtanding between *a* and *r* ſhows the weight of the body. As it hangs thus, let it be immerſed in the water of the veſſel O, when it will become much lighter, the ſcale *e* deſcending till the beam of the balance reſt on the ſupport *x*. Then, if 100 grains put in the ſcale *d* will exactly reſtore the equilibrium, ſo that the index T ſtands at the 36th diviſion above *a*, it is evident that the weight of an equal bulk of water would be exactly 100 grains.

In the ſame manner this balance may be applied to find the ſpecific gravities of fluids.

The Hydrometer.

The hydrometer diſcovers the ſpecific gravity of fluids only; for which purpoſe it is the moſt accurate, eaſy, and expeditious inſtrument of any.

This inſtrument conſiſts of a copper ball D (*fig.* 2), to which is ſoldered a braſs wire A B, a quarter of an inch in diameter. The upper part of this wire is filed flat, and marked *Proof* at *m* (*fig.* 3), becauſe it ſinks exactly to that mark in proof ſpirits. There are two other marks at A and B (*fig.* 2), to ſhow whether the liquor be one tenth above or below proof, according as the hydrometer ſinks to A or riſes to B, when a braſs weight, C or K, is ſcrewed to its bottom *c*. There are alſo other weights to ſcrew on, which ſhow the ſpecific gravity of ſeveral different fluids as low as common water.

The round part of the wire above the ball ſhould be marked ſo as to repreſent river water, when it ſinks to R W (*fig.* 3), the weight which anſwers to that water being then ſcrewed on. When it is put into ſpring water, mineral water, ſea water, and water of ſalt ſprings, it will riſe to the marks S P, M I, S E, and S A, reſpectively. On the contrary, when it is put into Briſtol water, rain water, port wine, and mountain wine, it will reſpectively ſink to the marks *b r*, *r a*, *p o*, and *m o*. Inſtruments of this kind are ſometimes called Areometers.—And, *Note*, that the globe D (*fig.* 2) ſhould be made of copper; for ivory imbibes ſpirituous liquors, and conſequently alters their gravity; and glaſs globes require too much attention.

This hydrometer was the invention of Mr. Clarke, and anſwers very well to diſcover the ſpecific gravity of ſpirituous liquors, and to ſhow whether any ſpirit be above or below proof, and how much.

But the moſt perfect hydrometer is that repreſented in figure 4, which may be made to ſhow the ſpecific gravity of fluids to the greateſt degree of exactneſs. It conſiſts of a large hollow ball B, with a ſmaller ball *b*, ſcrewed to its bottom, and partly filled with mercury or ſmall ſhot, in order to render it but little ſpecifically lighter than water. In the larger ball at C, there is a ſhort nick, into which is ſcrewed the graduated braſs wire A C, which has a ſmall weight A at the top, to cauſe the inſtrument to deſcend in the fluid.

When this inſtrument is immerſed in any fluid contained in a jar L M, the quantity of the fluid diſplaced by it will be equal in bulk to that part of the inſtrument which is under water, and equal in weight to the whole inſtrument. Therefore, if the weight of the whole inſtrument be 4000 grains, we can by theſe means compare the different dimenſions of 4000 grains weight of ſeveral ſorts of fluids; for if the weight A be ſufficient to ſink the inſtrument in rain water, till the middle point of the ſtem, marked 20, come to the ſurface of the water; and after that, if it be immerſed in common ſpring water, and the ſurface of the water ſtand at one tenth of an inch below the middle point 20, it is evident that the ſame weight of each water differs only in bulk by the magnitude of one tenth of an inch in the ſtem of the inſtrument.

Then, ſuppoſe the ſtem of the inſtrument to be ten inches in length, and to weigh juſt 100 grains; every tenth of

The ſame experiment was afterwards made by Sir Iſaac Newton, and others, with globes, made of gold and other metals; all which experiments were attended with the ſame phenomenon, and have tended to eſtabliſh the above theory.

As a great many of the phenomena in hydroſtatics depend upon gravity, it may be neceſſary to mention the laws of gravity, concerning bodies immerſed in fluids.

Gravity, in hydroſtatics, as well as in the other arts, is divided into abſolute and ſpecific gravity.

Abſolute gravity is that force with which the body tends downwards; and is always in proportion to the quantity of matter in the body.

Specific gravity is the relative, comparative, or apparent weight of any body compared with that of another body, of the ſame bulk or magnitude; and therefore ſignifies, that gravity or weight which is peculiar to each kind of body.

A body is ſaid to be ſpecifically heavier than another body, when it contains a greater weight than the other, under the ſame bulk or dimenſions; and thus reciprocally that body, which contains a leſs weight than another under the ſame bulk, is ſaid to be ſpecifically lighter than the other body. Thus, if there be two equal ſpheres, ſuppoſe one foot in diameter each; the one of lead, and the other of wood: as the leaden one is found to be heavier than the wooden one of the ſame ſize, it is ſaid to be ſpecifically heavier, that is, heavier in ſpecies, or in kind; and the wooden one is ſpecifically lighter.

Specific gravity is by ſome called relative gravity, to diſtinguiſh it from abſolute gravity.

The Laws of the ſpecific Gravities of Bodies.

1. Two bodies of equal bulk have their ſpecific gravities to each other, as their abſolute weights or denſities.

2. Two bodies of the ſame ſpecific gravity or denſity, have their abſolute weights in proportion to their magnitudes or bulks.

3. The ſpecific gravities in bodies of the ſame abſolute weight are reciprocally as their bulks.

4. The ſpecific gravities of all bodies are in a ratio compounded of the direct ratio of their weights, and the reciprocal ratio of their magnitudes.

5. The abſolute gravities, or weights of bodies, are in the compound ratio of their ſpecific gravities, and magnitudes or bulks.

6. The magnitudes of bodies are directly as their weights, and reciprocally as their ſpecific gravities.

7. When a body is immerſed in a fluid that is ſpecifically lighter than the body, the body loſes as much of its weight as is equal to the weight of a quantity of the fluid of the ſame bulk or magnitude.

Therefore, the ſpecific gravities of two bodies are as the abſolute gravities, under the ſame bulk. The ſpecific gravity of the fluid will be to that of the body immerſed in it, as the part of the weight loſt by the ſolid is to the whole weight. And the ſpecific gravities of two fluids are as the weights loſt by the ſame ſolid immerſed in them.

8. The ſpecific gravity of any fluid or ſolid body is found as follows:—Suſpend a globe of lead by a fine thread from one arm of a balance, and to the other arm faſten an equal weight. Immerſe the globe of lead into the fluid, and obſerve what weight it will require then to balance it, and conſequently what weight it has loſt, which is proportional to the ſpecific gravity of the fluid, compared with the whole weight of the ſolid. Thus the proportion of the ſpecific gravity of one fluid to another, is determined, by immerſing the globe ſucceſſively in all the fluids, and obſerving the weight the globe has loſt each time, which will be the proportions of the ſpecific gravities of the fluids.

This operation alſo gives the ſpecific gravity of the ſolid immerſed, whether it be a globe, or of any other ſhape, if the gravity of the fluid be known. For the ſpecific gravity of the fluid is to that of the ſolid, as the weight loſt is to the whole weight.

9. The ſpecific gravity of a ſolid that is lighter than the fluid in which it is immerſed, is found by the following proceſs. To the lighter body, whoſe ſpecific gravity is required, annex another body, that is much heavier than the fluid, ſo that the compound maſs may ſink in the fluid. Weigh the heavier body, and the compound maſs, ſeparately, both in water and out of it; then find how much each loſes in water, by ſubtracting its weight in water from its weight in air; and ſubtract the leſs of theſe remainders from the greater; then ſay, as this laſt remainder is to the weight of the lighter body in air, ſo is the ſpecific gravity of the fluid to the ſpecific gravity of that body.

10. The ſpecific gravities of bodies of equal weights are reciprocally proportional to the quantities of weight loſt in the ſame fluid. Hence is found the ratio of the ſpecific gravities of ſolids, by weighing in the ſame fluid, maſſes of them that weigh equally in air, and noting the weight loſt by each.

11. A body deſcends in a fluid that is ſpecifically lighter, but aſcends in a fluid that is ſpecifically heavier, with a force equal to the difference between its weight and the weight of an equal bulk of the fluid.

12. A body ſinks in a fluid that is ſpecifically heavier, ſo far, as that the weight of the body is equal to the weight of a quantity of the fluid of the ſame bulk as the part of the body which is immerſed in the fluid. Therefore, as the ſpecific gravity of the fluid is to that of the body, ſo is the whole magnitude of the body to the magnitude of the part immerſed. By this theorem is found the abſolute weight of any body immerſed in a fluid: for the weight of that quantity of the fluid which is diſplaced by the ſolid body is always equal to the whole weight of the ſolid; thus, if a boat on the water be ſo loaded, that it diſplace three cubical feet of water, its whole weight will be 3000 ounces or 187½ pounds, that being the weight of three cubical feet of water.

13. In equal ſolids the ſpecific gravities are as their parts immerſed in the ſame fluid.

The foregoing theorems have been ſufficiently demonſtrated by various authors, from the principles of mechanics: they are alſo exactly conformable to experience; as hath been ſufficiently proved from ſeveral courſes of philoſophical experiments.

Various tables have been given, by different authors, of the ſpecific gravities of many kinds of bodies. It will be ſufficient in this place to give the ſpecific gravities of ſome of the moſt uſeful bodies that have been determined with greater certainty and accuracy. The numbers in this table expreſs the number of avoirdupois ounces contained in a cubical foot of each body; that of common water being juſt 1000 ounces, or 62½ pounds.

Table of ſpecific Gravities.

1. *Solids.*

Platina, pure	23000	Crude antimony	4000
Fine gold	19640	Diamond	3517
Standard gold	18888	Granite	3530
Lead	11325	White lead	3160
Fine ſilver	11091	Iſland cryſtal	2720
Standard ſilver	10535	Marble	2705
Copper	9000	Pebble-ſtone	2700
Copper halfpence	8915	Rock cryſtal	2650
Gun-metal	8784	Pearl	2630
Fine braſs	8350	Green glaſs	2600
Caſt braſs	8000	Flint	2570
Steel	7850	Onyx ſtone	2510
Iron	7645	Common-ſtone	2500
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Treacle	1290	Proof ſpirits	931
Aqua regia	1234	Olive oil	913
Human blood	1054	Pure ſpirits of wine	866
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Again, what is the weight of a block of granite, the length whereof is 63 feet, and the breadth and thickneſs each 12 feet, being the dimenſions of one of the ſtones of granite in the walls of Balbeck? Here the ſolid content of this ſtone is 9072 feet; therefore, as 1 is to 9072, ſo is 3500 ounces to 31,752,000 ounces, or 885 tons 18 cwt. 3 qrs. the weight of the ſtone.

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To apply this inſtrument to uſe, let the weight L be taken away, and a wire *p n* be ſuſpended from the hook *s*; and let the bubble R be taken away from the other ſcale, and a weight ſuſpended in the room thereof. Suppoſe the weight to be ſufficient to keep the parts belonging to the other ſcale in equilibrium; and that the middle point of the wire *p n* is at the ſurface of the water in the veſſel N. And, *Note*, the wire *p n* is to be of ſuch a ſize, that the length of one inch ſhall weigh four grains.

It is evident, that as braſs is eight times heavier than water, for every inch the wire ſinks in the water, it will become an eighth part lighter, which is half a grain; and heavier in the ſame proportion, for every inch it riſes out of the water: thus, by ſinking it two inches below the middle point, or raiſing it as much above it, the wire will become one grain lighter or heavier. Therefore, when the middle point of the wire is at the ſurface of the water, if the balance be in equilibrium, and the index T ſet to the middle point *a* of the graduated wire *r s*, and the diſtance on each ſide *a r* and *a s* contain an hundred equal parts; then, if, in weighing bodies, the weight is required to the hundredth part of a grain, it may be eaſily found by the following method: let the body to be weighed be placed in the ſcale *d*, put the weight *x* in the ſcale *e*, and let this be ſo adjuſted, as one grain more ſhall be too much, and one grain leſs too little. Then, by moving the balance gently up or down by the ſcrew P, till the equilibrium be exactly ſhown at O; if the index T be at the middle point *a* of the wire *r s*, it ſhows that the weight put in the ſcale *e* is juſt equal to the weight of the body.

The foregoing method diſcovers the abſolute weight of the body; but to find the relative or ſpecific weight, it muſt be weighed hydroſtatically in water, as follows:—Inſtead of putting the body to be weighed into the ſcale *d* as before, let it hang by a horſe-hair, as the weight R, ſuppoſing the veſſel of water O were removed. Then the equilibrium being made, the index T ſtanding between *a* and *r* ſhows the weight of the body. As it hangs thus, let it be immerſed in the water of the veſſel O, when it will become much lighter, the ſcale *e* deſcending till the beam of the balance reſt on the ſupport *x*. Then, if 100 grains put in the ſcale *d* will exactly reſtore the equilibrium, ſo that the index T ſtands at the 36th diviſion above *a*, it is evident that the weight of an equal bulk of water would be exactly 100 grains.

In the ſame manner this balance may be applied to find the ſpecific gravities of fluids.

The Hydrometer.

The hydrometer diſcovers the ſpecific gravity of fluids only; for which purpoſe it is the moſt accurate, eaſy, and expeditious inſtrument of any.

This inſtrument conſiſts of a copper ball D (*fig.* 2), to which is ſoldered a braſs wire A B, a quarter of an inch in diameter. The upper part of this wire is filed flat, and marked *Proof* at *m* (*fig.* 3), becauſe it ſinks exactly to that mark in proof ſpirits. There are two other marks at A and B (*fig.* 2), to ſhow whether the liquor be one tenth above or below proof, according as the hydrometer ſinks to A or riſes to B, when a braſs weight, C or K, is ſcrewed to its bottom *c*. There are alſo other weights to ſcrew on, which ſhow the ſpecific gravity of ſeveral different fluids as low as common water.

The round part of the wire above the ball ſhould be marked ſo as to repreſent river water, when it ſinks to R W (*fig.* 3), the weight which anſwers to that water being then ſcrewed on. When it is put into ſpring water, mineral water, ſea water, and water of ſalt ſprings, it will riſe to the marks S P, M I, S E, and S A, reſpectively. On the contrary, when it is put into Briſtol water, rain water, port wine, and mountain wine, it will reſpectively ſink to the marks *b r*, *r a*, *p o*, and *m o*. Inſtruments of this kind are ſometimes called Areometers.—And, *Note*, that the globe D (*fig.* 2) ſhould be made of copper; for ivory imbibes ſpirituous liquors, and conſequently alters their gravity; and glaſs globes require too much attention.

This hydrometer was the invention of Mr. Clarke, and anſwers very well to diſcover the ſpecific gravity of ſpirituous liquors, and to ſhow whether any ſpirit be above or below proof, and how much.

But the moſt perfect hydrometer is that repreſented in figure 4, which may be made to ſhow the ſpecific gravity of fluids to the greateſt degree of exactneſs. It conſiſts of a large hollow ball B, with a ſmaller ball *b*, ſcrewed to its bottom, and partly filled with mercury or ſmall ſhot, in order to render it but little ſpecifically lighter than water. In the larger ball at C, there is a ſhort nick, into which is ſcrewed the graduated braſs wire A C, which has a ſmall weight A at the top, to cauſe the inſtrument to deſcend in the fluid.

When this inſtrument is immerſed in any fluid contained in a jar L M, the quantity of the fluid diſplaced by it will be equal in bulk to that part of the inſtrument which is under water, and equal in weight to the whole inſtrument. Therefore, if the weight of the whole inſtrument be 4000 grains, we can by theſe means compare the different dimenſions of 4000 grains weight of ſeveral ſorts of fluids; for if the weight A be ſufficient to ſink the inſtrument in rain water, till the middle point of the ſtem, marked 20, come to the ſurface of the water; and after that, if it be immerſed in common ſpring water, and the ſurface of the water ſtand at one tenth of an inch below the middle point 20, it is evident that the ſame weight of each water differs only in bulk by the magnitude of one tenth of an inch in the ſtem of the inſtrument.

Then, ſuppoſe the ſtem of the inſtrument to be ten inches in length, and to weigh juſt 100 grains; every tenth of an inch will weigh juſt one grain; and as the ſtem is of braſs, which is about eight times heavier than water, the ſame bulk of water will be equal to one eighth of a grain, and conſequently to one eighth of one four-thouſandth part, that is, to one thirty-two-thouſandth part of the whole bulk.

This inſtrument ſhould have two ſtems, either of which may be ſcrewed on, in a ſmall hole *a*. One of theſe ſtems ſhould be a ſmooth, thin, flat ſlip of braſs or rather of ſteel, having on one ſide the ſeveral marks or diviſions, to which it will ſink in different ſorts of water; as, rain water, river water, ſpring water, ſea water, &c. and on the other ſide, the diviſions to which it ſinks in various lighter fluids, as hot Bath water, Briſtol water, Lincomb water, Cheltenham water, port wine, mountain, madeira, &c. But in this caſe the weight A muſt be a little leſs than what is uſed for heavier waters. The other ſtem ſhould be cylindrical, to render it ſtronger and more ſteady, in order to try the ſtrength of ſpirituous liquors; and ſhould be ſo contrived, that when the inſtrument is immerſed, in what is called proof ſpirit, the ſurface of the ſpirit may be upon the middle point, marked 20. This is performed by adjuſting the ſmall weight A, and making the ſtem of ſuch a length, that when the inſtrument is immerſed in water, the ſurface of the water may juſt cover the ball B, and riſe to *a*; and when immerſed in pure ſpirit, it may riſe to the top A. Then, by dividing the upper part of the ſtem A 20, and the lower part *a* 20, into ten equal parts each, when the inſtrument is immerſed in ſpirituous liquor, it will immediately ſhow how much it is above or below proof, by the ſurface of the ſpirit riſing in proportion above or below the middle point 20, which marks proof ſpirits, or that ſpirit which conſiſts of half water and half pure ſpirit.

This is the moſt certain, and at the ſame time the moſt eaſy method of diſcovering the ſtrength of ſpirituous liquors; for it infallibly ſhows the difference of the bulks, and conſequently, the ſpecific gravities in equal weights of ſpirits, and that even to the thirty, forty, or fifty-thouſandth part: and is infinitely ſuperior to the common method uſed by exciſe officers and others, of ſhaking the ſpirits in a phial, and forming a judgment of the ſtrength by the breaking of the bubbles.

Of the Syphon.

The ſyphon is a bent tube uſed to decant fluids from any ſtanding veſſel; and ſerves to perform ſome curious experiments. It depends upon the preſſure of the air; and may be made in various forms.

If a ſmall ſyphon, whoſe legs are of equal length, be filled with water, and the ends turned downward, the water will remain ſuſpended in the ſyphon, as long as it is held exactly level; but when it is the leaſt inclined to either leg, whereby in effect one leg is made ſhorter than the other, the water will run out by the longer leg; for the air being a fluid, whoſe denſity near the ſurface of the earth is to that of water as 1 to 850; and according to the nature of all fluid bodies preſſing the ſurface of all things expoſed to it every way equally, it muſt neceſſarily follow, that the weight of the atmoſphere above, being kept off by the machine, and the air below bearing againſt and repreſſing the water, which endeavours to fall out of either leg with equal force, keeps it in ſuſpenſe, and prevents its falling. But if the ſyphon be the leaſt inclined to either leg, one of the legs is in effect ſhortened, and the other prolonged; by which advantage is given to the weightier fluid in the longer leg to preponderate, or overweigh the other part of the fluid in the other leg; and the water will all deſcend by the longer leg.

The leaſt inclination of the ſyphon to either end will be ſufficient to produce this effect, which may be proved by experiment, thus:—Hang a ſmall ſyphon, whoſe legs are of equal length, upon the edge of a jar filled with water; then from the ſloping of the jar, the external leg of the ſyphon will ſomewhat incline, and the ſyphon will ſoon begin to act, and the water will deſcend from the jar, through it. But in practice, one leg of the ſyphon is uſually made longer than the other leg; and the ſhorter leg is put into the liquor, when the fluid will be decanted by the longer leg.

If the two legs of the ſyphon (*fig.* 5) were of equal length, terminating in the plane A B, and the ſyphon held exactly level, and then filled with liquor, no motion of the fluid would follow, till an advantage in point of gravity be given to one ſide, by inclining it. But inſtead of which inclination, one of the legs is lengthened in general, perhaps a few inches, as from B to C; and which, previous to the operation, is generally filled, as well as the reſt of the ſyphon, with ſome fluid, many degrees heavier than air: by the gravity of which, the oppoſite ſide becomes greatly overbalanced, and the liquor in this machine is decanted very rapidly.

The ſyphon is ſometimes diſguiſed in a cup, when no liquor will flow through it, till the fluid be raiſed therein to a certain height; and when it has once begun to flow, it will continue till the veſſel be emptied; this is called a ſyphon diſguiſed. Thus, D D (*fig.* 6) is a cup, in the centre whereof is fixed a glaſs pipe or ſyphon C B, continued through the bottom at B; over this pipe is put a glaſs tube, made air-tight at top, by the cork C, but left ſo open at the bottom, by holes made about D D, that the water may freely riſe between the two tubes, as the cup is filled. When any fluid is poured into this cup, no motion will take place through the ſyphon, till the fluid in the cup ſhall have gained the top of the innermoſt pipe at C; but when the fluid is arrived to this height, it will begin to flow through the ſyphon, which runs through the bottom of the cup, and will continue to riſe up the inſide of the outer tube, and deſcend through the inner tube, till the whole fluid in the cup be run off; which is owing to the fluid at its firſt riſing through the tubes, expelling all the air from them, while the weight of the atmoſphere preſſes on the ſurface of the fluid in the cup.

This is ſometimes called Tantalus's cup, and has a hollow figure, repreſenting Tantalus, placed over the inner tube, of ſuch a length, that when the fluid is got nearly up to the mouth of the figure, the ſyphon begins to act, and empty the cup.

This is the ſame in effect as if the two legs of the ſyphon were both in the veſſel (*fig.* 7), when the water poured into the veſſel will riſe in the ſhorter leg of the ſyphon, to its own level; but when it has gained the bend of the ſyphon, it will begin to run off by the longer leg, and continue running till the veſſel be emptied as low as the extremity of the ſhorter leg of the ſyphon.

The Hydroſtatical Paradox.

Any quantity of fluid, however ſmall, may be made to counterpoiſe, and ſuſtain any weight, how large ſoever.—This is called the hydroſtatical paradox, and depends upon the equal preſſure of the parts of fluids every where at the ſame depth.

Let A B D G (*fig.* 8) repreſent a cylindrical veſſel, to the inſide of which is fitted a cover, which, by means of leather round the edge, will eaſily ſlide up and down in the veſſel, without permitting any water to paſs between its edge and the ſurface of the veſſel. In the cover is fixed a ſmall tube E F open at the top, and extending through the cover at the bottom. Then, the veſſel being filled with water, and the cover put on, and loaded with a weight, ſuppoſe of a pound, it will be depreſſed, and the water will riſe in the tube to E, and the weight will be ſuſtained. If another pound be laid on the cover, the water will riſe to F, and the weight alſo be ſuſtained: and thus the water will riſe higher in the tube in proportion to the weight that is laid on the cover. And though the weight of the water in the tube be but a few

And if he put his finger on the top of the pipe, he can ſupport himſelf up as long as he pleaſes, provided the bellows be air-tight.

This property of the upward preſſure of fluids furniſhes us with the means of rendering any body, how heavy ſoever, buoyant in water; and upon this principle, a piece of lead may be made to ſwim in the water, by immerſing it to a proper depth, and preventing the water from getting above it. Thus, let C D (*fig.* 11) be a glaſs tube open at each end, and E G a flat piece of lead, placed under the lower end of the tube, having a piece of wet leather between it and the tube, to make it keep cloſe to the tube. Let the piece of lead be about half an inch thick, and in breadth equal to the diameter of the tube; let it be held cloſe to the tube, by pulling the packthread H L upwards at L; and let the tube be immerſed in water, in the glaſs veſſel A B, to the depth of ſix inches below the ſurface of the water at K; then the leaden bottom E G will be plunged to the depth of upwards of eleven times its own thickneſs; at which depth ſetting looſe the thread at L, the lead will not fall from the tube, but be forced upwards againſt the tube, by the upward preſſure of the water below it, occaſioned by the height of the water at K, above the level of the lead; for as lead is 11.33 times as heavy as its bulk of water, and is in this experiment immerſed to above 11.33 times its thickneſs, in a depth of water, and no water getting into the tube between it and the lead, the column of water E a b c G below the lead is preſſed upwards againſt the lead, by the water K D E G I all around the tube; which water being a little more than 11.33 times as high as the lead is thick, it is ſufficient to balance and ſupport the lead, at the depth K E. If there be a little water poured into the tube, upon the lead, it will increaſe the weight upon the column of water which is under the lead, and cauſe the lead to fall from the tube to the bottom of the glaſs veſſel, as at *a b*. But if the tube be raiſed a little in the water, the lead will fall by its own weight, which will then be too great for the preſſure of the water round the tube upon the column of water below it.

Again, a piece of wood, however light, may be made to lie at the bottom of the water, by not ſuffering any water to get under it. Thus, having two pieces of wood, planed quite flat and ſmooth, ſo that no water may get between them, when they are put together; and cementing one of the pieces, as *a b*, to the bottom of the veſſel A B, place the other piece upon it, and let it be held down by a ſtick, while the water is poured into the veſſel; then, upon removing the ſtick, the upper piece of wood will not riſe from the lower one, being preſſed down both by its own weight, and the weight of all the water above it, while the contrary preſſure of the water upwards is kept off by the wood placed beneath it; but if the top piece of wood be raiſed ever ſo little at any part of its edge, ſome water will get under it, which will be forced by all the weight of the water above, and will immediately preſs it upwards; and being lighter than its own bulk of water, it will float upon the ſurface of the water.

To prove that all fluids weigh juſt as much in their own elements as they do in open air, put as much ſhot in a phial as, when corked, will make it ſink in water; then let it be weighed, both in the air and in the water, and the weight in each caſe wrote down; then, as the phial hangs ſuſpended in water, and counterpoiſed by another weight, pull out the cork, that the water may run into it, when it will deſcend and pull down that end of the beam. Next, put as much weight into the oppoſite ſcale as will reſtore the equilibrium; which additional weight will be found to anſwer exactly to the additional weight of the phial, when it is again weighed in the air with the water in it.

The velocity with which water ſpouts out of a hole, or through a tube in the ſide or bottom of a veſſel, is as the ſquare root of the depth or diſtance of the hole below the ſurface of the water. Therefore, in order to make double the quantity of water run through one hole, as through another of the ſame ſize, it will require four times the preſſure of water as the other hole hath; and conſequently, it muſt be four times the depth of the other below the ſurface of the water; and for the ſame reaſon, three times the quantity of a fluid running in the ſame time through a hole of the ſame ſize, muſt run with three times the velocity; and conſequently muſt be nine times as deep below the ſurface of the fluid. Thus, let C and *g* (*fig.* 12) be two pipes of equal diameters, fixed in the ſide of the veſſel A B, the pipe *g* being four times as deep below the ſurface of the water at *b*, as the pipe C is: while the water runs through theſe pipes, let more water be conſtantly poured into the veſſel, to keep the ſurface ſtill at the ſame height. Then, in the ſame time that a pint of water flows through the pipe C, through the pipe *g* will flow one quart.

The horizontal diſtance to which a fluid will ſpout from an horizontal pipe, fixed in any part of the ſide of an upright veſſel, below the ſurface of the fluid, is equal to twice the length of a perpendicular to the ſide of the veſſel, drawn from the mouth of the pipe to a ſemicircle, deſcribed upon the altitude of the fluid; and therefore the fluid will ſpout to the greateſt diſtance poſſible from a pipe at the centre of the ſemicircle; becauſe a perpendicular to its diameter (which is ſuppoſed parallel to the ſide of the veſſel) drawn from that point, is the longeſt that can poſſibly be drawn from any part of the diameter to the circumference of the ſemicircle. Thus, if the veſſel A B be full of water, the horizontal pipe D, in the middle of its ſide, and the ſemicircle N *d c b* be deſcribed upon D, as the centre with the radius D *d*. The perpendicular D *d*, to the diameter N *b*, is the longeſt that can be drawn from any part of the diameter to the circumference. And if the veſſel be kept full, the jet C will ſpout from the pipe D, to the horizontal diſtance N M, which is double the length of the perpendicular D *d*. If two other pipes, C and E, be alſo fixed in the ſide of the veſſel, at equal diſtances above and below the pipe D, the perpendicular C *c* and E *e*, from theſe pipes to the ſemicircle will be equal, and the jets F and H, which ſpout from them, will each go to the ſame horizontal diſtance N K; which is double the length of either the equal perpendiculars C *c* and E *e*.

CHAP. XIX.

HYDRAULICS.

SECT. I.

OF PUMP-WORK.

HYDRAULICS is that part of the doctrine of fluids which treats of the properties of fluids in motion, with a ſpecial attention to artificial water-works: and in this ſenſe it ſtands oppoſed to hydroſtatics, which concern fluids, as they remain at reſt.

The laws of fluid bodies, as given in the laſt chapter, obtain alſo in this, and therefore need not be repeated.

Of the pump, there are ſimply three kinds, viz. the ſucking, the forcing, and the lifting pump. By the former, the water is raiſed by the general preſſure of the atmoſphere on the ſurface of the water in the well, and cannot be raiſed to a greater height than 33 feet, as before obſerved; though, in practice, it is ſeldom raiſed above 28 feet, becauſe the air is not always denſe enough to ſupport a column of water of 33 feet. By the two latter, water may be raiſed to any height, having an adequate apparatus, and ſufficient power.

Of the Sucking Pump.

The ſucking pump is that in moſt common uſe, and conſiſts of a tube or pipe, open at each end; having within a ſliding piſton, as large as the bore of the pipe, and which fits the pipe ſo exactly, as to admit no air to paſs between it and the pipe. The pipe is called the barrel.

If the lower end of the barrel B be immerſed in water (*fig.* 1, *plate* 24), and the piſton D be raiſed, a vacuum will be made in the barrel, by lifting up the column of upper air from A to D, and thereby permitting the air in the lower part of the barrel to expand itſelf; and the atmoſphere preſſing upon the ſurface of the water in the well, will force it to follow the piſton, and that even to the height of 33 feet, if the ſtroke could be of that continued length. But when the piſton is let down again in the barrel, the water will fall with it; to prevent which, there is a valve fixed in ſome convenient part of the barrel, as at C, which valve conſiſts of a wooden frame A (*fig.* 2), exactly fitted to the bore of the barrel, and a leather flap B, lined with lead, in order to give it ſufficient weight and ſtrength. This valve opening with the upward motion of the water, and again cloſing when the piſton is let down, ſerves to retain the water above, which flows through it:

and at every riſe of the piſton, a freſh quantity of water flows through this valve.

Beſides this fixed valve, there is a moveable one placed in the piſton at D (*fig.* 1), which alſo opens the ſame way, and is called the bucket.

When the bucket deſcends, if the bore of the barrel be full of water, the reſiſtance of the water will open the bucket, and part of the water will riſe above it; and when the piſton is drawn upwards, the bucket will again cloſe under the incumbent weight of water, and the water will be raiſed by the force applied. So that whenever the bucket riſes, and lifts up the column of both air and water, which paſſes through it, the fixed valve C is diſcharged of its preſſure; and then a freſh quantity of water, exactly equal to that lifted up by the bucket, will, by the ordinary preſſure of the atmoſphere on the water in the well, be forced up through the valve C, to again ſupply the barrel. This alternate motion of the two valves may be ſeen to great advantage in the glaſs pumps.

But if there be no water in the barrel, before any water can be drawn from the well, the air in the barrel muſt be exhauſted, which may be done, if the piſton valve be tight, by the ordinary motion: but it is common to pour ſome water down the barrel, which is vulgarly called fetching the water, but which is of no other uſe than to ſupple the leather of the valves, and render them air-tight.

The firſt time the piſton is raiſed in the barrel, called the firſt ſtroke of the pump, it will make a vacuum in the barrel, and a part of the incumbent air is lifted away, upon which the air remaining in the barrel, from its natural ſpring, will become conſiderably dilated; when the atmoſphere preſſing with a greater force on the ſurface of the well water, than the dilated air does on the water in the barrel, it will cauſe the water in the barrel to riſe therein, ſo far as, together with the included air, ſhall juſt counterpoiſe the weight of the atmoſphere upon the outward ſurface of the water. A ſimilar

A ſimilar effect will be produced at the repetition of the ſtroke, till by degrees the water ſhall have reached the moving valve or bucket, and then the proceſs will go on as before deſcribed. Thus, water, even by this machine, may be raiſed to any height whatever, provided the power be adequate to the weight, and the pipe ſtrong enough to bear the fluid's natural preſſure.

The proportion of the preſſure of the water on the pipes in pump-work is according to the height of the water above the part conſidered: but the incumbent weight on the bucket of a pump, in action, is nearly proportional to that of a column of water raiſed; for though the weight of the atmoſphere on the ſurface of the water, when the bucket riſes, be really equal to the weight of 33 feet of water; yet this weight is exactly counterbalanced by the weight of the atmoſphere, ever incumbent on the ſurface of the water thereby raiſed. Thus, all the advantage to be obtained by the hydraulic machines, is ranging matters into a convenient method of being performed; the performance itſelf depending entirely upon the moving power, with all the diſadvantages of friction.

In this pump, if both the valves be placed towards the bottom of the pipe, the pump will work as eaſy, and require no greater power than if they were fixed 30 feet, or 33 feet, above the ſurface of the water.

It is generally found to be more advantageous in practice, to place both the valve and bucket pretty low in the barrel; for ſhould a leak happen beneath the bucket, which is often the caſe, in a great length of pipe, the air getting through, would render the pipe uſeleſs; whereas, ſhould a leak happen above the bucket, it will occaſion only the loſs of ſome of the water. And by placing the valves under water, they will always be found more ſupple and pliant, and, conſequently, be in a better condition for performing their offices.

There is another advantage of placing the pump-work (that is, the valve and piſton) near together, and at the bottom of the barrel: which is, it will in all caſes raiſe the water from the well by the ordinary pump, when another pump of an equal bore, but having its bucket higher, will not be able to raiſe any water, by reaſon of the ſhortneſs of the ſtroke, which cannot rarefy the air ſufficiently to bring the water up from the lower valve to the piſton. For example: having a ſmooth barrelled pump, 21 feet long, with its piſton placed above, fetching (ſuppoſe) a foot ſtroke, and the fixed valve at the other end of the barrel below. Here, by the playing of the piſton, ſuppoſing it poſſible for the water to riſe 11 feet (or, which is the ſame thing, let water be poured down the barrel to the height of 11 feet above the lower valve), there will ſtill remain nine feet of air between the bucket and the water, which cannot be ſufficiently rarefied by the ſtrokes of this bucket, to open the lower valve, or fetch up more water: for in this caſe, the air will be rarefied in the proportion of 9 to 10 only; whereas it ought to be as 9 to $13\frac{1}{2}$, to make only a bare equilibrium with the atmoſphere: for as 22 (which is the complement of 11 to 33 feet of water, the weight of the whole atmoſphere) is to 33 feet, or the weight of the atmoſphere, ſo is 9 to $13\frac{1}{2}$, to complete which, the ſtroke ſhould be at leaſt $4\frac{1}{2}$ feet long. However, in a pump of this conſtruction, by filling the whole barrel with water up to the bucket, a conſtant ſupply of water may be produced.

When the bucket is placed low, as here directed, inſtead of an iron rod, to which it is fixed, a piece of oak well ſoaked will work with leſs trouble, as it is nearly of the ſame ſpecific gravity with water.

When the pump cannot be placed perpendicularly over the well, the bottom of the barrel of the pump ſhould be placed as low as the ſurface of the water in the well is, and communicate with it by means of a pipe: thus, if water were to be raiſed from the well A (*fig.* 3), by means of the pump B, the barrel of the pump B communicating with the well by means of the pipe C, and the bucket playing in the barrel B C, the water will riſe as if the well had been perpendicular to the pump; becauſe the water in the well being forced by the natural preſſure of the atmoſphere, will repleniſh the barrel B, through the pipe A C.

But when it happens that the barrel of the pump cannot go down directly to the well, as in the laſt caſe, the water may be led about any other way by means of a pipe E, and thus be conveyed to the pump D. And by making this pipe of conveyance E leſs in diameter than the barrel of the pump, it will ſooner be exhauſted of air, by moving the piſton, and, conſequently, the water will ſooner follow.

But it will always be found more eaſy, in practice, to have the pipe of conveyance large, and of an equal bore throughout; becauſe the water will then have a velocity in it, and the friction will be leſs. This is the reaſon why the common pumps, made by the plumbers, do not work ſo eaſily as thoſe which are bored out of trees; for, by making the pipe ſo much leſs than the bucket, they, as it were, wire-draw the water. Therefore, in pumps that have a long pipe of conveyance, the diameter of the ſucking-pipe ſhould be nearly equal to half the diameter of the barrel. For, if the barrel be four inches in diameter, and the pipe of conveyance only one inch, the water will move 16 times as faſt through the pipe as it will through the barrel; which, conſequently, requires more labour, and is attended with a greater wear and friction of the machine.

It is alſo a great fault to bore a pump conically upwards, becauſe the water cannot with freedom run off ſo faſt, as a vacuum may be made by the moving piſton; and the reflection of the water from the ſides, will always be an hindrance in the operation.

branching or forcing pipe at E. Theſe valves ſhould alſo be air-tight, and ſo diſpoſed as to let the water freely riſe, but to prevent its returning back. The forcer C is leathered upwards, that it may withſtand the preſſure of the atmoſphere from above, and that by ſucking, when raiſed, it may bring up the water, to ſupply the barrel: and it is alſo leathered downwards, that, when repreſſed, it may reſiſt the weight of the water to be forced up.

When the forcer C is moved upwards in the barrel, it lifts up the incumbent air; and the air between that and the water having room to dilate itſelf, will be rarefied, and the water will riſe from the ſpring in the barrel A B, as in the ſucking pump: and continuing the motion of the forcer, the water will at length riſe up to the forcer, and fill the internal cavity of the pipes, between the two fixed valves D and E. And the water being prevented from deſcending again by the lower valve, will, by the forcer, be preſſed, and make its way through the upper valve E. When the forcer riſes, this preſſure will be intermitted, and the valve at E will immediately cloſe under the weight of the upper water, and thus prevent its return, while the forcer is riſing with a freſh ſupply. The ſame is repeated at every ſtroke of the forcer.

M. de la Hire's Pump, which raiſes Water both by the Aſcent and Deſcent of the Piſton in the Pump-barrel.

B and C are two pipes (*fig.* 6), having their ends in the well of water A A. The pipe B has a valve *b* at the top, and is ſoldered into the pump-barrel D. The pipe C alſo has a valve at the top, and is ſoldered into the pipe S. The pipes E and F have each of them a valve *e* and *f* at the ends, and communicate with the pump-barrel D, and the hollow box G.

K is a ſolid plunger or forcer, exactly fitted to the bore of much water forced up the pipe N, to the ſpout-hole O, by the deſcent of the plunger, as by its aſcent; and in each caſe as much water diſcharged at the ſpout-hole as fills that part of the pump-barrel through which the plunger moves.

P is a cloſe air-veſſel, fixed on the top of the pipe N, which compreſſes the air, when the water riſes up the pipe N, above the ſpout O: and this condenſed air acting on the water, cauſes it to run off by the ſpout-hole nearly in an equal ſtream.

The pipe S at the top of the pipe C ſhould never be above 32 feet above the ſurface of the water in the well; becauſe if the pipe C be entirely exhauſted of air, the preſſure of the atmoſphere on the water in the well, would not force the water up the pipe to a greater height than 32 feet, at the moſt; but if S be within 24 feet of the water in the well, the pump will work ſo much the better.

The pipe N may be of any ſize required; but the pump-barrel ſhould be made in proportion to the height of the ſpout-hole above the ſurface of the water in the well; as follows:

For ten feet of perpendicular height of the ſpout-hole O, above the ſurface of the water in the well, the diameter of the bore of the barrel ſhould be 6.9 inches: for 15 feet high, 5.6 inches: for 20 feet, 4.9 inches: for 25 feet, 4.4 inches: for 30 feet, 4.0 inches: for 35 feet, 3.7 inches: for 40 feet, 3.5 inches: for 45 feet, 3.3 inches: for 50 feet, 3.1 inches: for 55 feet, 2.9 inches: for 60 feet, 2.8 inches: for 65 feet, 2.7 inches: for 70 feet, 2.6 inches: for 75 feet, 2.5 inches: for 80 feet, 2.5 inches: for 85 feet, 2.4 inches, for 90 feet, 2.3 inches: for 95 feet, 2.2 inches: and for 100 feet, 2.1 inch, or at the moſt 2.2.

In pumps of this kind the pipes B and C ſhould be made ſufficiently large: for when they are too ſmall, the velocity of the water through them being great, the water will have too much friction to ſuffer the pump to be worked with much advantage.

If this machine be conſtructed according to the above dimenſions, it will raiſe more water than any perſon unſkilled in hydraulics would imagine.

The force required to work a pump will always be as the height to which the water is raiſed, and as the ſquare of the diameter of the pump-bore in that part where the piſton works. Thus, if two pumps be of equal heights, and one of them be twice as wide in the bore as the other, the wider will raiſe four times as much water as the narrower one; and conſequently will require four times the power to work it.

The piſton-rod of a pump is always raiſed by means of a lever, whoſe longer arm exceeds the ſhorter one, in length, generally five, or ſix times, and the power is applied at the end of the longer arm; by which means the rod is raiſed by a fifth, or ſixth part of the power, which would be required to raiſe the rod without it.

The following table ſhows the quantity of water which a common ſucking pump will diſcharge in one minute, the pump being of any given height above the ſurface of the well, from 10 to 100 feet incluſive; and the diameter of the bore of the barrel being from 6.93 inches to 2.19 inches incluſive.

SECT. II.

OF STEAM-ENGINES:—AND A DESCRIPTION OF MR. WATT'S STEAM-ENGINE.

Of all the uſes to which ſteam has been applied, there are none where it has been uſed with greater ſucceſs than in the application of it to raiſe water from any great depth, as in that machine called the ſteam-engine, otherwiſe denominated the fire-engine, on account of the fire employed in boiling the water to produce the ſteam.

The ſteam raiſed from hot water is an elaſtic fluid like air, and has its elaſticity proportional to its denſity, when the heat is the ſame; or proportional to the heat, when the denſity is the ſame. The ſteam raiſed from boiling water of an ordinary heat is near 3000 times rarer than water, or about 3½ times rarer than air, and its elaſticity is equal to that of common air. And it has been found by experiment, that water, when converted into very hot ſteam, will occupy 14000 times the ſpace that it occupied when in water, and, conſequently, it will become five times ſtronger than the atmoſphere. And by accidents that have happened, it has been demonſtrated that water, ſuddenly turned into ſteam by the immediate application of great heat, is vaſtly ſtronger than the atmoſphere, or even gunpowder.

The ſteam-engine is the moſt uſeful machine diſcovered in modern times; and were it not for this moſt important invention, we ſhould never have been able to work the coal-mines in England to the preſent advantage; as before the laſt century, for the want of this engine to draw the water, the attempts of our anceſtors to procure coals were always ineffectual.

Before

alternately turned by a man, in order to admit the ſteam, and to refill the veſſel with cold water.

Captain Thomas Savery having read the Marquis's book, conſtructed an engine, which, after ſeveral experiments, he brought to ſome degree of perfection; upon which he bought up, and deſtroyed, all the books of the Marquis he could procure, claimed the honour of the invention to himſelf, and obtained a patent for the ſame. His engine, however, would not raiſe water to any great height, or in quantities ſufficient to anſwer the purpoſe of draining a mine. The largeſt he ever erected was for the York Buildings Company, in London, for ſupplying the inhabitants of the Strand, and that neighbourhood, with water.

Several other gentlemen, both in England and France, attempted various improvements in the conſtruction and manner of working theſe engines; but with little ſucceſs, till the year 1705, when Mr. Newcomen, an ironmonger, and Mr. John Cowley, a glazier, both of Dartmouth, made a conſiderable improvement in theſe engines, by bringing the engine to work with a beam and piſton (which had never been then introduced), and where the ſteam, even at the greateſt depth of mines, is not required to be greater than the preſſure of the atmoſphere. Theſe gentlemen obtained a patent for the ſole uſe of this invention, for fourteen years; and the firſt engine they erected was in the year 1710, at a colliery at Griff, in Warwickſhire; the cylinder of this engine being 22 inches in diameter. The next engine they erected was in the year 1718, at a colliery in the county of Durham, which was alſo improved by Mr. Henry Beighton, F. R. S. who introduced the manner of opening and ſhutting the cocks, by the hanging beam, as at preſent uſed; and likewiſe made improvements in the pipes, valves, and ſome other parts of the machine.

When theſe engines came to be better underſtood, and their utility, particularly in draining mines, became more evident, from the great number of them every where erected, they pump-rods ſink by their ſuperior weight, and the piſton, at the other end of the lever, riſes; and when that ſteam is condenſed, the piſton deſcends, and the pump-rods riſe with their quantity of water; and ſo on alternately, as long as the piſton works.

As the piſton does not deſcend with a force exceeding eight or nine pounds upon every ſquare inch of its ſurface; and as, by reaſon of accidental frictions, and alterations in the denſity of the air, it is ſometimes leſs than this, it will be ſafeſt in practice to calculate the weight at ſomething leſs than eight pounds, viz. at about ſeven pounds ten ounces for every ſquare inch, or 7.64 pounds, which is ſix pounds upon every circular inch; and, as a gallon of water of 282 cubic inches weighs $10\frac{1}{5}$ pounds, we have the dimenſions of the cylinder, pumps, &c. for any ſteam-engine, as follows:

c = the cylinder's diameter in inches.
p = the pump's diameter in inches.
f = the depth of the pit in fathoms.
g = gallons drawn by a ſtroke of ſix feet.
h = the hogſheads drawn per hour.
s = the number of ſtrokes per minute.

Then c^2 is the area of the cylinder in circular inches; therefore $6\,c^2$ is the power of the cylinder in pounds.

And $\frac{p^2 \times .7854 \times 72}{282}$, or $\frac{1}{5}p^2$ is $=g$, the gallons contained in one fathom, or ſix feet of any pump; which, multiplied by f, gives $\frac{1}{5}p^2 f$ for the gallons contained in f fathoms of any pump, whoſe diameter is p.

Hence, $\frac{1}{5}p^2 f \times 10\frac{1}{5}$ pound gives $2\,p^2 f$, nearly, for the weight in pounds of the column of water, which is to be equal to the power of the cylinder, which was before found equal to $6\,c^2$. Thus, we have the ſecond equation, viz. $6\,c^2 = 2\,p^2 f$, or $3\,c^2 = p^2 f$; the firſt equation being $\frac{1}{5}p^2 = g$, or $p^2 = 5g$.

From which two equations any particulars may be determined.

The common Steam-Engine, (Fig. 9.)

A is the boiler.

B, the cylinder.

C, the injection-cock.

D, the ſteam-cock, or regulator.

E, the ſhifting clack.

F, the eduction pipe, or ſinking pipe.

G, the eduction valve.

H, the ſafety valve.

I, the piſton.

K, the lever beam.

L, weights to counterpoiſe the piſton, and preſs down the forcer in the pump-barrel M, to drive the water through the pipe N.

O, a ciſtern to hold the injection water.

P, an air veſſel which prevents the pipe N from burſting, and ſerves to keep up a regular ſtream.

The boiler A is filled with water to the height of *d e*, which being made to boil by a fire placed beneath it, will fill the upper part A D with a very elaſtic vapour or ſteam, which, when it is of ſufficient ſtrength, will force open the valve at H. This ſteam is let into the barrel or cylinder B, by turning the cock D; and by its elaſtic force raiſes the piſton I, which drives the air above it through a proper clack, placed at the top: and the weights L, at the other end of the lever, cauſe it to deſcend, and drive the piſton down the pump-barrel M. Then, in order to make the piſton I deſcend, a little cold water is let into the cylinder, at the bottom, from the ciſtern O, by turning the cock C, which, riſing in the form of a jet, condenſes the hot ſteam in the cylinder into water, whereby it occupies about 13000 times leſs ſpace than that it took up before; which creates a partial vacuum in the barrel, and thereby permits the piſton to

212 degrees is neceſſary to produce ſteam; and the difference of heat at which water boils under different preſſures increaſes in a leſs proportion than the preſſures themſelves; ſo that a double preſſure requires leſs than a double increaſe of the heat.

There are two principal defects in the common ſteam-engine: firſt, as the vacuum in the cylinder is produced by throwing in cold water to condenſe the ſteam, the water thrown in becomes hot, and produces a ſteam from itſelf, which greatly reſiſts the motion of the piſton downwards, and thereby leſſens the power of the engine. Secondly, upon attempting to fill a cold cylinder with hot ſteam, a great part of the ſteam will be deſtroyed; and the injection water that is let in to condenſe the ſteam, not only cools the cylinder, but remains there until it be extruded at the eduction pipe by the ſteam which is afterwards let into the cylinder, which ſteam will be condenſed into water as faſt as it enters, until all the matter it comes in contact with be nearly as hot as itſelf.

The great conſumption of fuel alſo has been a material object to theſe engines; for it is well known, that a ſteam-engine of an ordinary ſize will conſume near 3000 pounds worth of coals per annum, at any part near London.

Mr. Watt's Steam-Engine.

Mr. Watt has in a great meaſure, if not wholly, remedied the foregoing inconveniences: he preſerves an uniform heat in the cylinder of his engine, by ſuffering no cold water to touch it, and by protecting it from the air or other cold bodies, by a ſurrounding caſe filled with the ſteam, or with hot air, or water; and by coating it over with ſubſtances that tranſmit the heat very ſlowly. He makes his vacuum to approach nearly to that of the barometer, by condenſing the ſteam in a ſeparate veſſel, called the condenſer; which may be cooled at pleaſure, without cooling the cylinder, either by an injection of cold water, or by ſurrounding the condenſer with it, and generally by both. The injection water, and detached air, he extracts from the cylinder, or condenſer, by pumps, which are wrought by the engine itſelf; or he blows it out by the ſteam. As the entrance of air into the cylinder would ſtop the operation of the engine, and as it is hardly poſſible that ſuch enormous piſtons as thoſe of ſteam-engines can move up and down, and yet be perfectly air-tight; in the common engines a ſtream of water is always kept running on the piſton, which prevents the entry of the air; but this mode of ſecuring the piſton would be highly prejudicial in the new engine, though it be not hurtful in the common ones. The piſton of the new engine is therefore made more accurately; and the outward cylinder having a lid, which covers it, the ſteam is produced above the piſton; and when the vacuum is introduced under it, the ſteam acts upon it by its elaſticity, as the atmoſphere does on the piſton of the common engines by its gravity. This mode of working effectually excludes the air from the inner cylinder, and gives the advantage of increaſing the power by increaſing the elaſticity of the ſteam, as ſeen in the following deſcription:

A (*fig.* 10) is the boiler.

B, the ſafety valve.

C, the pipe which conveys the ſteam to the outer cylinder.

D, the outer cylinder.

E, the inner cylinder.

F, the piſton.

G, the valve that admits the ſteam from the outer cylinder into the inner cylinder, called the ſteam valve.

H, the valve that admits the ſteam from the inner cylinder into the condenſer, called the condenſing valve.

I, the condenſer.

K, the injection valve that admits a jet of cold water into the condenſer, to condenſe the ſteam.

L, the air-pump that exhauſts the condenſer both of air and the injection water that is let in at every ſtroke, and is fixed under water in the condenſing back M, which is full of water.

N, the lever beam.

O, the great water pump for clearing the mine, or raiſing water for any other uſe through the pipes, &c.

This new engine differs from the common ones only in the foregoing particulars: having the cylinder, the great beam, the pumps, &c. in their uſual poſitions.

The cylinder in this new engine is ſmaller than uſual in proportion to the load, and is very accurately bored; and is ſurrounded at a ſmall diſtance with another cylinder, furniſhed with a bottom and lid. The ſpace between the cylinders communicates with the boiler by a large pipe C, open at both ends, ſo that it is always filled with ſteam, and thereby preſerves the inner cylinder of the ſame degree of heat with the ſteam, and prevents the ſteam from condenſing within it, which would be more prejudicial than an equal condenſation in the outer cylinder.

The inner cylinder has a bottom and piſton as uſual; and as it does not reach up quite to the lid of the outer cylinder, the ſteam in the ſpace between them has always a free acceſs to the upper ſide of the piſton. The lid of the outer cylinder has a hole in the middle, through which the piſton-rod moves up and down; this hole is kept tight by a collar of oakum ſcrewed upon it.

There are two regulating valves at the bottom of the inner cylinder, one of which admits the ſteam to paſs from the ſpace between the two cylinders into the inner cylinder, below the piſton, and ſhuts it out at pleaſure; the other opens or ſhuts the end of a pipe that leads to the condenſer. The condenſer conſiſts of one or more pumps furniſhed with clacks and buckets (nearly the ſame as in common pumps), which are wrought by chains faſtened to the great working beam of the engine. To the bottom of theſe pumps

3 is

the piſton, which is therefore forced down by the full power of the ſteam from the boiler, which is ſomewhat greater than the preſſure of the atmoſphere.

In the common engines, when they are loaded to about ſeven pounds upon each ſquare inch of the piſton, and are of a middle ſize, the quantity of ſteam which is condenſed, in reſtoring to the cylinder the heat which it had been deprived of, by the former injection of cold water, is about one full of the cylinder, beſides what is really required to fill that veſſel; ſo that twice the full of the cylinder is employed to make it raiſe a column of water equal to ſeven pounds for each ſquare inch: or more ſimply, a cubic foot of ſteam raiſes a cubic foot of water about eight feet higher, beſides overcoming the friction of the engine, and the reſiſtance of the water to motion.

But in the improved engine of Mr. Watt, about one full and a fourth of the cylinder is required to fill it, becauſe the ſteam is one fourth more denſe than in the common engine. This engine, therefore, raiſes a load equal to 12¼ pounds upon each ſquare inch of the piſton; and each cubic foot of ſteam, of the denſity of the atmoſphere, raiſes one cubic foot of water 22 feet high.

Theſe engines work more regular and ſteady than the common ones; and the ſavings amount at leaſt to two thirds of the fuel; which is an important object where coals are dear. The new engines alſo will raiſe from 20000 to 24000 cubic feet of water, to the height of 24 feet, by only one hundred weight of good pit coals.

any falſe ſtrokes of the charcoal. The black and red lead pencils are uſed to draw out the draught the ſecond time, becauſe the lines drawn with theſe will not be liable to be rubbed out with the hand, when the lines are again drawn with the pen. The pens made of crow-quills (though another good pen may anſwer the purpoſe) are to finiſh the work. Rulers are to draw the ſtraight lines, triangles, ſquares, &c. which are to be done at firſt, till practice render them needleſs. The compaſſes ſhould have ſteel points, which will take out, in order to uſe a black or red lead pencil; their uſe is to draw circles, ovals, arches, &c.; alſo to meaſure, by the help of a ſcale of equal parts, the proportions of the figures.

The Precepts of Drawing in general.

There are no arts that depend leſs upon theory than thoſe of drawing and painting; in theſe it is principally practice and experience that can render any one a good artiſt. But here, as in every other art, a few rules may be of ſervice to the inexperienced ſtudent; and in attending to the following rules, the young artiſt ſhould be careful in following the outlines of the figure, which is the firſt proceſs. He muſt alſo content himſelf with copying parts of objects, before he aims at any finiſhed piece, and dwell upon each part; and never begin a ſecond till he thoroughly underſtands the proportions of all the outlines of the firſt. He muſt alſo be very ſlow in his firſt operations; and he cannot too often contemplate the length, breadth, and every other proportion of each object of his original. For this purpoſe he ſhould have it conſtantly in his eye, and cannot look at it too frequently.

1. The firſt part of drawing conſiſts of plain geometrical figures; as lines, angles, triangles, quadrangles, polygons, cones, and the like; for theſe are the foundation of the outlines of all other figures. The circle aſſiſts in all orbicular forms; as the ſun, moon, fruits, &c.: the oval, in giving a juſt proportion to the human face and mouth, the

mouth of a pot or well, any cylindrical body, the foot of a glaſs, &c. The ſquare confines the picture to be copied; and ſerves to draw a ſcale, to enlarge or contract the piece, &c. The triangle is uſed in the ſide face; the polygon, in ground plans, fortifications, &c. angles and arches in perſpective; the cone, in ſpires, ſteeples, &c.; the cylinder in columns, pillars, pilaſters, and their ornaments.

2. The next part of drawing conſiſts in the forming of fruits, which generally depart a little from the circular form; as apples, pears, cherries, grapes, &c.: the imitation of flowers, as roſes, tulips, &c.; herbs, as thyme, mint, &c.; trees, as oak, fir, &c.

3. The third part of the practice is the imitation of beaſts, fowls, and fiſhes, of which variety of prints may be purchaſed at reaſonable rates.

4. The fourth part conſiſts in imitating the human body, with all its lineaments; as the head, noſe, eyes, ears, cheeks, hands, arms, &c. which muſt be all exactly proportioned, both to the whole, and to one another.

5. The fifth part of the practice conſiſts in drapery, or imitating the clothing, and ſetting off in an artificial manner the outward coverings of the body; as cloth, ſilk, ſtuff, linen, &c. which ſhould have their natural and proper folds.

In every form it is neceſſary, firſt, to obſerve the greateſt exactneſs in the proportions of every figure; and alſo in the different parts of each figure. Secondly, the general outline of the whole work ſhould be accurately drawn, before the piece be ſhadowed, or finiſhed within.

When the artiſt has proceeded ſo far in the art as to attempt figures of a more uncertain form, where the circle and ſquare cannot be uſed; as in lions, horſes, &c. he muſt work by his own judgment, and be directed by a conſtant attention to his original, and ſo obtain the true proportions by daily practice.

The piece which is to be drawn, is to be firſt rudely ſketched with the charcoal; ſecondly, the lines are to be more exactly drawn with the lead pencil, rubbing out any falſe or imperfect ſtrokes of the charcoal; then having peruſed it well, mend it with the pencil, where there are any errors: this done, peruſe it again, correcting by degrees all the errors of leſs magnitude, even to the leaſt jot; then compare it with the original copy, uſing neither the rule nor the compaſs, but giving every part its due place and proportion according to judgment.

When the artiſt has arrived to ſome perfection in the art, he may begin to copy after life, for this is the moſt correct and complete method of drawing or painting; and here only he has the largeſt liberty of imitation. But there ought to be ſome perfection in the art before he aims at this.

Particular Directions for Drawing.

The print or painting which is to be copied, is to be placed ſo, that the gloſs of the colours, or ſhades, may not fall upon the eye, and thereby prevent a perfect view of the piece; but it is to be ſo placed, that both the light and the eye may fall obliquely upon it. It muſt alſo be placed at ſuch a diſtance, that the whole may be taken into the eye at once; for which purpoſe, the larger it is, the further diſtant it muſt be placed, and ſet a little reclining.

Make a ſmall point upon the paper, to repreſent the centre. And obſerve, as a general rule, always to begin with the right ſide of the piece; for by that means, what is finiſhed will not be hidden by the hand or pencil. Obſerve alſo the moſt perſpicuous and uppermoſt figures in the piece (if there be more than one), which are to be touched upon the paper in their proper places, by the charcoal; thus running over the draught, there will be had the ſkeleton of the work, which is to be afterwards finiſhed and filled up. But great care is to be taken in obtaining a true draught, and the more time there is beſtowed upon it, the more it will improve the learner.

To arrive at any tolerable proficiency in this art, the ſtudent muſt particularly attend to the directions given in the following leſſons. He muſt alſo perfect himſelf in the practice of the firſt before he attempts the ſecond, and in the ſecond before he proceeds to the third, &c.

LESSON I.

The firſt part in which the learner ſhould perfect himſelf is, in drawing the introductory lines with the ſeveral features and limbs of the human body ſeparately; in order to which, he ſhould be able, not only to imitate perpendicular and parallel lines, without uſing the ruler; but alſo the ſeveral ſorts of curved lines, with their different inclinations. When he can perform this with eaſe, he may proceed to draw the outlines of the features of the human face; as the eyes, noſe, mouth, and ears (*fig.* 16, 17, 18, 19, 20, and 21, *plate* 25); and from thence to the limbs and other parts of the human body (*fig.* 5 and 9). Theſe muſt be only faintly ſketched with the charcoal, that they may be eaſily rubbed out, in order to make the neceſſary alterations.

LESSON II.

When the learner has become maſter of the parts deſcribed in the foregoing leſſon, he may then, and not before, attempt the profile, or ſide face (*fig.* 1, 3, 12, and 13), ſtrictly obſerving the proportions both of the whole, and the ſeveral parts to each other. He may next deſcribe the full or oval face (*fig.* 2, 10, and 11), ſtill obſerving the bearings of every feature with reſpect to the reſt. The eye, ſeen in front, is divided into three equal parts (*fig.* 16), the centre part of which is the ſize of the ſight; and the opening of the eye is generally about one third of its length. The eye in profile (*fig.* 17) is half the ſize of the eye in front, being only one third and a half of the length of the eye in front. The noſe, ſeen in front, is of the ſame width as when ſeen in

profile,

profile, which is equal to the length of the eye (*fig.* 18 and 16), and the noſtril in height is nearly one third of the width of the noſe. Theſe proportions are to be attended to only during the learner's firſt practice: when he has arrived to a little proficiency, he is to follow his own judgment only, in the proportions, paying more regard to his original than to any verbal directions. When he is able to finiſh the outline of the profile and full face, he may proceed to the other different inclinations of the face, as ſeen in the plate.

LESSON III.

When the artiſt can eaſily imitate the different features and limbs, he may begin to attempt whole length figures; which is to be done in the following manner: ſketch the whole over lightly with the charcoal (or, if the learner be able, he may uſe the black lead pencil at firſt), beginning with the head, next the ſhoulders, then the body; after which, the arms and hands, then the hips, legs, and feet; then examine the proportion of the different parts, rubbing out any ſtrokes of the charcoal where neceſſary; and drawing the lines over again with the black lead pencil, to bring it as near as poſſible to the original. When this is done, proceed to finiſh the figure, by drawing it over again with the crow-quill pen, and Indian ink, departing from the black lead lines where it may be found neceſſary. Then rub out all the marks of the pencil with India rubber. The compaſſes are not to be uſed till after a very minute inſpection with the eye: when, if there be any fault that cannot be eaſily diſcovered, by applying the compaſſes, firſt to the original, and then to the copy, the fault will be ſoon found. To aſcertain the proportions of the ſeveral parts of the human body, a perpendicular line ſhould be firſt drawn through that part intended for the middle of the figure: which ſhould be divided into ſeveral equal parts, and from ſuch menſuration a ſcale may be formed to regulate the proportions of every part of the body. Some divide the length of the human body into eight heads or parts, and others divide it into ten.

When the human body is divided into eight heads (*fig.* 1, *plate* 26), the length of the head, which is the firſt diviſion of the figure, will extend from the crown or top of the head to the bottom of the chin. The ſecond diviſion will terminate in a line drawn through the paps of the breaſts. The third diviſion will fall a little below the navel. The fourth, acroſs the privities, which is exactly the middle of the figure. The fifth croſſes the middle of the thigh. The ſixth is juſt below the bend of the knee. The ſeventh falls a little below the calf of the leg. And the eighth extends to the bottom of the heel.—And obſerve, that the full extent from the end of the middle finger of the right hand to that of the left, when the arms are extended at full length in a direct line, is juſt the length of the whole figure, and from the middle of the collar-bone to the end of the middle finger is juſt four heads, or half the length of the figure, viz.:—The firſt head extends to the bend of the ſhoulder; the ſecond from thence to the elbow; the third to the wriſt; and the fourth to the end of the middle finger. Thus, from ſhoulder to ſhoulder, in a man of common ſize, meaſures exactly two heads. There is no preciſe ſtandard for the breadth of the limbs, for they vary according to the bulk of the perſon.

When the human body is divided into ten heads: the firſt diviſion extends from the crown of the head to the under lip; the ſecond, a little below the collar-bone, which is juſt even with the middle of the ſhoulder; the third will fall juſt below the paps of the breaſt; the fourth will reach juſt below the navel; the fifth, being the middle of the figure, will paſs acroſs the privities; the ſixth will paſs over the middle of the thighs; the ſeventh croſſes the bend of the knee; the eighth paſſes directly through the calves of the legs; the ninth extends half way from the calf to the bottom of the heel, where the laſt diviſion terminates.

The learner ſhould pay particular attention to theſe proportions, and retain them in his memory. It is alſo neceſſary that he have ſome knowledge of anatomy, as it will enable him to judge of the proportion and diſproportion of the human figure.

LESSON IV.

The drapery, or clothing of the figure, is next to be conſidered. Having drawn the outline of the figure faintly with charcoal, correcting every part that appears faulty; proceed to draw the outline of the drapery lightly with the charcoal, with the ſeveral folds, not ſuffering the folds to croſs each other. The quality of the drapery ſhould alſo be conſidered; as ſtuffs and woollen cloth are more harſh than ſilk, which is always flowing and eaſy. The drapery ſhould not ſtick too cloſe to the body; but ſhould appear to flow eaſily. If the drapery be ſuppoſed to be blown by a breeze of wind, it ſhould all flow one way: and the parts next the body ſhould be drawn firſt, before thoſe which fly off. The garments ſhould always bend with the figure; and the cloſer the drapery is to the body, the ſmaller muſt be the folds; and if it be quite cloſe to the body, there ſhould be no folds, but only a faint ſhadow, to repreſent that part of the body which it covers. The beſt rule in this caſe is, to remark the folds as they appear in the drapery of genteel perſons, if the figure be to have a modern dreſs; but a few particular rules may, however, greatly aſſiſt the learner:

1. Carefully avoid a ſuperfluity of drapery.—2. Let as much of the form of the body as poſſible appear under the drapery.—3. When the draperies are large, let them be thrown into large and graceful folds.—4. Drapery which is cloſe to the body, ſhould appear to be looſened by ſmall folds, judiciouſly placed: for want of this caution, the figure will have a certain ſtiffneſs, and appear as if wrapped round with a bandage, inſtead of being clothed.—5. If there be much drapery, let the greater part, if poſſible, be thrown into ſhadow.—6. Thoſe folds which fall in the light muſt have ſuch

The ſhades ſhould be rubbed with a piece of paper or glove-leather, rolled up hard, and cut almoſt to a point, like a pencil; which ſerves to blend the ſhades, and ſoften them into each other; and alſo to weaken them, where they appear too ſtrong. India ink is prepared for different ſhades, by rubbing it, more or leſs, in water on a marble ſtone, cut in hollows for that purpoſe, reſerving one of the hollows for the water. In ſhading a piece, as well as in drawing the outlines, the beſt rule is to follow nature, which will greatly improve the ideas of the learner in the diſpoſal of light and ſhade.

To enlarge, or contract, a Piece.

Divide the original into any number of equal ſquares; then divide the paper, upon which the piece is to be drawn, into the ſame number of equal ſquares, either greater or leſs, than thoſe in the original, according as the piece is to be enlarged or contracted, and draw the ſeveral parts of the piece as they fall in the ſquares of the original, in the correſponding ſquares of the paper: then outline it with India ink, and rub out the black lead marks of the ſquares; and ſhade the piece, &c. To prevent miſtakes, the ſquares may be numbered, both in the original and copy (*fig.* 14 *and* 15, *plate* 25).

Though the human body be divided into eight or ten heads, as before obſerved, and which may anſwer for ſmall figures, and where great exactneſs is not expected, yet it muſt be remembered, that theſe are not the exact proportions. Where the figures are large, the artiſt ſhould be more correct; for which purpoſe, it may be neceſſary to attend to the proportions of the ancients, as they are given by M. Audran, from the proportions of the Apollo Pythius, in the garden of the Vatican at Rome, and the Venus Aphrodite, belonging to the family of the Medicis: both which figures are ſuppoſed to ſtand upright, duly poiſed on both legs. The whole height of the former is divided into 31½ parts, being 7 heads 3 parts and 6 minutes; and the latter into 31 parts, being 7 heads and 3 parts, as follows:

Length of the Fore-Arm, or upper Extremities.

	Apollo.			Venus.		
	Hd.	Pts.	Min.	Hd.	Pts.	Min.
From the top of the ſhoulder to the elbow	1	2	3	1	2	3
The elbow to the hand	1	1	2	1	0	6
The joint of the hand to the root of the middle finger	0	1	8	0	1	6
The root to the tip of the middle finger	0	1	10	0	1	7
Length of the upper extremities	3	2	11	3	1	10
Breadth between the outward angles of the eyes	0	1	6	0	1	7
Of the face at the temples	0	2	2	0	2	2
Of the upper part of the neck	0	2	0	0	1	11
Over the ſhoulders	2	0	0	1	3	8
Of the body below the arm-pits	1	2	5	1	1	8
Between the nipples	1	0	7	0	3	8
From the bottom of the chin to the line croſſing the nipples	1	0	7	1	0	1
Of the body, at the ſmall of the waiſt	1	1	0	1	0	8
Over the loins	1	1	3	1	1	6
Over the haunches, or tops of the thigh bones	1	1	5	1	2	3
Of the thigh, at the top	0	3	0	0	3	1
Of the thigh, below the middle	0	2	8½	0	2	7
Of the thigh, above the knee	0	1	8	0	2	0
Of the leg, below the knee	0	1	6	0	1	10½
Of the calf of the leg	0	2	4	0	2	3
Below the calf	0	1	7	0	1	11½
Above the ankle	0	1	2	0	1	2
Of the ankle	0	1	4	0	1	3
Below the ankle	0	1	1½	0	1	1

Middle

	Apollo.			Venus.		
	Hd.	Pts.	Min.	Hd.	Pts.	Min.
Middle of the foot . .	0	1	4	0	1	3
At the roots of the toes .	0	1	7	0	1	7
Of the arm, over the biceps muſcle	0	1	8	0	1	9
Of the arm, above the elbow	0	1	6	0	1	5
Of the arm, below the elbow	0	1	10	0	1	7
At the wriſt . . .	0	1	1	0	1	0
Of the hand, over the firſt joint of the thumb . .	0	1	9	0	1	8
Of the hand, over the roots of the fingers . . .	0	1	7	0	1	6
Breadth over the ſhoulder blades .	1	2	0	1	1	4
Length of both arms and hands, each of Apollo's being 3 hds. 2 pts. 11 min. and each of Venus 3 hds. 1 pt. 5 min.	7	1	10	6	2	10
Breadth between the tips of the middle fingers of each hand, when the arms are extended horizontally . . .	8	3	10	8	0	2

Side View.

	Apollo.			Venus.		
	Hd.	Pts.	Min.	Hd.	Pts.	Min.
From the top of the head to the ſhoulder . . .	1	1	8	1	1	6
The top of the ſhoulder to the loins, above the hip .	1	3	3	1	1	7
The loins to the lower part of the hip	1	0	2	1	2	1
The hip to the ſide of the knee	1	2	0	1	0	11
The ſide of the knee to the bottom of the heel . .	2	0	5	2	0	11
Length of the figures . .	7	3	6	7	3	0

From

	Apollo.			Venus.		
	Hd.	Pts.	Min.	Hd.	Pts.	Min.
From the fore to the back part of the ſkull . . .	0	3	6	0	3	4
The wing of the noſe to the tip of the ear . . .	0	1	8½	0	1	6
The upper part of the neck .	0	2	0	0	1	11
The breaſt to the back, over the nipple . . .	1	0	6	1	0	6
The belly to the ſmall of the back . . .	0	3	6	0	3	7
The belly, above the navel, to the back of the loins .	0	3	9	1	0	2
The bottom of the belly to the round of the hip .	1	0	0	1	0	5
The fore part of the thigh to the bottom of the hip .	0	3	2	0	3	7
The middle of the thigh .	0	3	3	0	3	6½
The thigh, above the knee .	0	2	1	0	2	3
The middle of the knee .	0	2	1	0	2	2
The leg, above the knee .	0	1	9	0	1	11
The leg, at the calf . .	0	1	8	0	1	9
The leg, at the ankle . .	0	1	5½	0	1	4
The foot, at the thickeſt part	—	—	—	0	1	3
Length of the foot . . .	1	0	6	1	0	4½
The heel to the fore part of the bend of the foot .	—	—	—	0	2	2
The arm, over the biceps .	0	2	0	0	1	9
Over the elbow . .	0	1	6	0	1	6
Below the elbow . .	0	1	5	0	1	7
At the wriſt . . .	0	1	1	0	0	11
Below the joint of the wriſt .	0	1	0	0	0	10
The hand, at the roots of the fingers . . .	0	0	5½	0	0	5
At the roots of the nails .	0	0	2½	0	0	3

Theſe are the proportions of the two moſt admired ſtatues extant: the proportions of ſome other admired ſtatues differ ſomewhat from theſe; for example—the Laocoon meaſures 7h. 2p. 3m.; the Hercules, 7h. 3p. 7m.; the Pyramus, 7h. 2p.; the Antinous, 7h. 2p.; the Grecian Shepherdeſs, 7h. 3p. 6m.; and the Mirmillo, 8h.—But the porportions of each part of the ſame figure are allowed to be perfectly harmonious, and agreeable to the characters.

It muſt alſo be obſerved, that the centre, or middle part, between the extremities, or head and feet, of a new-born child, is in the navel; but that of an adult, or grown perſon, is in the os pubis. Painters and ſculptors generally divide the length of a child into four, five, or ſix parts, according to its age; and the head is one of thoſe parts. Thus, a child of two years old has about five heads in its whole length; but one of from four to five years has near ſix heads; and about the fifteenth or ſixteenth year, ſeven heads are the proper proportion or meaſure, and the centre is in the upper part of the pubis. Therefore, as the body increaſes in growth, there is a gradual approach to the proportion of an adult, who has near eight heads in his whole length, or rather upwards of ſeven and three quarters.

SECT. II.

OF ETCHING COPPER-PLATES.

ETCHING is a mode of engraving on copper, where the lines or ſtrokes are eaten in with aqua-fortis, inſtead of being cut with the tool, or graver.

Etching has ſeveral advantages over graving; as being done with more eaſe and expedition, requiring fewer inſtruments, and repreſenting faint, diſtant objects more agreeably, and according to nature. It is uſually done in all plates where there is a complexity of work, though they are generally finiſhed by the graver.

The inſtruments proper for etching are, needles, an oil-ſtone, bruſh pencils, a burniſher, ſcraper, compaſſes, ruler, tracer, and the graver; with the hard and ſoft varniſh, prepared oil, and aqua-fortis.

The needles ſhould be of a fine grain, and ſuch as will break without bending, of which there ſhould be ſeveral ſizes. They are to be fixed in firm round ſticks, about ſix inches in length, and the thickneſs of a large gooſe-quill; and may be fixed in ſuch ſticks as have a pencil at the other end. They ſhould ſtand at leaſt a quarter of an inch out of the ſtick.

The oil-ſtone is to whet the needles upon: and, *note*, if the points are to be round, they are to be whetted ſhort upon the ſtone, by turning them round; but if the points are to be ſloped, they are firſt to be blunted upon the ſtone, and then whetted, ſloping on one ſide only, till they come to a ſhort oval.

The bruſh pencil is to cleanſe the work, wipe off the duſt, and ſtrike the colours, even over the ground, when laid upon the plate.

The burniſher is a piece of tempered ſteel, ſomewhat round at the end, for ſmoothing and giving a luſtre to the plate.

The ſcraper is uſed for clearing the plate of any ſcratches, or ſtrokes, which the burniſher will not take out.

The compaſſes ſhould have ſteel points, and are chiefly uſed in ſtriking circles, meaſuring diſtances, &c.

The ruler is chiefly uſed to draw ſtraight hatches, or lines, upon the plate.

The tracer is uſed for drawing through all the outermoſt lines, or circumference of the print or drawing, which is called etching after.

The manner in which etching is performed, is, by covering the ſurface of the plate with a proper varniſh, or ground, capable of reſiſting the aqua-fortis; and then drawing the lines of the figure with needles, quite through the varniſh, upon the plate; the plate being then covered with aqua-fortis, the parts of the copper where the lines are drawn, being expoſed to the action of the aqua-fortis, will be corroded or eaten away by it; while the reſt of the plate, which is covered with the varniſh, will remain untouched.

There are two methods of etching, differing from each other in the varniſh uſed, and in the quality of the aqua-fortis; but the general methods of performing them are both alike. The varniſhes or grounds are diſtinguiſhed by the names of the hard and the ſoft varniſh; for their difference principally conſiſts in the reſiſtance they give to the needles, in working. The hard varniſh was formerly moſt in uſe, on account of its admitting the work to be made more correct, and appear more like engraving with the tool, or graver; and the firmneſs of the body of this varniſh gave more opportunity of retouching the lines, or enlarging them with the oval-pointed needles; for the excellency of etching was formerly thought to conſiſt in its near reſemblance to engraving; and the principal object in thoſe who practiſed it, was to render it as near as poſſible in appearance to engraving by the tool or graver; which, however, was the principal cauſe of obſtructing the improvement of the art, and cramped the talents of the ableſt maſters. The ſoft varniſh has, therefore, almoſt ſuperſeded the uſe of the hard, except in the caſe of ſome particular ſubjects, as it admits of a more free manner of working, and affords a power of expreſſion incompatible with the inflexibility of the hard varniſh, which confines the lines and hatches to ſuch a regularity and ſameneſs, as gives a ſtiffneſs of manner, and coldneſs of effect, to the work, particularly in hiſtorical engraving, where there is a greater opportunity of exerciſing the force of genius and fancy; and where the effect of the whole, more than the minute exactneſs of finiſhing the ſeparate parts, conſtitutes the principal value.

The manner of etching with the ſoft varniſh is now more frequently intermixed with the uſe of the graver than formerly: which is generally attended with great advantages, even where the whole is intended to paſs for the work of the graver; as it gives an opportunity of ſhowing the truth and ſpirit of the outline, and gives all the variety of ſhades which the different kinds of black can produce; while the exactneſs and regularity of the lines, which are requiſite for finiſhing many kinds of deſigns, are ſupplied by the graver; and by a judicious application of both, that complete finiſhing and effect is produced, which either of them alone would be incapable of affording.

The Preparation of the ſoft Varniſh, as directed by Mr. Lawrence, an eminent Engliſh Engraver at Paris.

"Take of virgin wax, and aſphaltum, each two ounces; of black pitch, and Burgundy pitch, each half an ounce: melt the wax, and pitch, in a new earthenware glazed pot, and add to them by degrees the aſphaltum, finely powdered; let the whole boil till ſuch time, as that (taking a drop upon a plate) it will break, when it is cold, on bending it double two or three times betwixt the fingers. The varniſh being then boiled enough, muſt be taken off the fire; and letting it cool a little, muſt be poured into warm water, that it may work the more eaſily with the hands, ſo as to be formed into balls; which muſt be rolled up, and put into a piece of taffety for uſe."

In boiling the ingredients, it muſt be obſerved, firſt, that the fire be not too violent, leſt they burn—a ſlight ſimmering will be ſufficient: ſecondly, while the aſphaltum is putting in, and even after it is mixed with them, the ingredients ſhould be ſtirred continually with the ſpatula: and, thirdly, the water, into which this compoſition is thrown, ſhould be nearly of the ſame heat with the compoſition, to prevent a kind of cracking, which will happen when the water is too times two or four candles are uſed together, for diſpatch; for the varniſh muſt be blackened before it grows cold; for if it grows cold during the operation, the plate muſt be heated again, that the varniſh be in a melted ſtate when that operation is performed: great care muſt be taken, not to ſcorch it; which may be perceived, when it happens, by the varniſh loſing its gloſs, and appearing burnt. Large plates are ſometimes ſuſpended from the ceiling by four cords, with an iron ring about four inches diameter at the end of each cord, to hold each corner of the plate. The plate being thus ſuſpended with the varniſh ſide downwards, may be blackened very conveniently.

In blackening the varniſh, the candle or flambeau ſhould be kept at a proper diſtance from the plate, that the wick may not touch the varniſh. If, after the operation, it appears that the ſmoke has not penetrated the varniſh, the plate muſt be again heated over the chafing-diſh, and as the plate grows hot, the varniſh will gradually melt and incorporate with the ſmoke, that lies above it, in ſuch a manner that the whole will be equally pervaded by it.

The greateſt caution is neceſſary in this operation, to keep a moderate fire all the time, to move frequently the plate, and change the place of every part of it, that the varniſh may be equally melted every where, and kept from burning, and to keep the varniſh entirely free from any filth, ſpark, or duſt, till it be entirely cold.

The Method of applying the hard Varniſh.

This is exactly the ſame as that of applying the ſoft varniſh; being ſpread equally over the warm plate with the taffety ball, and ſmoked in the ſame manner; but after it is ſmoked, it muſt be baked, or elſe dried over a gentle charcoal fire, till the ſmoke of the varniſh begins to decreaſe; obſerving not to heat the plate too much, which would burn, and ſoften the varniſh.

The plate being thus prepared, the outlines of the piece are to be drawn on the varniſh, as follows:—having an exact drawing of the outlines of the deſign upon a piece of thin paper, rub the other ſide of the paper with chalk, or Spaniſh white; or, which is better, with red chalk powdered, bruſhing all the looſe chalk off with a linen rag; then the ſtained ſide of the paper is laid upon the varniſh, the corners fixed to the plate with wax, or wafers, to prevent its moving; and the drawing is ſlightly traced through all the lines with a blunted needle, or pointer, which communicates to the varniſh the exact outlines of the deſign.

The deſign being thus chalked on the varniſh, the next proceſs is to draw the ſeveral lines with the needle through the ground upon the copper; obſerving, that ſuch parts of the plate as is not to be wrought, ſhould be covered with a ſheet of white paper, upon which is a ſheet of brown paper, in order to reſt the hand upon, to prevent its touching the varniſh.

In drawing the lines through the ground, as they conſiſt of ſo great a variety, being ſome ſtraight and others crooked, ſome ſmall and others large, there muſt conſequently be uſed ſeveral ſorts of needles, bigger or leſs, as the work requires. The large lines are made by leaning hard on the needle, the point of which ſhould be ſhort and thick; or by making ſeveral lines or hatches cloſe to one another, and paſſing them over again with a thicker needle; or by making them with a large needle, and permitting the aqua-fortis to lie longer thereon.

When the lines or hatches are to be of an equal thickneſs from end to end, the needle ſhould have the ſame force impreſſed upon it; but where the lines are required to be fine, or ſmall, the needles ſhould be leaned upon lighter; and where they are to be deep or large, the needle ſhould be leaned upon heavier. When the lines are too ſmall, they ſhould be drawn over again with a ſhort round-pointed needle, leaning ſtrongly where they ſhould be more deep.

The oval-pointed needle is moſt proper for making large and deep ſtrokes. It ſhould be held in the ſame manner as a pen, with the flat ſide next the thumb: though it may be uſed, with the face the other way: it muſt be held as upright and ſtraight in the hand as poſſible, ſtriking the ſtrokes freely and firmly, which render them neat and clear.

The fine needles with ſlender points, are proper for fine ſtrokes, and for the faint ſtrokes of thoſe places at the greateſt diſtance, in a landſcape; and alſo for thoſe places neareſt the light. And it is requiſite, when at work, to bruſh off all the looſe duſt which is worked up by the needles.

It is hardly neceſſary to obſerve, that the ſtudent ſhould be ſo far maſter of the art of drawing, as to be able to copy any print exactly, before he attempts etching. It is alſo neceſſary that he be able to hatch with a pen or pencil exactly, from good copies; and then he will be able to draw from plaiſter, or from the life.

In ſhading his piece, he muſt be careful to obſerve how the original is ſhadowed, how cloſe the hatches are joined, how they are laid, how they incline, and which way the light falls, which muſt always fall one way. If the light fall ſideways in the print, that ſide, which is fartheſt from the light, muſt be hatched the darkeſt.

In landſcapes the part next to the eye is to be hatched darkeſt; and the reſt to decline in its ſhadow gradually, the farther it is off from view. The ſame is to be obſerved in etching a ſky; for that which is neareſt the eye, muſt have the deepeſt ſhades; but in general, as ſoft and faint as poſſible, gradually loſing its ſhades as it comes nearer to the ground; and where they both meet, as it were, the ſky muſt be entirely loſt.

If any ſcratches, or falſe ſtrokes, happen in the working, they are to be ſtopped up with a hair pencil, dipped in the Venetian varniſh, mixed with lamp black, by which means theſe places will be defended from the aqua-fortis.

The next operation is that of eating, or corroding, the plate with aqua-fortis; in order to which, a border of ſoft wax, formed of bees-wax, melted, and mixed with a little tallow and Venice turpentine, muſt be raiſed round the edge of the plate, about an inch high, in the form of a little wall, or rampart, to contain the aqua-fortis. A gutter is uſually formed at one of the corners of this border, to pour off the aqua-fortis, when requiſite. The border being thus raiſed, mix ſome refiner's aqua-fortis, with half its quantity of common water, and pour it gently upon the plate, till it riſe about a finger's breadth above the ſurface of the plate, when it will be ſeen, that the aqua-fortis will ſoon exert itſelf in the hatches, which have been ſtrongly touched; but thoſe more weakly engraved, will appear at firſt clear, and of the ſame colour as the copper. The aqua-fortis muſt, therefore, be ſuffered to continue on the plate till its effects become viſible on the more tender parts. It muſt then be poured off, the plate waſhed with clean water, and dried before the fire; and all the lighter parts of the plate covered with a pencil dipped in the Venetian varniſh. The aqua-fortis muſt then be poured on again, and ſuffered to continue a longer or ſhorter time, according to the ſtrength of the menſtruum, and the nature of the engraving; when it muſt be again poured off as before, and the plate immediately waſhed with water.

While the aqua-fortis is on the plate, the verdigriſe that gathers in the hatches, by its action, ſhould be cleanſed away with a feather, which gives the aqua-fortis more room to exert itſelf.

When the plate is ſufficiently corroded, and waſhed with water, it muſt be warmed by the fire, and the wax border removed; after which, it muſt be heated, till the varniſh melt; then it muſt be well wiped with a linen cloth, and rubbed with oil of olives; when it will be ready to be retouched, and finiſhed by the graver.

The management of the aqua-fortis is the principal matter in the whole art of etching, and on which the ſucceſs of the work chiefly depends. For the exact ſtrength of the aqua-fortis, the time it is to continue on the plate, &c. no certain rule can be given; but practice and experience alone can inform the artiſt.

Of etching Letters.

To etch letters, the copper-plate is to have a ground of virgin wax, which is to be ſpread very evenly with a feather, all over the plate, while it is warm; then the letters being wrote on paper with a black lead pencil, the written ſide of the paper is to be laid upon the ground of the plate, the paper being faſtened at the four corners, as before directed. Then rub the back of the paper all over with a burniſher, taking care to rub every part of the paper; and taking the paper off the plate, the letters will all appear written on the wax, but reverſed; they are then to be drawn through the wax on the plate with a tracer, cleaning the work from the looſe wax with a linen rag, or pencil bruſh; then raiſing a border of wax, and pouring on the aqua-fortis, as before, the letters will be etched. The plate, being cleaned from the wax, is, in the next place, to be poliſhed, as follows:—take a piece of good charcoal, and pulling off the rind, put fair water on the plate, and rub it with the charcoal; and by this means the plate will be cleared from all the varniſh. But the charcoal ſhould have no knots, or roughneſs. After this, waſh the plate with a little aqua-fortis, added to twice its quantity of water. Laſtly, the plate ſhould be wiped dry, and rubbed with a little of the olive oil. Then, if any place require to be touched with the graver, it may be corrected; and the plate will be finiſhed.

SECT. III.

OF ENGRAVING.

ENGRAVING with the tool (ſo called in contradiſtinction to thoſe plates which are firſt etched, and which are called etchings) was the firſt kind of engraving practiſed, and is ſtill uſed for many purpoſes, particularly in portraits, where great regularity and exactneſs in the lines are required; as there every minute part muſt be expreſſed according to the original ſubject, without allowing the artiſt the leaſt indulgence of fancy.

The principal inſtruments uſed in engraving are the gravers, ſcrapers, a burniſher, an oil-ſtone, and a cuſhion for bearing the plates.

Gravers are made in ſeveral forms, with reſpect to the points; ſome being ſquare, others lozenge: the ſquare graver is uſed to cut broad deep ſtrokes; and the lozenge for more delicate, and fine ſtrokes. La Boſſe recommends a form betwixt the ſquare and lozenge; ſomewhat long, and ſmall towards the point, as the moſt generally uſeful.

The burniſher is uſed to aſſiſt in the engraving, on ſome occaſions; and alſo to poliſh the plates. This inſtrument is about ſeven inches in length, and of poliſhed ſteel. The burniſher is formed at one end, and the ſcraper at the other end of the ſame inſtrument. This ſerves to take out any ſcratches that may happen on the plate; or to leſſen the effect of any parts, that may be too ſtrongly marked.

The cuſhion is uſed for ſupporting the plate in ſuch a manner, that it may be turned every way with eaſe: it is formed of a bag of leather, filled with ſand, and ſhould be of the ſize that will beſt ſuit the plate it is intended to bear.

Of holding the Graver.

The graver ſhould not exceed the length of five inches and a half, including the handle, except it be uſed for ſtraight lines; and that part of the handle which is on the ſame line with the belly, or ſharp edge of the graver, ſhould be cut off flat, that it may be no obſtruction in working.

The handle of the graver ſhould be held in the hollow of the hand, with the fore-finger reſting upon the back of the graver, in order that it may be moved parallel to the plate. Great care muſt be taken that the fingers do not interpoſe between the plate and the graver, which would prevent the graver from being carried level with the plate, and render the ſtrokes not ſo clean.

Of laying the Deſign upon the Plate.

The plate being poliſhed ſmooth, is to be heated, ſo as to melt virgin wax, with which it is to be rubbed thinly and equally all over, and ſuffered to cool. The deſign to be laid on muſt be drawn on paper, with a black lead pencil, and laid upon the plate, with the pencilled ſide upon the wax; it is then to be preſſed cloſe to the plate, and rubbed over every part with a burniſher. Then taking the paper off the plate, every line drawn with the black lead pencil will appear upon the wax, which are to be traced through the wax upon the plate with a ſharp-pointed tool. The wax being taken off, the plate is to be engraved.

Of whetting and tempering the Graver.

Great care is required to whet the graver nicely; for which purpoſe, the two angles of the graver, which are to be held next the plate, are to be laid flat upon the ſtone, and rubbed ſteadily, till the belly riſes gradually above the plate, ſo that, when the graver is laid upon it, the light may be ſeen under the point; if it be not whetted in this ſhape, it will dig into the copper, and it will be impoſſible to prevent the point from breaking. In order to whet it, place the fore-finger of the left hand upon the graver, and rub it firmly upon the ſtone: then, to whet the face, place the flat part of the handle in the hollow of the hand, with the belly of the graver upwards, upon a moderate ſlope, and rub the face upon the ſtone till it has a very ſharp point, which may be tried on the thumb nail; obſerving that the ſtone be ſupplied with oil.

When the graver is too hard, the point will frequently break; it muſt therefore be tempered, by holding it upon a red hot poker, within half an inch of the point, till the ſteel change to a light ſtraw colour; then the point is put into oil, to cool it; or elſe, hold it cloſe to the flame of a candle till it change colour, and cool it in the tallow; but if it be heated too much, it will turn blue; it is then too ſoft, and muſt be tempered again. Sometimes a little whetting will bring it to a good temper.

The dry point, or needle, is now much uſed in engraving, and is a tool like an etching point, which being drawn hard on the copper, cuts a more ſoft and delicate ſtroke than can be made any other way; and the burr which it raiſes is to be ſcraped off.

All the art of engraving conſiſts in the proper uſe of the graver, and dry needle; for which there are no certain rules to be given, as the ultimate ſucceſs of the artiſt depends principally on his genius. However, a few general obſervations may not be improper. It need hardly be mentioned, that the perſon who attempts engraving, ſhould be a good maſter of deſign, and well acquainted with both perſpective and architecture: by which he may be enabled to throw back the figures and objects he deſigns to imitate; and to preſerve the due proportion of the ſeveral orders.

The plate being placed on the cuſhion, the graver muſt be held in the hand in a proper manner, and moved in the proper directions for producing the lines intended, obſerving, in forming ſtraight lines, to hold the plate ſteady on the cuſhion; preſſing more lightly, where they are to be fine, and leaning with greater force, where they are to be broad and deep. In making circular and curve lines, turn the plate upon the cuſhion againſt the graver. After ſome of the work is done, it is neceſſary to ſcrape off the roughneſs formed by the cutting of the graver, which is done with the ſcraper: or paſſing the graver over the plate in a level direction, taking care that it does not catch the copper. To render the work more viſible, it may be rubbed over with a roll of felt dipped in oil. It is neceſſary to learn to carry the graver as level as poſſible with the ſurface of the plate; for otherwiſe, if the fingers ſlip betwixt them, the line that is produced will become deeper and deeper in the progreſs of its formation, which will prevent making a ſtroke at one cut, that will be fine at the extremities, and larger in the middle; and renders it neceſſary to retouch it. Therefore it is neceſſary to acquire the habit of making ſuch ſtrokes, both ſtraight and curved, by lightening, or preſſing, the hand, according to the occaſion. And when the deſign is finiſhed, if any ſcratches or falſe ſtrokes appear in any part of the plate, they muſt be taken out by the burniſher.

In order to preſerve a due equality in the work, the principal objects of the deſign ſhould be ſketched out, before any of them be finiſhed. In working with the graver, the ſtrokes ſhould never be croſſed too much in the lozenge manner (particularly in repreſenting the fleſh of the human body), except in the caſe of a cloud, waves of the ſea, the ſkins of animals covered with hair, or the leaves of trees, where this method of croſſing may be admitted. In the diſpoſition of the ſtrokes, the action of the figures, and the diſpoſition of their parts, ſhould be conſidered; and alſo the manner in which they advance towards, or depart from, the eye of the obſerver. The graver ſhould be ſo guided, as to mark the riſing or cavities of the muſcles, making the ſtrokes wider and fainter in the light, and cloſer and bolder in the ſhades. Thus, the hand ſhould be lightened in ſuch a manner, that the outlines may be formed, and terminated, without being cut too hard; and though the ſtrokes break off where the muſcle begins, yet they ought always to have a connexion with each other, and in ſuch a manner, that the firſt ſtroke may, by its return, ſerve to make the ſecond, which will ſhow the freedom of the graver.

To produce the effect, in engraving the fleſh of any figure, in the lighter and middle tints, round dots, with long pecks of the graver, and ſometimes a few faint lines, may be judiciouſly intermixed. In engraving the hair and beard, the chief ſhades are firſt ſketched with a few ſtrokes, which may be afterwards finiſhed with finer ſtrokes towards the extremities.

In the repreſentation of edifices and large buildings, which are generally of ſtone, the work ſhould not be made very black, becauſe the colour does not produce very dark ſhades. In imitating ſculpture, the hair of the head or beard muſt not be repreſented flowing, as in nature, becauſe it cannot be ſo in ſculpture. Neither muſt white points be put in the pupils of the eyes, as it is in painting.

In engraving the drapery of figures, regard ſhould be had to the different kinds of clothing; linen ſhould be done with finer and cloſer lines than other ſorts of drapery, and be executed with ſingle ſtrokes. Woollen cloth ſhould be engraved wide, in proportion to the coarſeneſs, and with only two ſtrokes; and when the ſtrokes are croſſed, the ſecond ſhould be leſs than the firſt, and the third leſs than the ſecond. Shining ſtuffs, ſilks, and ſatin produce flat broken folds, and ſhould be engraved more hard and ſtraight than others, with one or two ſtrokes, according as the colours are bright or brown; and the firſt ſtrokes ſhould be interlined with ſmaller ones. Velvet and pluſh are alſo expreſſed in the ſame manner.

Metals, as armour, &c. are alſo repreſented by interlining by clear ſingle ſtrokes. In architecture, the ſtrokes which form the rounding object ſhould tend to the point of ſight; and when columns occur, it is proper to produce the effect by perpendicular ſtrokes. When a groſs ſtroke is made, it ſhould be at right angles, and wider and thinner than the firſt ſtroke. In engraving mountains, as there are ſharp and craggy objects, the ſtrokes ſhould be frequently broken; they ſhould alſo be ſtraight, in the lozenge manner, and accompanied with long points or dots. Rocks have ſome croſs ſtrokes in them more ſquare and even. Diſtant objects towards the horizon are very ſlightly ſhaded, as in drawing. Calm, ſtill waters ſhould be repreſented by ſtraight ſtrokes, parallel to the horizon, and interlined with finer ſtrokes; omitting thoſe places where the light caſts a ſhining reflection; and the forms of objects reflected from the water at a ſmall diſtance upon it, or on the banks of the water, are expreſſed by the ſame ſtrokes, retouched more ſtrongly or faintly, as it may be neceſſary: agitated waters, as the waves of the ſea, have the firſt ſtrokes in the figure of the waves, and are interlined; and the croſs ſtrokes ſhould be very lozenge. The firſt ſtrokes in caſcades ſhould follow the fall of the water, and be interlined. In clouds, that appear thick and agitated, the graver muſt be turned every way, according to their form and agitation. In dark clouds, where two ſtrokes are neceſſary, they ſhould be croſſed more lozenge than the figures, and the ſecond ſtrokes ſhould be rather wider than the firſt. The flat clouds, that are inſenſibly loſt in a clear ſky, ſhould be formed by ſtrokes parallel to the horizon, but a little waving; if ſecond ſtrokes be required, they ſhould be more or leſs lozenge, and ſhould be ſo lightened at the extremities, as to have no outline. The flat and clear ſky is repreſented by ſtrokes parallel and perfectly ſtraight.

In all landſcapes, in general, the trees, rocks, earth, and herbage ſhould be etched as much as poſſible, leaving nothing to be done by the graver, but the perfecting, ſoftening, and ſtrengthening. And obſerve, once for all, that the dry needle produces a more delicate effect, and may be uſed to much greater advantage than the graver can, particularly in be neatly imitated, if a plate be provided for every colour. And if it be well done, it will form ſuch a good deception, that an able connoiſſeur cannot, from the firſt inſpection, diſtinguiſh between the original drawing and the engraved imitation; therefore, this mode of engraving is very uſeful to multiply copies of drawings left by ancient artiſts who excelled in the uſe of chalks.

SECT. IV.

OF MEZZOTINTO AND AQUA-TINTA SCRAPING.

Mezzotinto prints have no hatches, or ſtrokes of the graver; but the lights and ſhades are more blended together than in etchings and engravings, and appear like a drawing of India ink.

This art is of late invention, but is greatly uſed, and is admired for the amazing eaſe with which it is executed, particularly by perſons who are deficient in drawing.

The principal tools uſed in this art, beſides thoſe uſed in etching and engraving, are the grounding tool and the ſcraper.

General Directions for laying the Mezzotinto Ground.

Leave a ſpace upon the bottom of the plate for the writing, coat of arms, &c. then, laying the plate upon a piece of ſwan-ſkin flannel, hold the grounding tool in your hand perpendicularly upon the plate, leaning upon it moderately hard; then rock your hand in a right line from end to end, and continue the rocking, till the plate be wholly covered with the marks of the tool in one direction: next, work the tool in the ſame manner acroſs the former ſtrokes; and laſtly, work the tool from one corner of the plate to the oppoſite corner, uſing all poſſible care, not to let the tool cut twice in a place in the ſame direction. The plate will now be full; or, in other words, all rough alike; and if an impreſſion were taken from it in this ſtate, it would appear perfectly black.

The ground being laid, as above, with a piece of rag, rub the ſcrapings of black chalk all over the plate; or with the flame of a flambeau, or candles, ſmoke the plate as before directed in Etching.

Then take the print or drawing, and having rubbed the back of it with red chalk duſt, mixed with white lake, proceed to trace it on the plate with a blunt needle or tracer, tracing only the outlines.

Directions for ſcraping the Picture.

With a ſcraper, ſcrape off the lights in every part of the plate, as clean and ſmooth as poſſible, and in proportion to the ſtrength of the lights and ſhades, in the original; ſcraping but very little and very lightly in thoſe parts where the ſhades run deep; and ſcraping more where the ſhades run lighter. And where there is to be no ſhade at all, the plate is to be ſcraped quite ſmooth, uſing great caution not to hurt the outlines, in the operation. A piece of tranſparent paper may be held in the left hand, ſloping juſt over the right hand, by which the artiſt will better ſee the different tints of the work.

The extreme light parts, where there is no ſhade, as the tip of the noſe, forehead, and that part of the linen, &c

where there is not to be any ſhade, are to be ſoftened or rubbed down with the burniſher, otherwiſe theſe parts will not appear clear, when the work is proved.

There is alſo another method, uſed by mezzotinto ſcrapers: which is, to etch the outlines of the original, with all the folds in the drapery, &c. marking the breadths of the ſhadows by dots; then having uſed the aqua-fortis, as in etching, and the ground being taken off the plate, the mezzotinto ground is to be laid, and the work finiſhed by ſcraping, as above.

When the work is to be proved, it is neceſſary to have ſome French paper, which has been wetted down four or five days, as no other paper will do for this work, and it is neceſſary for it to lie wet that length of time. A proof is then to be taken of the plate; when the proof is dry, correct it, by touching it with white chalk, where it ſhould be lighter; and with black chalk, where it ſhould be darker. In retouching the plate, proceed as before, where it ſhould be lighter, by uſing the ſcraper; and where it ſhould be darker uſe a ſmall grounding tool, as much as is thought neceſſary to give it its proper ſhade. Then it is to be proved again, and again corrected and retouched; and thus proceed to prove, and retouch it, till it be finiſhed.

It is to be obſerved, that the work ſhould be proved the firſt time, before it is the leaſt over-ſcraped in any part; as, by this caution, it will appear more elegant; for the ſmall grounding tool, which is uſed to deepen any ſhades that are over-ſcraped, generally gives the work a coarſe appearance.

Of Aqua-tinta Scraping.

Aqua-tinta is that method lately invented of etching, by which a ſoft and beautiful ſhade is given, reſembling a drawing in water colours, or India ink.

The principal operation is as follows: the etching ground is to be laid on the plate, as in common etching, and the outlines

lines of the deſign etched thereon, as directed before in Etching; the ground is then to be ſoftened with a little greaſe, which muſt again be wiped off with a rag, but leaving as much greaſe on the plate, as to take off the glare of the copper. Then upon the plate muſt be ſifted through a fine ſieve, the powder, which is made of equal parts of aſphaltum, and fine tranſparent reſin; after which ſtrike the ſide of the plate againſt the edge of the table, in order to ſhake off any looſe powder; then hold the back of the plate over a charcoal fire, till it become ſo hot, as to give pain, upon being held on the back of the hand, and the powder, which adhered to the greaſy ſurface of the plate, will now be fixed thereon. When the plate is cold, with a hair-pencil dipped in turpentine varniſh and ivory black, cover all the lights in the plate, or the places where there is no ſhade: raiſe a border of bees-wax round the plate; and having reduced the aqua-fortis to the proper ſtrength by vinegar, or water, pour it on the plate, and let it ſtand five minutes for the firſt or lighteſt ſhade; then pour it off, and having waſhed the plate with water, ſet it on its edge to dry. Then again, with the varniſh, ſtop up the next lighter ſhades, and pour on the aqua-fortis for the ſecond tint, letting it ſtand five minutes more. Proceed in the ſame manner for every tint, till the darkeſt ſhades are produced. If a bold open ground be wanted in any part, it muſt be effected by an after-operation; for which purpoſe, the ground muſt be laid on as the other, by ſifting on the powder; but in this proceſs the powder is much coarſer, and the plate muſt be more heated, in order that the particles of the powder may ſpread, and form ſmall circles: even good reſin alone will do for this.

In etching landſcapes, the ſky and diſtant objects muſt be etched by a ſecond operation, and the powder ſhould be ſomewhat finer. When any part of the fore-ground requires to be higher finiſhed, or when there are trees, &c. the plate muſt be entirely cleanſed from the greaſe by bread, and a ground laid, as in common etching: then it may be finiſhed as highly as requiſite, with the needle or point, by ſtippling with dots, and biting up thoſe parts; or by a rolling wheel.

The foregoing method will only ſerve for prints of one ſingle tint. When different colours are to be expreſſed, there muſt be as many different plates; each plate having only that part etched upon it, which is deſigned to be charged with its proper colour, unleſs (as ſometimes happens) ſome of the colours are ſo diſtant from each other, as to allow the printer room to fill them in with his rubber, without blending them; in which caſe, two or more different colours may be printed upon the ſame plate at once. When different plates are requiſite, there muſt be a ſeparate one, having a pin in each corner, to ſerve as a ſole or bottom to the aqua-tinta plates; and the aqua-tinta plates muſt be exactly fitted, having each a ſmall hole in their corners for paſſing over the pins of the ſole; the pins retain the plates in their due poſition, and alſo direct the printer in placing the paper exactly on each plate, ſo as not to ſhift; by which means each tint or colour will be exactly received on its proper place. This is the method practiſed by the Paris printers. Some ſubjects, however, ſuch as landſcapes, may be printed off at once in the different proper colours, by painting theſe upon the plate. Here the colours muſt be pretty thick in conſiſtence, and the plate carefully wiped in the uſual way, after laying in each tint, as well as wiped in general, when it is charged with all the tints.

In aqua-tinta plates, it muſt be obſerved, that the aſphaltum and reſin muſt be finely powdered, and well incorporated together, before they be ſifted on the plate; for this purpoſe, it is neceſſary to ſift each of them through a fine muſlin ſieve, ſifting firſt a layer of one on a ſheet of paper, and then a layer of the other; proceeding in this manner till the whole be finely ſifted, and well incorporated together.

This art has been hitherto kept as ſecret as poſſible; but a ſtrict attention to what has been delivered will enable the practitioner to finiſh his plate with ſucceſs.

Another method of etching in aqua-tinta is the following: —Having prepared the copper-plate as before directed, for etching, with the etching-ground laid upon it, etch the outlines of the deſign; then take the ground off, and clean the plate perfectly. This being done, ſprinkle ſome gum ſandrach, finely powdered, very thinly and evenly over the plate; then warm the plate juſt ſufficient to faſten the gum upon it; but great caution is requiſite that the plate be not ſo hot as to melt the gum; if it be, the gum muſt be taken off, and the operation repeated. When the plate is cold, with a ſmall bruſh pencil, dipped in ſpirits of nitre, go over every part that is intended to be ſhaded, for the firſt time. This operation is to be repeated as often as is neceſſary, to bring the ſhadows to a proper depth. And, *Note*, if the ſpirits be too ſtrong, they may be diluted with a little water.

It is by this method that Mr. P. Sandby finiſhed ſome excellent prints, which have been greatly admired by the beſt judges of the art.

SECT. V.

OF CRAYON PAINTING.

Of the Materials neceſſary for Crayon Painting.

To execute a picture with ſucceſs in crayon painting, it is abſolutely neceſſary that the crayons ſhould be ſoft, otherwiſe the artiſt will fail in his performance; therefore the greateſt care ſhould be obſerved in preparing them, to prevent their being hard. And in all compoſitions of colours for crayons, flake white and white lead ſhould be wholly avoided, as the ſlighteſt touch with either of theſe will always turn black, for theſe whites will ſtand only in oils. Therefore, when white is required, I would recommend the ſtudent to make uſe of common whiting, prepared in the following manner :—

" Put ſome whiting in a large veſſel of water, mixing them well together; when this has ſtood about half a minute, pour off the water into another veſſel, and throw the gritty ſediment away; after this water has reſted about a minute, pour it off as before, which will purify the whiting from all dirt and grittineſs. This being done, let the whiting ſettle, and pour the remainder of the water from it; after which lay it on the chalk to dry, and when dry, it will be fit for uſe; either for making white crayons, or for preparing tints with other colours. And, *Note*, if the ſtudent make the crayons of the whiting immediately after it is waſhed, it is not neceſſary to dry it on the chalk; for it may be mixed with any other colour inſtantly, whereby much trouble will be ſaved. All colours of a heavy or gritty nature, particularly blue verditer, muſt be waſhed in this manner.

The ſtudent muſt be provided with a large flexible pallet-knife, a large ſtone, and muller, to levigate the colours; two or three large pieces of chalk, having large ſmooth ſurfaces, to abſorb the moiſture from the colours, after they are levigated; and a piece of flat glaſs, to prevent the moiſture from being abſorbed too much, till the colours are rolled into form. Theſe implements being provided, the ſtudent may proceed to form his crayons from the following colours:

Reds are formed either from carmine, lake, or vermilion, or a compoſition of two or more of them; though it muſt be obſerved, that it is difficult to procure either good carmine or lake: good carmine is inclined to the vermilion tint, and ſhould be an impalpable powder: a good lake ſhould incline to the carmine tint.

The carmine crayons are prepared by mixing a ſufficient quantity of good carmine with ſpirits of wine with the levi-

Lake crayons are ſomewhat difficult to form, on account of the harſhneſs of the lake, therefore the ſtudent ſhould obſerve the following rules in the formation of theſe crayons:—Take about half the quantity of the lake intended for the crayons, and grind it very fine in ſpirits of wine; when dry, pulverize it; and take the other half, and grind it with ſpirits of wine; after which, mix it with the pulverized lake, and lay it out directly in crayons on the chalk: this colour will not bear rolling. The ſimple colour being thus prepared, proceed with the compound crayons, as before directed in the carmine crayons, and in the ſame degree of gradation.

Vermilion crayons are formed by mixing the vermilion on the ſtone with the ſpirits of wine, or even ſoft water; after which it may be rolled into crayons. The different tints are produced by mixing the ſimple colour with whiting, according to the proportions given in the carmine. And, *Note*, that theſe crayons will ſometimes be ſo ſoft, that they cannot be held in the fingers, but will break, and return to powder: which may be remedied by mixing the colour with ſome thin water-gruel, well ſtrained, which will give it ſufficient coheſion.

Blues are formed of Pruſſian blue, and blue verditer.

Pruſſian blue crayons are formed in the ſame manner as the lake crayons: but as the Pruſſian blue is very apt to bind, it is ſomewhat more difficult to be ſoftened than either lake or carmine. It is neceſſary to grind a large quantity of this colour, as it is chiefly uſed in draperies. The different tints may be made according to the fancy of the painter.

Blue verditer crayons are ſomewhat more difficult to form, on account of the coarſe gritty nature of the verditer, which requires ſome binding matter to unite it, otherwiſe it will never adhere together. Therefore, to a quantity of blue verditer, ſufficient to form two or three crayons, muſt be added a little ſifted plaſter of Paris, about the ſize of a pea: theſe are to be mixed well together, and the crayons formed upon the chalk, the blue verditer being firſt well waſhed. This is a very brilliant blue, and is uſed to good effect in heightening draperies, &c. The different tints are formed by whiting, as before directed; and are very uſeful to produce thoſe pearly tints in painting fleſh, ſo much admired in crayon pictures. Common water will be ſufficient to mix theſe crayons.

Yellows are formed of yellow ochre, king's yellow, and Naples yellow.

The king's yellow crayons are formed by levigating with ſpirits of wine, and forming the different tints with whiting, as in the carmine crayons.

Good yellow crayons may alſo be formed with yellow ochre, and Naples yellow ground with ſpirits of wine, which may alſo have their different tints.

Orange crayons are produced by a mixture of king's yellow and vermilion, ground together with ſpirits of wine; and the tints formed as in other caſes.

Greens are formed by a mixture of yellow and blue, in different proportions; but brilliant greens are produced with great difficulty, and are therefore generally procured of thoſe whoſe buſineſs it is to prepare them.

Good green crayons are formed ſeveral ways: 1. By grinding yellow ochre in ſpirits of wine, and mixing it with Pruſſian blue; then tempering it with the knife, and laying the crayons on the chalk without rolling them.—2. King's yellow, mixed with Pruſſian blue.—3. Brown ochre and Pruſſian blue.—4. Roman ochre and Pruſſian blue, mixed in different proportions, will alſo be found very uſeful, as they may be rolled. Various tints may be produced by the foregoing crayons, to partake more or leſs of the blue or yellow, as found neceſſary.

The brilliant green crayons are abſolutely neceſſary to complete the ſtudent's ſet; but in the crayons procured from the ſhop, there is generally a mixture of flake white, particularly in the light green ones, which will turn black on the pictures, if the leaſt damp come to them: though the dark colours will remain perfect.

In order to diſcover whether there be any flake white in the crayons, the following experiment may be made: having bruiſed the crayon to a powder, mix it with an equal quantity of charcoal duſt; put the whole into a crucible, which muſt be placed in a fierce fire till the charcoal duſt be conſumed; and if the crayon have any flake white in it, the lead will return to its original metallic ſtate.

Browns are originally produced from Cullen's earth, or umber.

Cullen's earth crayons are of a fine dark brown, and ſeveral rich tints may be produced from a mixture of this colour with carmine, in various degrees: alſo, this colour mixed with black and carmine, makes uſeful tints for painting the hair. Several gradations may be made from each of theſe, by a mixture with whiting. Roman and brown ochre alſo form an excellent colour, either mixed together, or compounded with carmine. Whiting, tinged in ſeveral degrees with either of theſe, will prove very ſerviceable. Common ſea-coal, ground to a fine powder, and mixed with carmine, forms a very fine brown.

Umber crayons are formed in the ſame manner as the above; but it is neceſſary to levigate the umber with ſpirits of wine.

Purples are formed by a mixture of blue and red. Good purple crayons may be formed with Pruſſian blue, ground with ſpirits, and mixed with pulverized lake. Alſo Pruſſian blue and carmine produce a deep purple of an excellent hue. From either of theſe compounds various tints may be made, by a mixture with whiting.

Black crayons are formed of lamp black, as no other full black can be uſed with ſafety, all others being ſubject to mildew. But as lamp black is liable to great adulteration, the ſtudent will find it convenient to prepare it himſelf; as follows:

Fix a tin cone over the flame of a lamp, at ſuch a height that the flame may juſt reach the cone, for the ſoot to gather within it. When a ſufficient quantity of ſoot is collected, take it out, and burn all the greaſe from it in a crucible. It muſt then be ground with ſpirits, and laid upon chalk to abſorb the moiſture, when it may be rolled into crayons.

Greys are formed of a mixture of lamp black with whiting, with which a variety of tints may be made. A very good blue black is alſo formed by black chalk finely ground.

Beſides theſe colours, there are a variety of compound colours formed from two or more of the foregoing full colours, as carmine and black, which form a good compound, of which five or ſix gradations ſhould be made; ſome partaking more of the black and others of the carmine; beſides ſeveral tints formed by a mixture with whiting. Cinnabar and black form another very uſeful compound, which has all its different tints. Black and Pruſſian blue alſo form a compound very uſeful in painting the drapery.

Note, The Cullen's earth, yellow, brown, and Roman ochre, Naples yellow, umber, black chalk, and ſea-coal duſt, ſhould all be well ground, and waſhed with boiling water; otherwiſe they will injure the picture, as they are apt to throw out a white kind of ſalt.

Of rolling the Crayons.

The colours being prepared, as above directed, the compoſition muſt be cut into a proper magnitude, in order to be rolled into paſtils. The crayon ſhould be formed in the left hand, with the ball of the middle finger of the right hand; firſt formed cylindrically, and then tapered at each end. If the compoſition be too dry, dip the finger in water; and if too wet it muſt be laid upon the chalk again, to abſorb more of the moiſture. They ſhould be rolled as quick as poſſible; and when finiſhed, muſt be laid upon the chalk again to abſorb the remaining moiſture. When all the crayons of one colour are formed, the chalk and grinding ſtone ſhould be well ſcraped, and waſhed with water, before they are uſed for another colour.

When a ſet of crayons is completed, they ſhould be ranged in ſome thin drawer, divided into a number of partitions, and diſpoſed according to the ſeveral gradations of light. The bottom of the partitions ſhould be covered with bran, to preſerve the crayons clean, and prevent them from breaking.

The box in which the crayons are placed for uſe, and which ſhould be held in the lap when the ſtudent paints, ſhould be about a foot ſquare, having nine partitions. In the upper corner, on the left hand, the black and gray crayons are uſually placed, as they are the moſt ſeldom uſed; in the ſecond partition are placed the blues; in the third the greens and browns: in the firſt partition, on the left hand of the ſecond row, the carmines, lakes, vermilions, and all deep reds are depoſited; the yellows and orange are in the middle partition; and in the next are placed the pearly tints, which, being of a delicate nature, muſt be kept very clean, that the different gradations of colour may be eaſily diſtinguiſhed: in the laſt row, the firſt partition contains a piece of linen rag to wipe the crayons with, while they are uſing; the ſecond partition holds the pure lake and vermilion tints; and the laſt partition contains all thoſe compounded tints, which cannot be claſſed with any colour.

Directions for the Artiſt.

To arrive at excellence in this art, the ſtudent ſhould be as particular in the outline of the work, as in the diſpoſal of his colours; for however juſt the colouring may be, if the work be deficient in ſymmetry and proportion, it will exhibit nothing but a glaring deformity.

The ſtudent being provided with ſome ſtrong blue paper (the thicker the better), he is to level all the knots in it with a penknive or razor; then the paper muſt be paſted very ſmooth on a linen cloth, which is previouſly ſtrained on a deal frame; on this the picture is to be executed: but the ſubject ſhould be firſt dead coloured, as it is called. The manner of doing this is as follows: lay the paper, when dead coloured, on its face, upon a ſmooth board or table; then, with a bruſh, cover the back ſide of the paper with paſte; the frame, with the ſtrained cloth, muſt then be laid on the paſte ſide of the paper, after which, turn the painted ſide uppermoſt, and lay a piece of clean paper upon it; then, by ſtroking it gently with the hand, all the air between the cloth and the paper will be expelled.

By paſting the paper in the frame, the crayons will adhere to it much better than in any other way, and the ſtudent will be able to give a firmer body of colour and greater luſtre to his piece. The ſtudent is, however, not to proceed with the painting, till the paſte be perfectly dry.

To take a correct Copy of a Picture.

Take a piece of tiffany or black gauze, ſtrained tight on a frame, which lay upon the picture to be imitated, and with a piece of ſketching chalk trace all the outlines of the piece on the tiffany or gauze: then place the tiffany with the chalk lines upon it upon a piece of canvas, rubbing it over with a handkerchief, in order to make the chalk lines upon the tiffany adhere to the canvas; and there will be an exact outline of the picture upon the canvas. This is the method generally uſed by painters, and anſwers very well when the ſubject to be imitated is in oils; but if it be a crayon picture, the following method muſt be uſed, on account of the glaſs.

The picture being placed upon the eſel, draw all the outlines upon the glaſs with a ſmall camel's hair pencil, dipped in lake, ground very fine in oils; then take a ſheet of paper, and place it on the glaſs, ſtroking over all the lines with the hand, by which means the colours will adhere to the paper, which is then to be pierced with pin-holes pretty cloſe to each other in all the outlines. The paper intended for the drawing is then to be laid upon the table, and the pierced paper to be laid upon it; then, with ſome fine powdered charcoal, tied up in a piece of lawn, rub over all the pierced outlines, which will give an exact outline of the piece, upon the paper under it. This is not to be bruſhed off till the whole is drawn over with ſketching chalk; which is a compoſition made of whiting and tobacco-pipe clay, rolled like a crayon.

But when the ſtudent paints immediately from life, it is beſt to make a correct drawing of the outlines on another paper, which he may trace by the firſt method: for if there be any falſe ſtrokes of the ſketching chalk, they will prevent the crayons from adhering to the paper.

The ſitting poſture is the moſt proper for painting with crayons, having the box of crayons in the lap. That part of the picture which the ſtudent is at work upon, ſhould be below his face; for when it is placed too high, it will fatigue the arm. The windows of the room, in which the artiſt works, ſhould be darkened to the height of ſix feet from the ground, and the ſubject to be painted ſhould be ſituated in ſuch a manner, that the light may fall on the face to the greateſt advantage, avoiding too much ſhadow, which ſeldom has a good effect in this kind of painting, particularly if the face have much delicacy.

In painting, as well as drawing, the ſtudent cannot be too attentive to the ſubject; he muſt alſo learn to appropriate the action or attitude to the ſubject; thus, if it be a child, let the action or attitude be childiſh; if a young female, let the figure expreſs more vivacity than in the ſtately beauty of a middle-aged woman; while a perſon far advanced in years, ſhould have a greater degree of gravity. The embelliſhments of the piece, and the introduction of birds, flowers, animals, &c. ſhould be regulated by the rules of conſiſtency and propriety.

Of the Features of the Face.

The features of the face being carefully ſketched with chalk, the ſtudent muſt firſt carefully draw the noſtril and edge of the noſe next the ſhadow, with a crayon of pure carmine; then, with the fainteſt carmine firſt, lay in the ſtrongeſt light upon the noſe and forehead, which muſt be executed broad. He is then to proceed gradually with the ſecond tint, and the ſucceeding ones, till he arrives at the ſhadows, which muſt be covered brilliant, enriched with much lake and carmine, a little broken with brilliant green. Theſe colours will be a good foundation towards producing a pleaſing effect, when the piece comes to be finiſhed; though at firſt they will rather offend the eye from their crude appearance; for colours are more eaſily ſullied when too bright, than raiſed into a brilliant ſtate when the firſt colouring is too dull. Thoſe pearly tints, diſcernible in fine complexions, muſt be imitated with blue verditer and white, which anſwers to the ultramarine tints uſed in oils: but when theſe pearly tints are uſed in the ſhade, the crayons compoſed of black and white muſt be ſubſtituted in the place of others. And it muſt be obſerved, that though all the face, when firſt coloured, ſhould be laid in as brilliant as poſſible, yet each part ſhould be kept in its proper tone, which will preſerve the rotundity of the piece.

Whatever colour the iris of the eyes is, the eyes muſt be firſt drawn with a crayon, inclined to the carmine tint; the colour muſt be laid in brilliant at firſt, and executed lightly, not meddling with the pupil yet. The light of the eye ſhould incline very much to the blue caſt; for if a ſtaring white appearance is once introduced, it can ſeldom be altered: a broad ſhadow ſhould alſo be thrown on the upper part by the eyelaſh. The eyebrows ſhould be executed at firſt like a broad glowing ſhadow, on which is to be painted, in the finiſhing, the hair of the eyebrows, by which the former tints will ſhow themſelves through, and produce a pleaſing effect; but a black heavy tint is always to be avoided in firſt forming the eyebrows.

The lips ſhould be begun with pure carmine and lake, ſhading them with carmine and black, and laying on the ſtrong vermilion tints afterwards. Great caution is neceſſary to avoid ſtiff, harſh lines: each colour is to be gently intermixed with the neighbouring colour; the ſhadow beneath ſhould be broad, and enriched with brilliant crayons; The corner of the mouth is formed with carmine, brown ochre, and greens, variouſly intermixed. If the hair be dark, it is neceſſary to uſe a good quantity of the lake and deep carmine tints therein, which may be eaſily overpowered by the warmer hair tints, and which, as in the eyebrows, will produce a richer effect when the piece is finiſhed, than if the lake and carmine be neglected.

When the ſtudent has dead-coloured the head, he is to ſweeten the whole together, by rubbing it over with his finger, beginning at the ſtrongeſt light upon the forehead, and paſſing his finger very lightly to the next tint, to unite them together; which he muſt continue to do, till the work is ſweetened together, frequently wiping his finger on a towel, to prevent ſullying the colours. In this proceſs the ſtudent muſt be careful not to ſweeten his picture too often; as that would produce a thin and ſcanty effect, and the piece would have more of the appearance of a drawing than a ſolid painting, as nothing but a body of rich colours can produce a rich effect. Therefore, it is often neceſſary, in ſweetening a picture, to repleniſh it with more crayon.

The head being brought to ſome degree of perfection, the back ground is to be laid in. This muſt be performed in a different manner; for it is to be covered as thin as poſſible, and rubbed into the paper with a leather ſtump. Near the face the paper ſhould be almoſt free from colour, which will give the head and face a good appearance. Crayons which have whiting in their compoſition, ſhould not be uſed in the back ground, but with great caution: the beſt crayons for this uſe are the moſt brilliant and the leaſt adulterated. The ground ſhould alſo be very thin next the hair, whereby the edge of the hair may be painted over it in the finiſhing touches.

When the face, hair, and back ground are entirely covered, the ſtudent muſt carefully view the piece at ſome diſtance, remarking in what reſpect it is out of keeping, that is, what parts are too light, and what too dark, marking particularly thoſe parts which have a white or chalky appearance, which muſt be ſubdued with lake and carmine.

The painting now will have the appearance of a painting principally compoſed of three colours, viz. carmine, black, and white, which is the beſt preparation that can enter into a compoſition for producing a fine crayon picture.

The back ground and the hair are next to be completed; for, if the face be finiſhed firſt, the duſt in painting thoſe will fall on it, and injure it. Next, proceed to finiſh the forehead, finiſhing each part in its order, proceeding downwards, till the whole be completed.

A great deal of ſkill is required in forming the back ground; though by young artiſts it is often neglected; for a great part of the beauty and brilliancy of the piece, particularly the face, depends upon the tints being well ſuited and adapted to each other, the darks being kept in their proper places; and the whole being perfectly ſubſervient to the beauty of the face. This is requiſite even in a ſimple back ground, where there is but one object in the piece; but more attention is required in the back ground of a picture which has ſeveral objects.

A great variety of colours are uſed for back grounds; but they ſhould always be ſuited to the complexion of the figure. A ſtrong-coloured head generally ſhould have a weak and tender-tinted ground; and, on the contrary, a delicate complexion requires ſtrong and powerful tints in the ground, by which proper contraſt between the figure and the back ground, the picture receives great force.

But when ſeveral objects are introduced into one piece, as hills, trees, buildings, &c. the general rule to be obſerved is, that each grand object be diſpoſed ſo, as to contraſt each other, not merely in their forms, but in their colour, light, ſhade, &c. For example; ſuppoſe a figure in the piece, receiving the ſtrongeſt light, and behind this figure, and near at hand, ſuppoſe there be ſtems of ſome large trees: theſe ſtems muſt have ſhade thrown over them, either from a driving cloud, or ſome other interpoſing object; behind theſe ſtems or trees, and at a diſtance, ſuppoſe there are ſeen trees on a riſing ground; theſe again ſhould receive the light, whereby they will ſerve as a contraſt to the former; and the ſame may be obſerved in all other caſes. The ſame rule holds good in an architectural back ground; as, ſuppoſe a building at a moderate diſtance, and behind this building, the figure which receives the light; a column, or ſome other object in the ſhade, intervenes to preſerve proper decorum in the piece; or, what will have the ſame effect, a ſhadow may be thrown over the lower part of the building. In a word, it muſt be remembered, that the light muſt be always placed againſt the dark, and the weak againſt the ſtrong; and *vice verſa*, in order to produce force and effect.

Of finiſhing the Features.

When the features are to be finiſhed, the forehead is to be painted over firſt, and for the laſt time, beginning the higheſt light with the moſt faint vermilion tint, in the ſame place where the faint carmine was laid at firſt, and keeping it broad in the ſame manner. The next ſhade ſucceeding the lighteſt muſt be worked with ſome light blue tints, compoſed of verditer and white, intermixing with them ſome of the deeper vermilion tints, ſweetening them together with great caution. (This direction, however, is only to be obſerved in the fineſt complexions, for the ſtudent muſt vary his colouring according to his ſubject.) Some brilliant yellows may alſo be uſed ſparingly; and towards the roots of the hair it is neceſſary to uſe ſtrong verditer tints intermixed with greens. Theſe muſt be followed with cooling crayons, compoſed of black and white, which ſhould gradually melt into the hair. Pearly tints are to be preſerved beneath the eyes, under the noſe, and on the temples; alſo beneath the lips: the tints being compoſed of verditer and white, mixing them with light greens and vermilions.

It is neceſſary to uſe greens and blues in the face, in this kind of painting (though it may appear ſtrange to thoſe unacquainted with the art), as they ſerve to break, and correct, the other colours.

In the dead colouring, the carmine ſhould predominate, as that is the beſt preparation for the ſucceeding tints; but the crudeneſs of this preparation renders it neceſſary to be intermixed with greens, blues, and yellows; the degree of carmine, and the complexion of the figure, will determine which of thoſe colours are to be uſed. The blue and yellow are diametrically oppoſite to each other; therefore they ſerve to oppoſe one another; and alſo to correct the reds; the greens, being compounded of both colours, are of peculiar ſervice, where the tranſition is not to be violent.

In finiſhing the complexion, the ſtudent ſhould be particularly attentive to Nature herſelf: for whoever carefully examines a clear and tranſparent ſkin, will diſcover a pleaſing variety of colours on the ſurface, and diſcernible, through it, which will be greatly increaſed by the effect of light and ſhade; one part will appear to incline to the vermilion, another to the carmine or lake, one to the blue, another to the green, and another to the yellow, &c. Now, in order to produce theſe effects, a good artiſt will apply thoſe colours correſponding to the tints, uſing, as often as he can, the compounded colours, inſtead of the ſimple colours; as, blue and yellow, inſtead of green, blue and carmine, inſtead of purple, and red and yellow for orange. In all other circumſtances the compounded crayons already mixed ſhould be uſed; but in this caſe no abſolute rule can be given; the ſucceſs of the piece depending upon the experience and diſcretion of the artiſt. And, obſerve, that it is impoſſible to give any ſet of rules for forming the complexion, that will hold in every caſe, the circumſtances that require different treatments are ſo many and various; but great advantage will be derived, in the commencement of this art, by an able maſter, to direct the ſtudent, and point out the deformities and beauties of a piece, as they occur in practice; which, to a good capacity, will ſoon become clear and intelligible.

In finiſhing the cheeks, uſe the pure lake tint, which will clear them from any duſt they may have contracted from the other crayons, mixing with the lake ſome bright vermilion; and laſtly, (if the ſubject require it,) give a few touches of the orange crayons, but with great caution. This being done, ſweeten the part with the finger as lightly as poſſible, leſt it produce a heavy diſagreeable effect on the cheeks; for the only method of imitating a beautiful complexion, conſiſts in one colour ſhowing itſelf through, or rather between, other colours.

The eye is next to be executed. This is generally found the moſt difficult feature in the face, as every part muſt be expreſſed with the greateſt nicety, and with the true proportion. The ſtudent ſhould, therefore, uſe his crayons, in ſweetening this feature, as much, and his finger as little, as poſſible. When it is neceſſary to have a point to the crayon, to touch a ſmall part with, he may break off a little of his crayon againſt the box, which will give it a ſharp corner. If the eyelaſhes be dark, he muſt uſe ſome of the carmine, and brown ochre, and the crayon of carmine and black; and with theſe he may alſo touch the iris of the eye (if brown or hazel), making a broad ſhadow, cauſed by the eyelaſh. The corners of the eye are executed with red tints, of vermilion, carmine, and lake; taking care that the eyelids are not too red, otherwiſe they will have a diſagreeable appearance. The pupil of the eye muſt be made of pure lamp black. Between the pupil and the lower part of the iris, the light is apt to catch very ſtrong; but it muſt not be made too ſudden, but be gently diffuſed round the pupil, till it be loſt in ſhadow. The eyeballs being ſufficiently finiſhed, the ſmall ſhining ſpeck muſt laſtly be made with a pure white crayon, which ſhould be firſt broken to a point; the ſpot is then to be laid on firmly; and if it ſhould happen not to be perfectly round and neat, it may be corrected with a pin, taking off the redundant parts, which will render it perfectly neat.

The noſe is next to be finiſhed, in which the chief difficulty conſiſts in determining the lines ſo artfully, and blending them into the cheek, ſo as to give it its true projection, and to leave no real line diſcernible, upon the cloſeſt examination. In ſome caſes, it ſhould be quite blended with the cheek, which is to appear from behind it, and determined entirely with a ſlight touch of red chalk. The ſhadow cauſed by the noſe is generally the darkeſt in the whole face, and has no reflection from its ſurrounding parts. The beſt colours for this, are carmine and brown ochre, carmine and black, and ſuch brilliant crayons.

The lips, being firſt prepared with the ſtrongeſt lake, carmine, &c. muſt now, with the ſame colours, be made completely correct: and in the finiſhing, have a little of the ſtrong vermilion; but with great caution, as this colour is very predominant. This colour, if properly uſed, will give the lips an appearance equal, if not ſuperior, to thoſe executed in oils, notwithſtanding the great advantage the latter have, by glazing, of which the former is deſtitute.

In painting the neck, the ſtudent ſhould carefully avoid giving too much expreſſion to the muſcles, in the ſtem; and alſo be careful that the bones appear not too prominent on the cheſt, as either of them has an unpleaſing effect, and denotes a violent agitation of the body, which is ſeldom neceſſary in portrait-painting. The moſt neceſſary part to be expreſſed, is a ſtrong marking juſt above the place where the collar-bones unite. This ſhould always be expreſſed even in the moſt delicate ſubjects; and if the head be thrown much over the ſhoulders, the muſcle that riſes from behind the ear, and is inſerted into the pit, between the collar-bones, ſhould be faintly marked. But, in general, all inferior muſcles ſhould be quite avoided, and not noticed. Many artiſts, in the portraits of thin perſons, mark the muſcles of the neck too evidently. The neck ſhould, in general, have a ſmall addition to the length, as few necks are too long; and nothing is more ungraceful than a neck too ſhort; the ſtem of the neck ſhould have a pearly hue; and the light ſhould not appear too ſtrong upon the cheſt. The breaſt alſo (if any part appears) ſhould be expreſſed by pearly tints, but blended with beautiful vermilion in the upper part thereof.

Of Drapery.

The drapery, by many young artiſts, is thought to require very little attention; but this is an egregious miſtake. An eminent painter being aſked, what part of the picture he thought the moſt difficult to execute? he anſwered, The drapery;—and the beſt judges of the art have univerſally allowed it to be a very difficult part to execute with taſte. It is not ſufficient that the ſtudent be able to give the effect of ſilk, ſatin, cloth, &c. ſo as to deceive the vulgar eye. This, the ſervile copyiſt may effect by the mere dint of labour, and to ſuch perfection, as to make the imitation paſs for reality; but the eſſential attributes of good drapery are, to make the folds in ſuch a judicious manner, as to give grace and dignity to the figure: to clothe it, uninfluenced by prejudice, faſhion, or caprice, ſo as to bear the teſt of ages: theſe it is that require the greateſt exertion of genius, and diſplay all the powers of a refined taſte.

APPENDIX.

NECESSARY RECEIPTS FOR THOSE WHO PAINT IN WATER COLOURS.

To make Gum Water.

DISSOLVE one ounce of pure gum-arabic, and half an ounce of double-refined ſugar, in a quart of ſpring water: ſtrain it through a fine ſieve, or piece of fine muſlin, and bottle it up for uſe, to keep it from the duſt.

Or, ſecondly, take ſome of the whiteſt ſort of gum-arabic, bruiſe it, and tie it up in a piece of woollen cloth; and ſteep it in ſpring water till it be diſſolved. If it be too ſtiff, add more water; and if it be too thin, more gum.

With this water, moſt of the colours are to be mixed; and in ſuch a proportion, that the colour may not rub off, when dry. If the colour ſhine, it is a ſign there is too much gum in the water.

To make liquid Gold for painting on Vellum, &c.

Grind ſome of the fineſt leaf gold with ſtrong gum water, very finely, adding more gum water in the grinding, as you ſee neceſſary. When it is ground very fine, waſh it in a large ſhell; then temper it with mercury ſublimate, and bind it in the ſhell with a little diſſolved gum, ſhaking and ſpreading it equally all over the ſhell. When uſed, it is mixed with fair water only. The ſhells are alſo ſold, ready prepared, at the colour-ſhops.

To make liquid Silver for the ſame Purpoſe.

The proceſs for this is the ſame with the foregoing: but in uſing this, it muſt be tempered with glare of eggs, inſtead of water.

To make the Glare of Eggs.

Take the whites of eggs, and beat them with a ſpoon till they riſe in a froth: let them ſtand all night, and they will be clarified into a good glare.

To make Colours for waſhing Maps.

Colours proper for waſhing maps are obtained by boiling different kinds of wood, or ſtaining ſubſtances, in water, as logwood for purple, cochineal for red; Brazil, madder, turnſol, &c.

To keep Water Colours from ſinking.

Wet the back of the print with a ſolution of four ounces of roach alum in a pint of ſpring water, before the colours are laid on; letting the paper firſt be dried from the alum water. This will prevent the colours from ſinking, alſo give them an additional beauty and luſtre; and likewiſe preſerve them from fading. If the paper is not good, it ſhould be waſhed three or four times with the water, drying it every time.

To make Size for painting Scenes, or other Candle-light Pieces.

When the colours, mixed with gum water, are laid upon any ſurface, they are apt to produce a glare by candle-light; to prevent which, the colours ſhould be mixed with the following ſize, while it is warm:—Steep a quarter of a pound of the cuttings of white leather for ſome time in water; or for the ſpace of two or three days: then take them out, and boil them in three quarts of water, till it be conſumed to one pint, and ſtrain it through a cloth. If it feel firm under your hand when it is cold, it is a ſign it is of a ſufficient ſtrength.

To lay Mezzotinto Prints upon Glaſs.

Having a clear plate of glaſs, as ſtraight as poſſible, and a little larger than the print to be laid upon it, ſoak the print in warm water for about an hour, and with a thin, flexible pallet knife, ſpread ſome Venice turpentine, or good varniſh, very thinly and evenly over one ſide of the glaſs, obſerving to keep the glaſs warm, that it ſpread the better, and taking care that there be not the leaſt ſpeck in the glaſs uncovered with the turpentine; then take the print out of the water, and ſpread it between two cloths, or ſeveral folds of ſoft paper, in order to abſorb the ſuperfluous water. Next lay the print on the glaſs by degrees, beginning at one end, and ſtroking outward that part which is faſtened to the glaſs, that no wind or water may lie betwixt the print and the glaſs, as that would cauſe bliſters. The print being laid in the moſt exact manner upon the glaſs, it is next to be rubbed with the finger, or a piece of linen cloth, until all the thickneſs of the paper rolls off in ſmall rolls, and nothing is left on the glaſs but a thin film, like a ſpider's web, which will be held faſt to the glaſs by the turpentine. When the print is ſo large that ſome part of it becomes dry, before it be rubbed; it ſhould be wetted with a little water on the end of the finger, which muſt be done as often as is requiſite, to keep the paper moiſt: for when it is dry it will not rub off. Great care is neceſſary in rubbing the paper off, that no holes are made in the print, particularly in the lights, which are the moſt tender parts. When the print is rubbed, till it appears tranſparent on the back, it ſhould be ſet up to dry for three or four hours; after which, varniſh it over with turpentine, or maſtic varniſh, two or three times, or till it appear tranſparent; then, it having ſtood a day or two to dry, it is to be painted with ſome of the following colours:—

Colours proper for painting upon Glaſs.

The colours neceſſary for this proceſs are to be procured at the colour-ſhops, prepared in ſmall bladders, at a reaſonable rate; and are generally formed of flake white, lamp-black, umber, vermilion, maſticote, Pruſſian blue, verdigriſe, &c. Ultramarine for blue, and carmine for red, are beſt kept in powder, as being leaſt liable to waſte in that ſtate, and when wanted for uſe, a ſmall quantity may be mixed up with a drop or two of nut oil, with the pallet knife; as may alſo any of the other colours, where they cannot be had, ready prepared from the ſhops.

To get the Colours out of the Bladders.

Prick a ſmall hole near the bottom, and preſs the bladder until enough run out for preſent uſe, for if they ſtand open they are apt to ſpoil.

With theſe colours any tints or ſhades whatever may be exactly imitated, by the different ways and methods of mixing them, according to judgment.

To uſe the Colours.

The lighter colours are to be firſt laid on the lighter parts of the print, and the darker colours are next laid over the ſhaded part, and in the regular order in which the ſhades deepen; for when the brighter colours are once laid on, it is not material if the darker colours be laid a little over them: as the colour firſt laid on will always hide thoſe laid on afterwards. The colours are not to be laid on too thick: and if any of them be too thick in conſiſtence, they ſhould be thinned, before they are uſed, with a little oil of turpentine.

If any of the colours be too ſtrong, or dark, they may be lightened to any degree, by mixing more or leſs white with them on the pallet; or if they be too light, they may be darkened to any degree, by mixing them with a deeper ſhade of the ſame colour.

Note. It is neceſſary to have a pencil for each colour; but that pencil which has been uſed for green ſhould never be uſed for any other colour, without firſt waſhing it well with oil of turpentine, as green will always appear predominant when the colours are dry. And it is alſo neceſſary to waſh all the pencils in oil of turpentine after uſing them.

To

To waſh any Powder very fine for Colours.

Fill a large wine-glaſs with clear water, and put therein half an ounce of the colour you intend to waſh; ſtir it well with a knife: then after it has ſtood about ten ſeconds, in order to let the gritty parts ſettle to the bottom, pour it into another glaſs, and there let it ſtand till the next day; then pour off the water, and the powder will be left very fine, which is to be dried and put up for uſe. Some powders will require a longer time to ſettle in the water, and therefore muſt be permitted to ſtand longer.

To make the beſt drying Oil for painting in Oil Colours.

Boil ſome linſeed oil, in which is a little litharge of gold, over a ſlow fire, obſerving that it does not boil too much, otherwiſe it will prove too thick, and will not be fit for uſe.

To make the Turpentine Varniſh.

Put one ounce of Venice turpentine into an earthen pot or pipkin, which place over a ſlow fire, and when it is diſſolved, add thereto two ounces of oil of turpentine: when it boils, take it off the fire; and when it is perfectly cold bottle it up for uſe: for this, as well as all other varniſhes, ſhould be cloſe ſtopped up, to ſecure it from the air. With this varniſh, prints on glaſs, or other things, may be varniſhed, in order to render them tranſparent: if the varniſh ſhould prove too thick, it may be thinned with a little oil of turpentine; and if too thin, add a little Venice turpentine.

To make the Maſtic Varniſh.

Put two ounces of the cleareſt gum maſtic, finely powdered, into a bottle, with ſix ounces of oil of turpentine: ſtop the bottle cloſe, and ſhake them well together, in order to incorporate them with each other. Then hang the bottle in a veſſel of boiling water for half an hour, taking it out three or four times to ſhake it. If it be neceſſary to make the varniſh ſtronger, it may hang a quarter of an hour longer in the boiling water.

To make Camp Paper, with which a Perſon may write or draw, without Pen, Ink, or Pencil.

Mix ſome hard ſoap with lamp-black and water, into the conſiſtence of a jelly; with this mixture bruſh over one ſide of the paper, and let it dry. When you uſe the paper, put it between two ſheets of clean paper, with its black ſide downwards: then with a pin, a ſtick, or any other ſubſtance with a ſharp point, draw, or write upon the clean paper; and where the point has touched, there will be the impreſſion upon the lowermoſt ſheet of paper, as if it had been drawn or written with a pen.

This camp paper may be made of any other colour, by mixing the ſoap with different colours.

By this paper alſo any print or drawing may be exactly copied, by laying it under the ſame, and tracing the outlines, &c.

The Method of taking Impreſſions from Moulds, Medals, &c. in Plaſter of Paris, and thereby counterfeiting the ſame.

Oil the ſurface of the mould, or medal, with a piece of cotton, or camel-hair pencil, dipped in oil of olives; put a hoop of paper, or paſteboard, round the medal, exactly equal to the thickneſs you would chooſe your impreſſion to be made: then mix ſome plaſter of Paris with water, to the conſiſtence of a thick cream, and with a bruſh lay it on the ſurface of the mould or medal, and immediately after lay on more plaſter, to make it of a ſufficient thickneſs. By rubbing it on the ſurface of the medal with a bruſh, it will entirely prevent any air-holes from appearing on the ſurface of the impreſſions. After it has ſtood about half an hour, it will be ſo hard, that it may be ſafely taken off without breaking; then pare it ſmooth on the back, and round the edges, and it will be done. If the weather be cold or damp, it ſhould be dried before a briſk fire. Alſo, in the operation, when the plaſter is laid on to a ſufficient thickneſs, it ſhould be ſprinkled with ſome of the dry powder of plaſter of Paris, which makes it harder, and dry ſooner. If the face of the medal only be covered with the fine plaſter, the coarſer ſort will anſwer to fill it up, which will be a conſiderable ſaving. And, *Note*, no more plaſter ſhould be mixed at one time than is uſed, otherwiſe that which is mixed will thicken and ſpoil; for, adding water to it to thin it, will totally prevent it from ſetting hard a ſecond time.

Having taken an impreſſion from a medal, &c. according to the foregoing directions, a plaſter mould may be prepared from it, according to the following receipt, whereby a metallic or wax impreſſion may be again taken from the plaſter mould, ſo as to exactly reſemble the original.

To

To prepare a Plaſter Mould ſo as to take an Impreſſion from it.

Having prepared a plaſter mould, according to the foregoing receipt, and letting it be quite dry, dip it in the following mixture: half a pint of boiled linſeed oil, and one ounce of ſpirits of turpentine; theſe are to be mixed well together in a bottle, and when wanted, the ſurface of the mould is to be dipped into it, and then ſuffered to dry. When the mould has ſucked up the oil on its ſurface, it is to be dipped again in the oil. This operation is to be repeated till the mould will imbibe no more oil, and the oil begins to ſtagnate upon it; then, with a little cotton wool, rolled up hard, wipe all the looſe oil off the mould, and put it in a dry place for a day or two, to dry, and the mould will acquire a very hard ſurface from the effect of the oil. When it is to be uſed, it muſt be oiled with oil of olives, in the ſame manner as before directed. By theſe two methods, any medal, ſeal, or impreſſion, may be ſo exactly imitated, that the new medal can ſcarcely be diſtinguiſhed from the original.

The Method of caſting Brimſtone, and of giving it a metallic Gloſs.

Melt ſome ſtone brimſtone over the fire, in an iron ladle, and let it flame for about five or ſix minutes, then take it off the fire, and extinguiſh the flame, by covering the mouth of the ladle with a piece of board; when it is a little cool, ſo as not to feel gluey, or run ropy, it is then fit for uſe, and may be poured into the mould, in which it ſhould ſtand five or ſix minutes, and then be taken off; part it as before, and rub the ſurface of the impreſſion over with ſome cotton and the beſt black lead in powder, which will give it a very fine metallic gloſs.

To make Sulphur red or green, and caſt it in Moulds like Marble.

Melt two ounces of the beſt ſtone brimſtone over a gentle fire, without permitting it to flame; when melted, mix it well with one ounce of vermilion. Pour the compoſition over the ſurface of the mould, and immediately pour it off again, and fill up the mould to a proper thickneſs, with common brimſtone; let it ſtand the ſame time as before, then pare and rub over the ſurface with ſome clean cotton, which will give it a poliſh. The more impreſſions there are made at once, melting the brimſtone, the better it will be, becauſe the brightneſs of the red fades the oftener it is melted. If it be required to be green, it is done in the ſame manner, but adding a ſmall quantity of ſmalt inſtead of vermilion; and it requires more ſtirring to make it mix. It may alſo be made to imitate a beautiful marble, by mixing ſeveral colours ſeparately, and made in ſmall ſquares of equal ſizes, which break into exact lengths, and diſpoſe them according to your fancy; after which, melt them together, and the colours will unite in a pleaſing manner, and each will appear diſtinct. And, *Note*, when the brimſtone is melting, be careful not to ſhake it; and ſuffer it to cool by degrees.

FINIS.

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